Special Exchange Rates for Capital Account Transactions

Rudiger Dornbusch

The governments of developing countries are constrained in the effective implementation of domestic policy by the interlinkages of national and international financial markets. Domestic macroeconomic conditions are influenced by the interaction of national and world interest rates and prices, and through the impact of real exchange rates on employment. The domestic responses to changes in these factors are often strong and rapid. In an attempt to sever these ties, governments have adopted dual exchange rate systems in which capital account transactions are conducted at a depreciated exchange rate while an otherwise overvalued rate is maintained for commercial trade. This article suggests that dual rates can indeed be used successfully as a strictly transitory policy to offset sudden shocks in capital markets. The article develops models which indicate why these dual systems are able to prevent inflationary or recessionary pressures caused by a misaligned exchange rate in the short term. While free capital account rates can cut the flow of capital flight, however, a dual rate system cannot prevent a possibly equivalent loss of foreign reserves that will ultimately result because of the impact of the overvaluation of the commercial rate on the trade balance. In the longer term, a dual rate system with a misaligned commercial rate exacerbates the government’s deficit; ultimately, real wages must be cut and real interest rates raised to generate sufficient foreign exchange to finance the external debt. Thus a dual rate works well if the commercial rate is maintained close to the equilibrium level.

The starting point for any discussion of special asset transaction exchange rates is the high mobility of capital. Assets markets are linked internationally in terms of risk and expectations-adjusted returns, and that linkage is potentially tight and rapid. That implies severe restrictions on the scope of government strategy. Policies must be such as to give asset holders the world rate of return, or they will seek to purchase assets abroad with one of three results: under a fixed rate, the stock of reserves will be depleted; under a flexible rate, the exchange rate will be depressed to a level where home returns are again in line with those abroad; or because of the threat of these responses, policies will be aligned with the requirements of asset markets rather than with governmental objectives and priorities.

The author is at the Massachusetts Institute of Technology. This article is part of a research project on exotic exchange rate arrangements conducted for the World Bank.

Copyright © 1986 by the International Bank for Reconstruction and Development / THE WORLD BANK.
This article examines the experience of various Latin American countries with the use of dual and multiple exchange rate systems to delink these markets, and it develops models to explain the macroeconomic outcomes of such systems. These models analyze the impact of dual rates on the balance of trade, foreign reserves and asset holdings, inflation, the government budget, and the supply of and demand for traded versus nontraded goods. The article also looks at overvaluation and the effects of expected depreciation on interest rates and investment. While the relative benefits of these systems for asset holders and wage earners are apparent at many points in the analysis, this is not an essay in applied welfare economics, and empirical investigation would be required to determine the distributional effects of specific uses of the various types and levels of controls.

Section I briefly describes the ties between domestic and world markets, the rationale for initiation of exchange controls, and the forms such controls may take. Section II looks briefly at the extent of exchange rate differentials which occurred in Mexico and Venezuela in the early 1980s and develops various models of dual market systems. The implications of multiple exchange rates and the development of illegal markets for foreign exchange transactions are examined in section III.

I. LINKAGES BETWEEN MARKETS: RATIONALE AND METHODS FOR SEVERING THE TIES

The problem of asset market integration can be understood by looking at three linkages between an economy and the rest of the world. These are the linkages between interest rates, the interaction of prices, and the impact of real exchange rates on employment. These relationships are shown as:

\[ i = i^* + \dot{e}/e + R \]

\[ P = f(eP^*) \]

\[ N = N(eP^*/P) \]

where \( i \) and \( i^* \) are home and foreign interest rates, \( \dot{e}/e \) is the expected rate of depreciation of the exchange rate expressed in domestic currency, and \( R \) is the risk premium. \( P \) and \( P^* \) are home and foreign prices, and \( N \) is employment.

Equation 1 states that home interest rates are equal to those abroad, adjusted for anticipated depreciation and the risk premium that emerges from political and exchange rate risk. This equation can be viewed as the constraint on financial policies: in integrated asset markets, the home interest rate must be set high enough or savings will be transferred to foreign assets and the currency will come under attack. Equation 2 points out that domestic prices will be affected by the exchange rate: a rapid depreciation of the exchange rate would cause an increase in home inflation. Equation 3 emphasizes that a change in the (real)
exchange rate will influence employment. In the long run, real depreciation is likely to raise employment. But in the short run, the adjustment process may make the effects run the other way.

These three linkages then mean that asset markets are internationally integrated and that this integration places restraints on policy, or that lack of attention to these constraints has negative implications for inflation and for employment. Moreover, because the reactions are strong and rapid, the issues are of foremost importance. They cannot be disregarded, because reserves are often in short supply and depreciation of the exchange rate can be politically difficult; however, running the world to the tune of asset markets may be undesirable. Hence the interest in institutional arrangements that delink asset markets and free policies to be directed to a government’s true priorities.

There are any number of examples of countries where exchange rate movements or capital flows became an inconvenience or more for policymakers. For example, in the United States in 1980–85, the dollar appreciation, for safe haven reasons or because of the U.S. monetary-fiscal mix, led to overvaluation and an unprecedented shock to manufacturing. Very soon there was talk of renewing the import surcharge that had been adopted to cope with the overvaluation of the early 1970s, and even a renewal of interest equalization taxes came into discussion.

But, of course, the shock can also run the other way when capital flight leads to a fall of the exchange rate and, as a result, an inflation burst. The best example would be the onset of the German hyperinflation in the 1920s. The “balance of payments school” at that time saw the confidence-induced collapse of the exchange rate as the source of domestic inflation, which in turn led to budget deficits; these deficits reinforced the escalating rate of price increases. For a while, the government managed to stabilize the exchange rate, and prices remained stable. Then a loss in confidence (related to the reparations problem) ensued, and in a few weeks the exchange rate increased sevenfold. The exchange rate depreciation raised import prices, wages, and the budget deficit, which opened all mechanisms for uncontrolled price rises and hyperinflation.

The importance of the capital market integration issue has also been highlighted in the aftermath of the debt problem. Much of the accumulation of Latin American external debt reflects the financing of capital flight (see World Bank 1985). This is strikingly the case for Argentina, Mexico, and Venezuela, where the amounts are extraordinarily large. Once again, the issue arises whether alternative capital market arrangements would have been an effective means to stop capital flight and tax evasion.

There are several ways in which asset markets can be delinked. The first is to decide on the scope for capital controls. One possibility is to maintain the international integration of capital markets (given by equation 1) by keