THE TOWN-VERSUS-COUNTRY PROBLEM: OPTIMAL PRICING IN AN AGRARIAN ECONOMY

Avishay Braverman, Raaj Kumar Sah (Consultant), Joseph E. Stiglitz (Consultant)

CPD Discussion Paper No. 1982-3
August 1982

CPD Discussion Papers report on work in progress and are circulated for Bank staff use to stimulate discussion and comment. The views and interpretations are those of the authors.
ABSTRACT

This paper develops a simple model of a less developed country, focussing on the relationships between the rural and urban sector, and on the effects of pricing on migration and on incentives within the rural and the urban sectors. Within this model, we derive simple formulae for optimal pricing in the two sectors, relating the magnitude of taxes on the one hand to parameters which are, in principle, observable (e.g., demand elasticities in the urban sector, supply elasticities in the rural sector, and migration elasticities) and, on the other hand, to explicit value judgments concerning the relative weights attached to the welfare of those living in the urban sector and those living in the rural sector; those living today and those living at future dates. The framework is also used to analyze the value of the shadow wage, and, when the government can control migration, the optimal pattern of migration. We show that the shadow wage depends on (a) the terms of trade and the resource flow between the urban and rural sectors; (b) migration between the two sectors; (c) the distribution of welfare between the urban and rural sectors, and the welfare weights associated with the two sectors relative to investment; and (d) the level of investment (availability of savings) in the economy. Previous studies are shown to be limiting cases of our more general analysis, and the results that they obtain may, as a consequence, be misleading.
I. INTRODUCTION

Within the last few years, a central issue of the policy debate in China has been the "appropriate" difference between the relative prices of agricultural and industrial goods in the rural sector and in the urban sector. Though cast in different language, the issue of the correct terms of trade between the urban and rural sectors (the desirability of food subsidies in the urban sector, or of taxes on the output of the urban sector) has been an important one in numerous other developing countries.\(^1\) Moreover, some of the most crucial debates in the history of economic thought - Malthus versus Ricardo and Preobrazhensky versus Bukharin - have taken place around the question of the internal terms of trade. It is quite surprising, therefore, that this issue has received remarkably little attention in the more recent development economics literature.\(^2\) In particular, there is no simple statement of the important considerations that should be relevant in the descriptive and normative analysis of the internal terms of trade. The objective of this paper is to fill this lacuna.

There are, in addition, several other motivations for this study. The analysis of the optimal growth and employment policies of the late 60's and early 70's [Dixit (1968, 1969, 1971, 1973), Dixit and Stern (1974), Marglin (1976), Newbery (1972, 1974), Sen (1968), and Stern (1972)] focussed on an important set of trade-offs between current employment and consumption, on the one hand, and investment and growth on the other. There were, however, several important limitations to these models. First, the models paid little attention to the agricultural sector; they focussed almost exclusively on the industrial sector. Implicitly or explicitly, most of these studies employed a
version of the Lewis (1954) model, in which it was assumed that a surplus of labor exists in the rural sector; hence changes in urban employment left the output in the rural sector unchanged. The issue of incentives in the rural sector, which seems so central to the town-versus-country policy debate referred to earlier, was thus completely ignored. Second, most of the existing studies did not explicitly specify the nature of economic linkages between the rural and the urban sector through the exchange of the distinct product of each sector, and through the division of rents and profits. Third, issues of migration from the rural to the urban sector, which have played such a large role in the more static models of employment policy [Harris and Todaro (1970), and Stiglitz (1982a)], played no role in these dynamic models which were concerned with investment policy. The failure to model explicitly the rural sector not only makes these earlier studies incomplete, but it also means that a number of the central conclusions of these studies may be seriously misleading. As we shall show, the relationship between the market wage and the shadow wage may differ markedly from that implied by these earlier studies.

Another important limitation of most of these earlier models was the absence of an explicit analysis of taxation. In the past decade, there have been many studies of the optimal structure of taxation for neoclassical economies [For a survey of this literature, see Atkinson and Stiglitz (1980)]. But there have been few attempts to apply the basic insights of this literature to less developed economies, with the particular set of distortions associated with dualism. There are two lessons of this literature which are central to our concern. First is the recognition that in most countries, the government can not directly control economic activity, but through taxation
and pricing policy, it can exercise a form of indirect control. Second, the
basic framework of analysis can apply to a variety of institutional settings.
Stiglitz and Dasgupta (1971), for example, show that the problem of the
determination of optimal prices in a socialist economy and the problem of the
optimal structure of taxation are formally equivalent. We shall see, however,
that even though the same framework of analysis does apply both to socialist
and non-socialist economies, there are certain internal features of the
economy which have distinct and different influences in determining the
internal terms of trade. These features of the economy will play a critical
role in what follows.

The task of the present paper, therefore, is to develop a simple
model of a less developed country, focussing on the relationship between the
rural and urban sector, and on the effects of pricing both on migration and on
incentives within the rural and the urban sectors. Within this model, we
derive simple formulae for the optimal pricing (the optimal terms of trade),
relating the magnitude of the taxes on the one hand to parameters which are,
in principle, observable, and on the other hand, to explicit value judgements
concerning the relative weights attached to the welfare of those living in the
urban sector and those living in the rural sector; those living today and
those living at future dates. The framework can, at the same time, be used to
analyze the value of the shadow wage and, when the government can control
migration, the optimal pattern of migration.

This paper is divided into six parts. In Part-II, we examine the
question of pricing in the simplest model in which there is no unemployment,
the urban wage is rigid and there is no migration. We then show how this
model can be used to analyze optimal migration policy (Part-IIIa). In Part-IIIb, we take up the question of pricing in a mixed economy in which migration is endogenous. The shadow wage is then examined for these different kinds of economies in Part-IV. Part-V contains comments on some other economic systems. Conclusions follow at the end of the paper.

In the analysis of the internal terms of trade, several features of the economy play an important role. In particular

(1) The organization of the rural sector: whether production occurs on state farms, family farms, collectives, etc.; whether workers receive a wage which is equal to the value of their marginal product or their average product (or some other number); to whom the rents on the fixed factors accrue, and how they are spent; etc.

(2) The organization of the urban sector: whether production is directly controlled by the state, or by domestically owned private enterprises, or by international corporations; whether the wage is fixed institutionally, or is determined endogenously; and if determined endogenously, whether the labor market clears, or whether, because of, say, efficiency wage considerations, the wage is set at a level above the market clearing level [see, Libenstein (1957), Mirrlees (1975b), Stiglitz (1976) and Bliss and Stern (1978)]; also, whether there is serious land congestion in the urban areas or not [Williamson (1982)].

(3) The relationship between the urban and rural sector: whether workers can freely migrate to the urban sector; whether they can search for jobs in the urban sector while employed in the rural sector; or whether, to obtain a high paying urban job, they must first migrate to the urban sector.
This list is by no means exhaustive, and yet it is clear that to pursue all, or even a few, of the various possible economic structures would be impossible. We have chosen, instead, to focus on a few central cases. We have chosen these cases, partly because of their relative analytical simplicity, but also because we believe that these cases best convey the central insights which are to be obtained from an analysis of this kind.

II. THE BASIC MODEL

We begin our analysis with a model in which rents from the rural land accrue to the rural sector and are spent on consumption. (Moreover, the intra-rural sector distribution is kept out of the picture.) This conforms to a rural setting with family owned farms, cooperatives or an agriculture owned by feudal lords who spend their entire income on consumption. The opposite extreme, in which the entire rural rent goes out of the rural sector, is taken up in Part-V.

The government cannot control the rural sector directly; it can exercise only indirect control through instruments such as prices. In contrast, the government can potentially exercise direct control on the urban sector. The present model, thus, refers to socialist economies which, as the past experience has shown, have been unable to control agriculture directly. Further, in this section we wish to focus on the pricing decision alone. For this reason the urban wage is taken as a fixed parameter, and migration is ruled out. The policies relating to the urban wage and migration are taken up later. Also, we ignore the phenomena of underground economy.
(a) The Rural Sector

Output of the rural sector is a function of the total number of man-hours supplied, which is the product of a number of workers, \( N^1 \), and the number of hours, \( L^1 \), which an individual spends on work. In addition, the output is a function of the total land area \( A \) which is a fixed factor. With constant returns to scale in the production technology, the output per worker is \( \frac{5}{6} \)

\[
(1) \quad X = X\left(\frac{A}{N^1}, L^1\right) = X(a, L^1)
\]

where \( a = \frac{A}{N^1} \) is the land area per rural worker.

We assume that the rural sector acts as if it were maximizing a utility function of the form

\[
(2) \quad u^1 = u^1(x^1, y^1, L^1)
\]

where \( x^1 \) is the per capita consumption of the rural good, and \( y^1 \) is the per capita consumption of the urban good. If \( p \) is the price of the agricultural good in terms of the urban good (as faced by the farmers), then the budget constraint is

\[
(3) \quad px^1 = p^1 + y^1, \text{ or } pQ = y^1
\]

where \( Q \) is the per capita surplus of the rural sector.
\[ Q = x - x^1 \]

We can derive a modified indirect utility function giving the level of utility of the representative worker in the rural sector as a function of the prices he faces and the rural population. This is done by maximizing (2) under the constraint (3). The indirect utility, \( V^1(p,N^1) \), is given by

\[ V^1(p,N^1) = \text{max}_{x^1,y^1,L^1} u^1(x^1, y^1, L^1) + \lambda^1[pX(A/N^1, L^1) - px^1 - y^1] \]

The change in the rural welfare due to a change in the parameters is obtained from Roy's formula

\[ \frac{\partial V^1}{\partial p} = \lambda^1 Q > 0, \quad \frac{\partial V^1}{\partial N^1} = -\lambda^1 pX a/N^1 < 0 \]

where \( \lambda^1 \) is the (positive) marginal utility of income for a person in sector 1.

(b). **The Urban Sector**

We shall assume that the government has the capability to control directly the urban sector. Moreover, for simplicity, we shall take the number of hours for which an individual works as fixed (e.g. due to the production technology) at \( L^2 \). The wage \( w^2 \) (in terms of the urban good) is also taken at present as a parameter.
The urban production function, net of depreciation, has constant returns to scale in the two inputs, capital, \( K \), and total labor, \( N^2L^2 \). The per worker output is

\[
Y = Y\left(\frac{K}{N^2}, L^2\right) = Y(k, L^2)
\]

Where \( k = \frac{k}{N^2} \) is the capital stock per urban worker. The properties of \( Y \) are analogous to those of \( X \) (see Footnote 5). The representative worker maximizes his utility, \( U^2(x^2, y^2, L^2) \), subject to the budget constraint

\[
qx^2 + y^2 = wL^2
\]

where \( q \) is the relative price of the agricultural good in the urban sector and \( x^2 \) and \( y^2 \) are the urban per capita consumption of the rural good and the urban good. The indirect utility is

\[
V^2(q, w^2) = \max_{x^2, y^2} U^2(x^2, y^2, L^2) + \lambda^2[wL^2 - qx^2 - y^2]
\]

\( V^2 \) is also a function of \( L^2 \), but we suppress this dependence for notational convenience, since \( L^2 \) is kept fixed throughout the analysis. The welfare effect of a price change is obtained from (9) as

\[
\frac{\partial V^2}{\partial q} = -\lambda x^2 < 0
\]

Note that the consumption vector \((x^2, y^2)\) does not depend on \( N^2 \), while the per
capita output does

\[ \frac{\partial Y}{\partial N^2} = - \frac{Y_k}{N^2} < 0 \]  

(c). Optimal Pricing

In this simple model, there are three resource constraints: on labor, agricultural output, and urban output. We require

\[ N^1 + N^2 = N \]

where \( N \) is the entire working population, and full employment is guaranteed by the socialist government.

\[ N^1 Q = N^2 x^2 \]

Output of agricultural goods equals consumption of agricultural goods; and

\[ \cdot K = N^2 Y - N^2 y^2 - N^1 y^1, \]

urban goods are either consumed or used for investment.

The nature of the equilibrium depends critically then on what the government controls. In this section, we assume that

(i) \( N^1 \) and \( N^2 \) are fixed, and
(ii) the government can control the relative price of agricultural goods in both the urban and rural sectors. The two controls, however, cannot be exercised independently, as we shall now see.

If the government specifies \( p \), it determines the level of output and the per capita consumption of the rural good within the rural sector; thus, by (13), there is a value of \( q \) which is required if the urban demand for food is to equal the marketed surplus. This value of \( q \), from (14) results in a particular level of investment. The change in \( q \) corresponding to a change in \( p \) is obtained from (13)

\[
\frac{dq}{dp} = \frac{N_1}{N_2} \frac{\partial Q/\partial p}{\partial x^2/\partial q}
\]

Because consumption goods are normal, \( \partial x^2/\partial q < 0 \) in (15). The sign of \( \partial Q/\partial p \), however, can not be predicted from the standard assumptions of consumer behaviour. There is strong empirical evidence nevertheless to believe that the rural surplus increases with own price. In the rest of this paper we therefore take \( \partial Q/\partial p > 0 \). Expression (15) then says that \( \frac{dq}{dp} < 0 \). That is, as an increase in the rural price increases the rural surplus, it must lead to a decrease in the urban price to clear the market.

Next, substitution of (3), (8), and (13), in (14) gives

\[
X = N_1(q - p)Q + N_2[y - w^2L^2]
\]
Investment is equal to profits from the urban enterprises plus the 'tax' revenue from the price difference between the two sectors.

Differentiating (16) with respect to \( p \) and substituting (15) in it, we have

\[
\frac{dK}{dp} = N^1(q - p) \frac{\partial Q}{\partial p} + N^1 Q \left[ \frac{N^1}{N^2} \frac{\partial Q}{\partial x^2} \right] \frac{\partial x^2}{\partial q} - 1
\]

The second term above is clearly negative. The first term is negative if \( q - p < 0 \), i.e., if the rural price exceeds the urban price. Investment thus declines with an increase in the rural price if the price difference \( q - p \) is negative or zero, or if it is positive but not very large.

Let us look now at the trade-offs involved in increasing the rural price \( p \). An increase in \( p \) obviously increases the welfare of the rural population, as (6) shows. It also increases the welfare of urban workers through (10) and \( \frac{dq}{dp} < 0 \). Now, if \( \frac{dK}{dp} > 0 \), then there is no loss to the society by increasing the price \( p \), and this should certainly be done until we are in the region \( \frac{dK}{dp} < 0 \). And it is this region in which a real trade-off arises between consumption and investment. A precise examination of this crucial trade-off forms the major part of this section.

We have already obtained an important result: a sufficient condition for the nonoptimality of the existing pricing regime is that \( \frac{dK}{dp} > 0 \) at the existing prices. If this is so, then the welfare of current economic agents as well as investment can be increased by increasing the rural price.

Rearranging (17), and using (13), this condition becomes

\[
q - p > \frac{Q}{\partial q/\partial p} - \frac{x^2}{\partial x^2/\partial q} = \frac{p}{\xi_{0p}} + \frac{q}{\xi_{xq}}
\]
where,

$$e_{q_p}^1 = p \frac{\partial Q}{\partial p} / Q, \quad e_{x_q}^2 = q \left| \frac{\partial x^2}{\partial q} \right| / x^2$$

are the elasticity of rural surplus and the elasticity of urban consumption of the rural good with respect to the sectoral prices, respectively. This is a simple test to discover whether the existing regime can be improved without hurting anyone. Only the demand and surplus elasticities are needed. If the above inequality is met, then the correct policy response is to increase $p$, and keep on doing so until the above inequality disappears.

The identification of an inefficient regime, if it exists, and the correction of it certainly constitutes an important first step; but this in itself does not provide the policies which are socially most desirable. To obtain these, we now turn to the question of the optimal pricing. Let $\bar{W}$ be an additive social welfare function, with a current value

$$N^1 \overline{W}(V^1(p,N^1)) + N^2 \overline{W}(V^2(q,w^2))$$  \hspace{1cm} (19)

If $\gamma$ is the discount rate for social welfare, $\delta e^{-\gamma t}$ is the costate variable for (14), and $\eta e^{-\gamma t}$ is the Lagrange multiplier for the constraint (13), then the current value Lagrangian is

$$H = N^1 \overline{W}(V^1(p,N^1)) + N^2 \overline{W}(V^2(q,w^2))$$

$$+ \delta [N^2 y - N^2 y^2 - N^1 y^1] + \eta [N^1 q - N^2 x^2]$$  \hspace{1cm} (20)
Note from the above that $\eta/\delta$ is the shadow price of the rural good. The problem of finding the optimal prices can thus be considered as a problem of finding two "wedges", a "rural wedge" and an "urban wedge". The rural wedge, $p - \eta/\delta$, is the public loss on every unit of rural surplus. The urban wedge, $q - \eta/\delta$, is the public gain on every unit of urban consumption of the rural good. The price difference, $q - p$, is simply the urban wedge minus the rural wedge.

The first order conditions of (20) with respect to the controls $(p, q)$ are:

$$\frac{\partial H}{\partial p} = N^{-1} \frac{\partial W}{\partial y^1} - \delta N^{-1} \frac{\partial y^1}{\partial p} + \eta N^{-1} \frac{\partial Q}{\partial p} = 0$$

$$\frac{\partial H}{\partial q} = N^{-2} \frac{\partial W}{\partial y^2} - \delta N^{-2} \frac{\partial y^2}{\partial q} - \eta N^{-2} \frac{\partial x^2}{\partial q} = 0$$

Let $\beta^i = \frac{\partial W}{\partial y^i}$ be the marginal social utility of individual income in the two sectors. Differentiating the budget constraint (3) with respect to $p$, we have

$$\frac{\partial y^1}{\partial p} = q + p \frac{\partial Q}{\partial p}$$

Substitution of (6) and (23) in (21) gives $\beta^1 Q - \delta [q + p \frac{\partial Q}{\partial p}] + \eta \frac{\partial Q}{\partial p} = 0$.

The rural wedge follows from the last expression.

$$p - \frac{\eta}{\delta} = (\frac{\beta^1}{\delta} - 1) \frac{Q}{\partial Q/\partial p}$$
Similarly, differentiation of (8) with respect to $q$ yields

$$\frac{\partial y^2}{\partial q} = -[x^2 + q \frac{\partial x^2}{\partial q}]$$

Substitution of (25) in (27) gives

$$-\beta^2 x^2 + \delta[x^2 + q \frac{\partial x^2}{\partial q}] - \eta \frac{\partial x^2}{\partial q} = 0.$$  

The urban wedge is obtained as

$$q - \frac{\eta}{\delta} = (\frac{\beta^2}{\delta} - 1) \frac{x^2}{\partial x^2/\partial q}$$

The optimal price difference is obtained immediately from (24) and (26)

$$q - p = (\frac{\beta^2}{\delta} - 1) \frac{x^2}{\partial x^2/\partial q} - (\frac{\beta^1}{\delta} - 1) \frac{\partial}{\partial q/\partial p}$$

These formulae can be easily rewritten in the elasticity form.

$$\frac{(p - \frac{\eta}{\delta})}{p} = (\frac{\beta^1}{\delta} - 1)/\varepsilon_{qp}$$

$$\frac{(q - \frac{\eta}{\delta})}{q} = - (\frac{\beta^2}{\delta} - 1)/\varepsilon_{xq}^2$$

$$\frac{q}{p} = \frac{[1 - (\beta^1 - \delta)/\delta \varepsilon_{op}^1]}{[1 + (\beta^2 - \delta)/\delta \varepsilon_{op}^2]}$$

The left hand sides of (28) and (29) are the rates of rural subsidy and urban tax respectively.

The above equations have identified the four parameters that should determine pricing. Two of these are behavioral parameters, the price elasticity of the rural surplus and the price elasticity of demand for rural goods in the urban sector. The other two represent summaries of
distributional judgments: the welfare weights $\beta^1$ and $\beta^2$ (measured relative to the value of a dollar of investment, $\delta$) of the urban and rural sectors. These welfare weights, in turn, reflect judgements about the intertemporal distribution of income $\delta$.

We can now make several observations concerning the desirable price structure:

(i) Whether there should be a "tax" or subsidy in the rural sector (i.e., whether price $p$ should be less than or greater than the shadow price $\eta/\delta$) depends simply on whether the marginal evaluation of a dollar of investment is greater or less than the marginal social evaluation of a dollar of income in the rural sector. Similarly, whether there should be a tax or subsidy in the urban sector depends on whether the marginal evaluation of a dollar of investment is greater or less than the marginal social evaluation of a dollar of income in the urban sector. The magnitude of the tax or subsidy in the former case is inversely proportional to the price elasticity of the surplus while the magnitude of the tax or subsidy in the latter case is inversely proportional to the price elasticity of the demand for agricultural goods.

(ii) If capital is very scarce, then the marginal social evaluation of investment will be high compared to the welfare weights (i.e. $\delta > \beta^1$). In this case, the urban price should exceed the rural price. Also, the more important incentives are in the rural sector (to supply the surplus, i.e. $\varepsilon^1_{Qp}$), and in the urban sector (not to consume, i.e., $\varepsilon^2_{xq}$), the smaller the tax should be. The effect of the surplus elasticity in the rural sector is natural; that of the demand elasticity in the urban sector requires some
explanation. We have already noted that a decrease in the rural price leads to an increase in the urban price. This, in turn, shifts consumption towards urban goods, and thus reduces what is left over for investment. The larger the demand elasticity, the more important this effect is, and hence the less desirable is a tax on the rural sector.

(iii) If the marginal social evaluation of investment and consumption of all individuals is the same, then, not surprisingly, there should not be any taxation \( q = p \).

The decision rules obtained above bear some resemblance to the familiar Ramsey formula (1927), and its subsequent generalization to many persons [Diamond and Mirrlees (1971), and Atkinson and Stiglitz (1976)]. There are, however, several distinctions between this analysis and that of the standard tax model: (a) In the present problem, there are distinctly different groups of individuals acting as producers and consumers. The distributive effects of taxation, therefore, cannot be ignored even as a first approximation. (b) In most of the existing tax literature all individuals face the same prices. Here, prices are different according to location, which in turn is related to the source of income. (c) The tax literature typically ignores rents which accrue to different individuals. Unless land is abundant, these rents can not be ignored in agricultural production, and we shall see that the rent term will play an important role in the later analysis.
A final comment before closing this section. The present model will have to be enlarged to address the important question of dividing new investment between the urban and the rural sector. Specifically, capital would appear as the third factor of production in (1), and a rule for the disposal of rent on capital will have to be specified. The pricing rules, however, would remain largely unchanged, except for a modification of the rent terms when they appear in the subsequent sections. We do not build in this additional complexity because our attention in the present paper is on determining pricing rules, and not on the division of investment.

III. MIGRATION AND PRICES

(a). Relocation in a Socialist Economy

Socialist governments frequently exercise control over the location of their populations. Entry into the urban sector is often restricted. In addition, the urban population has sometimes been transferred into the rural sector as, for example, in China during the cultural revolution. Surprisingly, this instrument of control has received little attention in the analytical literature on economic planning. This section, we hope, will go some ways toward correcting this shortcoming.

To begin our analysis, we take the sectoral populations as the only control variables, while the rural and urban prices are taken as given parameters. From (12), and from \( \frac{\partial N^1}{\partial N^2} = -1 \), the first order condition of (20) is

\[
\frac{\partial H^2}{\partial N^2} = w(v^2) - w(v^1) - N^1 \frac{\partial w}{\partial v^1} \frac{\partial v^1}{\partial N^1}
\]
\[ + \delta [y - y_2 + y_1 + n^2 \frac{\partial y}{\partial n^2} + n^1 \frac{\partial y}{\partial n^1}] \]
\[ + \eta [-Q - x^2 - n^1 \frac{\partial Q}{\partial n^1}] = 0 \]

We first substitute (3), (6), (8), (11) in (31). Also substituted are
\[ y = y_L L^2 + y_k k, \]
which follows from the constant returns to scale in the production technology, and
\[ \frac{\partial y}{\partial n^1} = \frac{\partial Q}{\partial n^1}, \]
which follows from (3). This yields,
\[ (32) \quad [W(v^1) - W(v^2)]/\delta = \frac{\partial 1}{\delta} px_a + [y_L - w^2] L^2 \]
\[ + (p - \eta) Q(1 - \epsilon_{Qa})^1 + (q - \eta) x^2 \]

where \( \epsilon_{Qa} = \frac{a}{Q} \frac{\partial Q}{\partial a} \) is the elasticity of per capita surplus with respect to per capita land. A small value of \( \epsilon_{Qa} \) will imply that land is not too scarce. If land is surplus, then, \( \epsilon_{Qa} = 0 \).

There are three distinct effects of moving a person from the rural to the urban sector: (i) additional land area \( a = A/N^1 \) becomes available to the remaining workers in the rural sector, (ii) there is one less person in the rural sector to produce the surplus, and (iii) this person now joins the urban sector and acts both as a worker and a consumer. The left side of (32) is the social value of the utility lost by the person who has been moved. The right side of (32) can now be examined term by term. \( px_a \) is the private income gain to the entire rural sector from the extra land that has now become
available; and the first term represents the social value of this gain. Obviously, this term is positive because there is less congestion on land.

The second term above is the direct gain (or loss) from the newly arrived urban worker because of the difference between what he produces and what he receives. Next, to interpret the third term, recall that \((p - \eta/\delta)\) is the social loss on each unit of rural surplus collected. The direct reduction of surplus from the migration is \(Q\). But, the rural sector has now a unit of extra land available, from which the indirect gain in surplus is \(aQ/\partial a\). The net reduction then is \(Q(1 - \frac{1}{Qa})\) of which the social value is represented in the third term. Finally the newly arrived urban worker also consumes \(x^2\) of the rural good, whereas \((q - \eta/\delta)\) is the social gain on each unit of this good sold in the urban sector. The final term therefore is the public gain on this trade.

We can also derive some insights on the difference between individual utilities in the two sectors, based on (32), if the social welfare function is utilitarian. We find from (32) that the sectoral difference in utilities increases if: (i) the social weight on the rural income increases, (ii) the urban price increases, (iii) the urban wage payment decreases, and (iv) if the rural land is not too scarce, i.e., \(1 > \frac{1}{Qa}\), and the rural price increases. All these effects make clear economic sense in view of the earlier discussion.

Now we can readily examine the situation in which the relocation decision is being taken in conjunction with the decision on the prices. Substituting (28) and (29) in (32)
(33) \[ \frac{[W(v^1) - W(v^2)]}{\delta} = \frac{\beta^1}{\delta} p x_a a + [(y_L - w^2)L^2] \]

\[ + \left( \frac{\beta^1}{\delta} - 1 \right) \left( 1 - \frac{\epsilon}{Q_a} \right) \frac{pQ}{\epsilon q_p} - \left( \frac{\beta^2}{\delta} - 1 \right) \frac{q x^2}{\epsilon x q} \]

We can note several new economic effects from the above expression. In these interpretations, we are assuming that the rural land is not too scarce, whereas capital is scarce. First, a rural social weight which is higher than the urban social weight, in itself, is by no means sufficient to guarantee that the rural utility should exceed the urban utility. Second, even if both sectors are fully controlled by the government, i.e. \( \beta^1 = \delta \), it may not be desirable that the utilities be equal across the sectors. In fact, the rural sector should be kept better-off, if there is any scarcity of land and the urban workers are being paid less than or equal to their marginal product.

(b). **Mixed Economies with Unemployment**

Now, we shift our attention to a typical developing economy which has far fewer instruments of policy available to it than those in the idealised socialist economies just described. In particular, we assume that migration can not be controlled directly by the government but is influenced indirectly by the government policies. To analyze migration and urban unemployment, we employ a simple adaptation of the Harris-Todaro model [Harris and Todaro (1970)]. As in the preceding analysis, the urban wage \( w^2 \) is taken as a parameter. The industrial producers, being competitive, pay labor its marginal product. Urban employment thus is a function of the wage and the capital stock. Employment in the urban sector, \( N^u \), is obtained from
\[ y_L(x/N^e, L^2) = \varphi^2 \]

In the urban sector the utility of an employed person is \( V^2(q, \varphi^2) \), and that of an unemployed person is \( V^2(q, 0) \). The utility of a person in the rural sector is \( V^1(p, N^1) \). Migration is endogenous; it occurs until rural utility equals the expected urban utility. If \( S \) is the number of unemployed in the urban sector, then the probability for a migrant to find urban employment is \( \frac{N^e}{N^e + S} \). The sectoral populations are determined by

\[ V^1(p, N^1) = \frac{S}{N^e + S} V^2(q, 0) + \frac{N^e}{N^e + S} V^2(q, \varphi^2) \]

and

\[ N = N^1 + N^e + S, \quad N^2 = N^e + S \]

These can be rewritten as

\[ NV^1(p, N^1) = N^1V^1(p, N^1) + (N - N^1 - N^e)V^2(q, 0) + \alpha V^2(q, \varphi^2) \]

From (34), \( N^e \) does not depend on \( (p, q) \). We can, therefore, solve (37) to obtain \( N^1 \) as a function of \( p \) and \( q \). Contrast this situation with our earlier analysis in which we could not choose \( p \) and \( q \) independently of one another.

Now, differentiating (37) with respect to \( p \) and \( q \) and substituting (6), (10) and \( x^2(q, 0) = 0 \), we obtain:

\[ \frac{dN^1}{dp} = \mathcal{N}^2 V^1_1/\mathcal{V}^1_1 \]

\[ \frac{dN^1}{dq} = N^e \mathcal{N}^2 V^2_1/\mathcal{V}^2_1 \]

\[ \frac{dN^2}{dp} = N^e \mathcal{N}^2 V^2_1/\mathcal{V}^2_1 \]

\[ \frac{dN^2}{dq} = N^e \mathcal{N}^2 V^2_1/\mathcal{V}^2_1 \]
in which, \( v_u^2 = v^2(q, 0) \), for notational simplicity, and \( N^2 = N^e + S \) is the urban population. Note that both \( \frac{dN^1}{dp} \) and \( \frac{dN^1}{dq} \) are positive from (6), and because \( v^1 > v_u^2 \). An increase in the rural price makes it more attractive to stay in the rural sector. An increase in the urban price does the same by making it less attractive to stay in the urban sector. We define,

\[
m_p = \frac{d\ln N^1}{dp}, \quad \text{and} \quad m_q = \frac{d\ln N^1}{dq}
\]

as the elasticities of migration with respect to the rural and urban price respectively.

As individual behaviour is being determined here, in part, by expected utilities, the usual question arises whether the social welfare function should be sensitive to the ex-ante or the ex-post utilities. The difference between the two disappears if a utilitarian social welfare function is used, as we shall do in this section. Using (35), the current value Lagrangian is given by

\[
H = NV^1(p, N^1) + \delta[N^e y - N^e y^2 - N^1 y^1] + \eta[N^1 Q - N^e x^2]
\]

The first order conditions with respect to the control variables \( p \) and \( q \) can now be obtained as

\[
\frac{\partial H}{\partial p} = N \frac{\partial V^1}{\partial p} + N \frac{\partial V^1}{\partial N^1} \frac{dN^1}{dp} \]

\[
- \delta \frac{d(N^1 y^1)}{dp} + \eta \frac{d(N^1 Q)}{dp} = 0
\]
\[ \frac{\partial H}{\partial q} = N \frac{\partial V^1}{\partial N^1} \frac{dN^1}{dq} \]

\[ - \delta N^1 \frac{\partial y^2}{\partial q} - \eta N^1 \frac{\partial x}{\partial q} - \delta \frac{d(N^1 y^1)}{dq} + \eta \frac{d(N^1 Q)}{dq} = 0 \]

From (3), we have \( \frac{d(N^1 y^1)}{dp} = N^1 Q + p \frac{d(N^1 Q)}{dp} \). Substitution of this and (6), in (41) gives an expression, which is comparable to (23):

\[ \frac{(p - \frac{m}{\delta})}{p} = [(\frac{\lambda^1}{\delta} - 1) + \frac{\lambda^1 N^2}{N^1} - \frac{N^1 X a}{Q} \frac{m_p}{\varepsilon^1} \varepsilon^1_{Qp} \]

where \( \varepsilon^1_{Qp} \) is the elasticity of total rural surplus with respect to its price:

\[ \varepsilon^1_{Qp} = \frac{\frac{dN^1 Q}{dp}}{\frac{dln(N^1 Q)}{dp}} = \varepsilon^1_{Qp} + (1 - \varepsilon^1_{Qa})m_p. \]

(recall that \( Q = Q(p, N^1) \))

It is intuitively obvious that we should now be interested in the overall rural surplus response to prices rather than the per capita response, because the rural population itself is sensitive to prices in the present model. It is also clear from (44) that \( \varepsilon^1_{Qp} \gg \varepsilon^1_{Qp} \) as \( \varepsilon^1_{Qa} \ll 1 \), because \( m_p > 0 \). Now, as in our earlier discussion, we focus on the case where \( \varepsilon^1_{Qa} < 1 \). In this case, migration increases the price sensitivity of the rural surplus and, comparing (23) to the first term in the right side of (43), we find that endogenous migration results in a reduction in the absolute value of the rural "tax rate".
A redefinition of the surplus response alone, however, is only part of the story because there are additional influences of endogenous migration on the rural wedge. The second term in (43), representing the direct social gain due to the utility increase of those who are in the urban sector, is positive. Because of the equalization of expected utilities, this gain is identical to the gain of those in the rural sector (contrast this with the previous model in which only the urban price influenced the welfare of the urban sector). The final term is the familiar congestion effect. The price induced in-migration into the rural sector increases congestion on the rural land, and reduces the utility of the entire rural population. This utility loss, however, is shared also by the urban population through the equalization of expected utilities, and its influence is to reduce the rural wedge.

A simplified expression is obtained for (43) if land is abundant. In this case

\[(45) \quad (p - \frac{\eta}{5})/p = \left(\frac{\lambda^1 N}{N^1} - 1\right)/\varepsilon^1_{Qp}\]

where \(\varepsilon^1_{Qp} = \varepsilon^1_{Qp} + m_p\). If \(\lambda^1 > \frac{N^1 S}{N}\), then the term in the square bracket is positive, and \(p > \eta/5\). Now, take an example. If half of the population is in the urban sector, and if the private marginal utility of income in the rural sector is more than one half of the social weight on investment, then the rural price should exceed the shadow price of the rural good.

The urban wedge is obtained from (42) by using (6) and (25). Also used are: \(d(N^1 y^1)/dq = pd(N^1 Q)/dq\), obtained from (3), \(d(N^1 Q)/dq = Q(1 - \varepsilon^1_{Qa}) dN^1/dp\), and \(N^1 Q = N^2 x^2\). Substitution yields

\[(46) \quad (q - \frac{\eta}{5})/q = \left[1 - \frac{N}{N^2} \frac{\lambda^1 X^a}{s} \frac{p}{q} m_q - [(p - \frac{\eta}{5})/p] (1 - \varepsilon^1_{Qa}) \frac{p}{q} m_q \right]/\varepsilon^2_{xq}\]
The middle term in (46) is, once again, a congestion effect arising from the induced migration due to change in the urban price. This term is negative, and it reduces the urban wedge. The last term in (46) is a new term. If \( l > \varepsilon_{Qa} \), then this term has the same sign as \( \eta/\delta - p \). This term represents the gain (or loss) on the government trade in the extra surplus produced by the induced migrants.

The first term above, which is positive, is somewhat intriguing, and it requires explanation. Compared to (29), it would appear as if the government does not care any more about the direct private welfare loss of the increased price on the urban workers. This indeed is true! The reason is as follows. From (37), an increase in the urban price does not change the economy-wide expected utility, because it does not change the rural workers' utility. It merely transfers welfare from the urban employed to the urban unemployed who migrate to the rural sector. Given the utilitarian welfare function, the government is well advised to ignore this transfer!

If the land congestion effect in the rural sector is ignored, then (46) simplifies to

\[
(47) \quad (q - \frac{\eta}{\delta})/q = [1 - \{(p - \frac{\eta}{\delta})/p\} \frac{p}{q} m_q]/\varepsilon^2 x_q
\]

From this, \( q > \frac{\eta}{\delta} \) if \( \frac{\eta}{\delta} > p \); which also implies that \( q > p \). That is, if the rural sector is being underpaid for its surplus, or not being overpaid by a large amount, then it is essential that the urban consumption should be overpriced.

The optimal "tax rate" can be obtained in the general case by first substituting (43) into (46), and then subtracting (43) from (46). This being quite a lengthy expression, we present below the case in which land congestion
effect is ignored.\textsuperscript{14/}

\begin{equation}
\frac{q-p}{p} = \frac{1}{\varepsilon^2_{\text{xx}}} - \frac{p}{q} \left(1 + \frac{m_{\text{d}}}{\varepsilon_{\text{xx}}} N \lambda \frac{l}{\delta} - 1\right) \frac{1}{\varepsilon_{\text{QP}}}
\end{equation}
IV. WAGE EFFECTS AND SHADOW WAGE

(a). Shadow Wage in a Socialist Economy

The general methodology for calculating shadow wage is by now well understood. The simplification found in much of the existing literature is to calculate the loss of total output if one person were to be provided new employment in the urban sector [see Stiglitz (1982a) for a review of this material]. This approach is meaningful only in the context in which prices remain unchanged. In contrast, in a full general equilibrium model, such as the present one, the question has to be: what is the loss of aggregate social welfare, while fully respecting the constraints of the economy, if one person is to be provided a new urban job? We find an answer to this question by differentiating the "value" (in welfare terms) of national income, $H$ in (20), with respect to urban employment, $N^2$, while keeping urban output, $N^2Y$, constant (that is because the fruits of the new employment must not be counted while finding its cost). Also we take $(p, q)$ as parameters because we wish to see the influence of suboptimal as well as optimal pricing on the shadow wage. Finally, we express the shadow wage in terms of a numeraire. The most natural numeraire is the urban good, of which the shadow price is $\delta$. The expression for shadow wage, $w^sL^2$, is then given by

\[
(49) \quad w^sL^2 = -\frac{1}{\delta} \left. \frac{\partial H}{\partial N} \right|_{N^2Y} = \frac{1}{\delta} \left[ W(V^1) - W(V^2) \right] - \frac{1}{\delta} p X_a a \\
- (p - \frac{\eta}{\delta}) q(1 - \frac{1}{q_a}) - (q - \frac{\eta}{\delta}) x^2
\]
We shall now use the above expression to contrast it, and heuristically relate it, to the received wisdom on shadow wage.

Most of the existing models for shadow wage have two features. First, the price distortions are completely ignored (or it is assumed that they do not exist), which means that the last two terms are missing from (49). Second, in the one-sector model (as well as in the grossly simplified two-sector models with uniform prices), the individual welfare levels $V^1$ and $V^2$ are treated as synonymous to the rural consumption and the urban wage (consumption), respectively. Further, if there are no distinctions between investment and consumption, i.e., $\delta$, $\beta^1$ and $\beta^2$ are all unity, then, from (49), the shadow wage is the rural per capita consumption minus the land congestion effect. With surplus labor, i.e., $X_a = X$, these two terms exactly cancel each other. All these assumptions together, thus, lead to the doctrine of zero, or quite low, shadow wage which was proposed during the fifties. The contribution of Sen and Marglin, among others, during the sixties and early seventies was to make the economic distinction between consumption and investment. Their results, once again, can be obtained as a very special case of (49). Using the same simplifications as above, we find that the shadow wage increases as $\delta$ becomes larger than one, i.e., the shadow wage is higher if investment is dearer. The shadow wage exactly equals the urban wage payment if $\delta \to \infty$, i.e., if investment is infinitely more desirable than consumption.

It is obvious now that much of the existing literature has missed several important points. First, distortionary pricing directly influences the shadow wage [see (49)]. This influence can be quite important since price
differentiation and distortions are often substantial in most developing economies. This effect holds even if prices are equal in the two sectors, and even if land is abundant. Second, the price distortions can exert an influence on the shadow wage even if the prices are optimally set. This is obvious from (24) and (26), because optimal prices would rarely imply zero distortions, unless both sectors are fully controllable. Thirdly, it is quite insufficient to take wages as representing welfare levels. This is because price differences and the work hour differences between the two sectors can take much of the welfare meaning out of wages. Further important differences arise if the government cannot directly control migration, as we shall shortly show.

(b). Wage Setting in a Socialist Economy

Thus far, the urban wage has been taken as a given parameter. A socialist government, however, would want to control the urban wage, the implication of which we study in this subsection.

From the urban consumer's problem (9), it is obvious that control over \((q, w^2)\) amounts to a control over \((x^2, y^2)\). If the latter are used as control variables then (22) is replaced by

\[
\frac{\partial w}{\partial u^2} \frac{\partial u^2}{\partial x^2} - \eta = 0, \quad \frac{\partial w}{\partial u^2} \frac{\partial u^2}{\partial y^2} - \delta = 0
\]

From the first order conditions in the maximization (9), we have \(\frac{\partial u^2}{\partial x^2} = \lambda^2 q\), and \(\frac{\partial u^2}{\partial y^2} = \lambda^2\). Substituting these in (50), we have \(\beta x^2 = \delta\), and \(\beta y^2 = \eta\). This yields
\[ (51) \quad \beta^2 = \delta, \text{ and } q = \frac{n}{\delta} \]

The expression (24) continues to hold for \( p \). The pricing formula, however, simplifies from (27) to

\[ (52) \quad q - p = (1 - \beta^1/\beta^2) \frac{\delta}{\delta Q/\delta p}, \text{ or, } (q - p)/p = (1 - \beta^1/\beta^2)/\varepsilon Q \]

With a fully controlled urban sector, it is obvious that distortionary pricing is not necessary anymore in the urban sector. But the same does not hold for the rural price. The rural price is less than the urban price if the urban social weight \( \beta^2 \) is higher than the rural social weight \( \beta^1 \), and vice-versa. Also, if the urban weight is higher, then the price differential decreases with an increase in the rural surplus response. The urban consumption response does not play any direct role, as would be expected.

The fact that the rural sector is not directly controllable (and we strongly suggest that it should not be considered controllable, even as a first approximation), leaves intact most of the considerations which go into the determination of shadow wage. The shadow wage with optimally set prices, and fully controlled urban sector is found by substituting (52) into (49).

\[ (53) \quad \bar{w}^2 L^2 = \bar{w}^1 L^2 + [\bar{w}(v^1) - \bar{w}(v^2)]/\delta - \frac{\beta_1}{\delta} p X \]

\[ + (1 - \beta^1/\beta^2) (1 - \varepsilon Q) \frac{p_0}{\varepsilon Q} \]
An interesting result arises from the above expression. In much of the existing literature it is believed that the shadow wage increases if the shadow price of investment, $\delta$, increases. Note from (53), that this will happen if the urban utility is higher than the rural utility. Just the opposite would be true, however, if the rural utility is higher, and if the land congestion is not significant.

(c). **Shadow Wage in a Mixed Economy**

When migration cannot be directly controlled, then the shadow wage becomes

\[
\lambda^2 \Sigma^2 = \lambda^2 \Sigma^2 + \frac{N}{N^1} \lambda^1 \frac{\partial}{\partial p} X_a \frac{\partial N^1}{\partial N^a} + (p - \frac{\eta}{\delta}) Q(1 - \frac{\epsilon}{Q_a}) \frac{\partial N^1}{\partial N^a}
\]

\[-(q - \frac{\eta}{\delta}) \lambda^2 \Sigma^2 \]

This is obtained from (40), while keeping $N^e Y$ fixed, and making familiar substitutions. $\frac{\partial N^1}{\partial N^a}$ for the present model can be obtained from (37) as

\[
\frac{\partial N^1}{\partial N^a} = - \frac{(v^2 - v^1)}{[v^1 - v^2 - u^2 \frac{\partial v^1}{\partial N^1}]} \]

\[
\frac{\partial N^1}{\partial N^a} \text{ can take any negative value. This can be seen by using (5) and } v^2 > v^1 > v^2 \text{, which follows from (35). Further, if land is not scarce then more than one person migrates out of the rural sector if a new urban job is created. This also follows from (55). On the other hand, if the rural land}
\]
is scarce, then it is quite possible that less than one rural worker would migrate out under the same situation. This is because the migrant rural worker releases land for the remaining rural workers, and this reduces the incentive to migrate for those who are left behind in the rural sector.

We can first show how under certain limiting assumptions we can derive the kinds of expressions that other researchers have obtained earlier. The most significant result of the models with endogenous migration is that the shadow wage can be quite close to the actual urban wage. [An entire class of existing models which lead to such results is summarized in Stiglitz (1982a)]. If two assumptions are made, namely, there is no price distortion, and the land is surplus, then the above result follows from (54) because all terms in it, except the first, vanish. The reasoning is obvious. The rural utility, and the expected urban utility, remain unchanged due to migration because land is surplus [see, (40)]. Another way to come to the same conclusion is to derive the increase in the social welfare from creating employment for one person. This increase is $V^2 + V_u^2[-\frac{dN^1}{dN^e} - 1] - V^1[-\frac{dN^1}{dN^e}]$. Using (55), it is seen that this increase is zero. Hence, the only way the value of national income is affected is through investment. As investment in this special case equals $N^e[Y - w^2L^2]$, it follows that the shadow wage is equal to the actual urban wage.

This result generalizes that of Stiglitz (1974, 1982a) where the relative price between the two sectors is fixed, the hours worked in both sectors are fixed at the same level, and when individuals are risk neutral [so that the expression corresponding to (35) is $w^1 = N^e \bar{w}^2/(N^e + S)$]. It should be noted from (54), however, that even if there are no price distortions, but
if land is not in surplus, then the shadow wage is less than the urban wage, and indeed, if $\lambda^1 > \delta$ and $X_a$ is sufficiently large, then it may even be less than the rural per capita product.

Another interesting use of the expression (54) is to see how, in the presence of price distortions, the shadow wage with endogenous migration differs from the case of controlled migration. First take the effect on the public trade in rural surplus, i.e., the fourth term in (49) versus the third term in (54). The latter is a larger negative number than the former if $\frac{dN^1}{dN^e} < -1, 1 > e_{Qa}^1$, and $p > \eta/\delta$, i.e., if the rural sector is being underpaid for its product. On account of the surplus trade, therefore, endogenous migration reduces the shadow wage. The economic meaning is clear: if the government is 'losing' on the rural surplus purchase, then a higher migration response (i.e., a higher reduction in the rural surplus) reduces this surplus more.

Now take the influence of migration on land congestion. We find that under most circumstances, endogenous migration will reduce the shadow wage on this account. To see this, compare the third term in (49) to the second term in (54). It is quite unlikely that an absolute value of $\frac{N}{N^1} \frac{dN^1}{dN^e}$ will be less than one; in fact, it could be much larger. (The presence of term $\frac{N}{N^1} \frac{dN^1}{dN^e}$ simply reflects the equalization of welfare effects). The economic reasoning behind this reduction in the shadow wage is also intuitive. Out-migration from the rural sector releases land for the remaining rural population. The gain from this, however, accrues to the entire population due to endogenous migration. This in turn reduces the overall economic cost of creating a new job.
V. PRICING OTHER ECONOMIC SYSTEMS

We have argued that the problem of the internal terms of trade can be best examined as an indirect control problem of the kind that has become familiar in the modern theory of taxation. The desirable policies are, therefore, dependent on two aspects: what is it that the government can control, and what are the characteristics of the economic system? Further, the optimal policies can be systematically related to each of these two aspects in a clearly understandable manner. We have already seen the importance of what it is that the government controls in our comparison of policy rules with endogenous migration versus policy rules when migration is controlled. It may now be useful to provide some comments on how the policy rules will change under different economic systems.

First consider a rural sector in which the entire land rent accrues to the government. This sector, however, is competitive in the sense that labor receives the value of its marginal product. This extreme characterization is of interest for two reasons. First, we may be interested in looking at the case in which the landlord's consumption, which has received such prominence in the historical debates, is altogether taken out of the picture. Second, this characterization of the rural sector along with the characterization discussed earlier (in which the entire rent accrued to the rural sector) brackets all those situations in which land rent accrues to both sectors in some proportions.
There are two new welfare effects of increasing the rural price. First, the government gains on the rural rent which it now controls. Second, the government may gain or lose through the changes in the wage payment induced by the price increase. Formally, the rural indirect utility function is given by $V^1(p, \bar{w}(p, a))$ instead of (5). The wage $\bar{w}(p, a)$ is determined through the labor market. The rural wedge in the case of controlled population can be obtained by substituting the above indirect utility function in (20), differentiating it with respect to $p$, and making appropriate substitutions.

$$\frac{(p - \eta)}{\delta} \frac{dQ}{dp} = \left(\frac{\bar{w}^1}{\delta} - 1\right)Q - \left(\frac{\bar{w}^1}{\delta} - 1\right)\left[aX_a + L^1x_L(1 - \varepsilon_{wp}^1)\right]$$

where, $\varepsilon_{wp}^1 = \frac{p}{w} \frac{dw}{dp}$ is the elasticity of rural wage with respect to the rural price.$^{15}$ Contrasting the above with (24), it is clear that the new expression differs only with respect to the two specific effects mentioned above. Further, if capital is scarce, and if the wage does not rise too rapidly with an increase in the rural price, i.e., $1 > \varepsilon_{wp}^1$, then the last term is positive, which has an effect of increasing $p - \eta/\delta$. This has an intuitively clear meaning. If the rural sector is sending its rent to the urban sector, then it may be desirable to provide a somewhat higher price for its surplus, in comparison to the situation in which the rural sector is consuming its rent.

We conclude this section by providing comments on a rural sector in which labor is surplus; a model which has provided much ground for discussion in development economics. (This is the opposite extreme to the land surplus
rural sector, which has been occasionally mentioned in this paper.) While the
literature has ignored the effects of incentives in the rural sector, our
purpose here is to point out that these effects may be important even with
surplus labor in the rural sector.

The two principal elements of the Lewis tradition of a labor surplus
rural sector are: (i) changing the number of rural workers does not change
the rural output, and (ii) each rural worker gets an average of the sector
output. A simple way to model these two elements is the following. The
production technology is characterized by fixed proportions, in which land is
a fixed factor. An individual worker is rationed to provide average number of
work hours, and receives an income, which is the value of the average
output. Now we can examine the consequence of government policies on the
two variables of the rural sector which are of greatest interest to us,
namely, marketed surplus and the rural welfare. First consider the impact of
an increase in the rural price. This changes the budget constraint of a
worker, which, in turn, alters his consumption pattern. Next, consider the
impact of a person moving out of the rural sector (either due to controlled
migration, or due to induced migration). This increases the number of work
hours which the rural workers must provide. The rural welfare and the
marketed surplus will change in either case. The responses of the rural
sector, thus, can not be ignored even if this sector is characterized by a
surplus of labor.
VI. CONCLUSIONS

This paper has had several objectives. Its primary objective was to develop a simple, but general, framework, for the analysis of a number of important policy issues facing many less developed countries concerning the relationship between the urban and the rural sector — a framework which, at the same time, would represent a synthesis and an extension of two important strands in the economics literature of the past quarter century: modern public finance theory and modern development economics. We have used this model to analyze several questions.

First, we have attempted to identify the critical parameters in determining the optimal terms of trade between the urban and rural sectors. At one level of analysis, the results are hardly surprising. The optimal terms of trade depends on (i) the welfare weights associated with individuals in the two sectors; (ii) the welfare weights associated with the present generation versus the future generation (as reflected in the shadow price of investment); (iii) the price elasticity of the rural surplus; and (iv) the price elasticity of demand for rural goods in the urban sector; (v) the migration response to changes in welfare in the two sectors; and (vi) the elasticity of output with respect to land in the rural sector. Although these are the factors that one might have expected to find among the principal determinants of the optimal terms of trade, the precise way that these factors enter is far from obvious, and the simplicity of the optimal pricing formulae in certain cases is perhaps somewhat surprising.

Secondly, we have attempted to derive a more general expression for the shadow price of labor, taking into account the effects of hiring
additional urban laborers on (a) the terms of trade and the resource flow between the urban and rural sector; (b) migration between the rural and urban sectors; (c) the distribution of welfare between the urban and rural sectors; and (d) the level of investment (availability of savings) in the economy. We have shown that previous studies can be viewed as special, and quite limiting, cases of our more general analysis, and the results that they obtain may, as a consequence, be misleading.

The analysis, though admittedly complex, has had to abstract from a number of important considerations. We have ignored, for instance, the distribution of income within each of the sector, the effect of changes in prices on incentives for investment within the rural sector, and the effects of that, in turn, on innovation and technical progress in agriculture. We suspect that some of these considerations, at least, when properly taken into account, would result in a tax on the rural sector which is smaller than what might be suggested by the analysis of this paper. But these are questions which we hope to take up in greater detail on a subsequent occasion.
1/ This problem is related to, but different from, the question of price "scissors", which was a crucial issue in the Soviet industrialization debate [Dobb (1966), Erlich (1960), and Preobrazhensky (1965)]. The typical institutional structure for the scissors question has identical prices in both the urban and the rural sectors, and the size of scissors is the ratio between the industrial and agricultural prices. The institutional structure in the present paper, on the other hand, entails different prices in the two sectors. That is, there is an internal "tax" border between the two sectors. The problem of price scissors is taken up in a separate paper.

2/ Lipton (1977) is perhaps unique among the recent scholars to have confronted this normative issue directly. He provides a vivid description of various ideological positions, as well as evidence to show that the recent development policies have been marked by an economically unwarranted bias against the rural sector.

3/ For a more extensive discussion of how some of these factors affect the shadow price of labor within a static model of the economy, see Stiglitz (1982a).

4/ Though this may seem a natural assumption, it is not as innocuous as it might seem. Agricultural output depends not only on the total number of labor hours supplied, but also on when they are supplied. If there are certain times of the year when all laborers are working at full capacity, then reducing the number of workers, keeping the total number of hours
worked over the course of the year unchanged, will still have an effect on output. See Stiglitz (1969).

5/ If the total output is denoted by $X$, then $X = X(A, N^1L^1)$, where $X$ is strictly increasing, strictly concave and homogeneous of degree one in the inputs. Then, $X = N^1X(a, L^1)$, and $X_a > 0$, $X_L > 0$, $X_{aa} < 0$, $X_{LL} < 0$. The subscripts denote the partial derivatives with respect to the factors. Also, $X_{aL} > 0$ from homogeneity.

6/ We fully recognize the importance of capital investment and of capital accumulation in the rural sector. We do not, however, build it into our present model for the reasons described at the end of Section II.

7/ There are two kinds of empirical evidence. First, following the pioneering work of Behrman (1968), there are many econometric studies which estimate the price elasticity of marketed surplus for a large number of crops in many LDCs. Second, there are recent micro-econometric studies which jointly estimate the production and the consumption functions [Ahn, Singh and Squire (1982), and Barnum and Squire (1979)]. All of these studies suggest a positive price elasticity of surplus.

8/ If $p$ is the price paid to farmers for a unit of rural surplus, $\eta/\delta$ is its marginal social value. If $p > \eta/\delta$, then the price exceeds the social value, and $p - \eta/\delta$ is the 'public loss'. It could be either positive or negative. Similarly, $q - \eta/\delta$ is the public gain on a unit of urban consumption, because it is the excess of the price charged over the shadow price.
In this analysis we assume an interior solution to the problems like the maximization of (20), and examine the first order conditions. The standard caveats, as well as the standard justification, for doing so apply.

The derivation of these requires solving an explicit intertemporal maximization, yielding the differential equation of the form \[\dot{\delta} = \gamma - Y_k\], which, together with the transversality conditions, can be solved for \(\delta\). We do not pursue this in the present paper.


An extra first order condition would give the allocation of investment, apart from the conditions due to the non-shiftability of capital across sectors. Now, if we limit ourselves to the regime in which investment is positive in both sectors, and if rent on rural capital accrues to the rural sector, then it can be seen that (24), (26), and (27) will remain entirely unchanged.

The zero consumption of the urban unemployed is obviously a simplifying device to abstract from intra-family transfers and borrowings. The additional effects created by these arrangements, however, are likely to be quite insignificant compared to the effects on which we have focussed our analysis.
Before closing this section, it may be useful to comment briefly on the two-sector neoclassical model, in which all resources are fully employed. From a descriptive point of view, Cheetham, Kelley, and Williamson (1974) and Kelley, Williamson and Cheetham (1972) provide the growth path of such an economy when prices are uniform across sectors, and the demand and the production functions take specific functional forms. From a normative viewpoint, Atkinson and Stiglitz (1980, p. 191-192) provide a static tax incidence analysis with uniform prices and fixed factors supplies, and Mirrlees (1975a) provides a static optimal tax analysis with fixed incomes and uniform prices. The question of internal terms of trade, however, does not get raised in these models.

An expression for $\varepsilon_{wp}^1$ can be obtained by solving the labor market equations, which gives, $\varepsilon_{wp}^1 = (1 + \varepsilon_{Lp}^1)/(1 - \varepsilon_{Lw}^1)$, where $\varepsilon = Lx_{LL}/x_L$ and $\varepsilon_{Lp}$ and $\varepsilon_{Lw}$ are the partial elasticities of the labor supply with respect to the price of the rural good, and the wage rate.

If $a^a$ is the land-output coefficient and $a^L$ is the labor-output coefficient, then $X = \min \left\{ \frac{a}{a^a}, \frac{L^1}{a^L} \right\}$. As land is the fixed factor, the output is $X = a/a^a$. The amount of labor that a worker provides is $L^1 = a^L/a^a$. The indirect utility is then expressed as

$$v^1(p,N^1) = \max_{x,y} u^1(x^1,y^1, \frac{aa^L}{a^a}) + \lambda^1 [p\frac{a}{a^a} - px^1 - y^1]$$
REFERENCES


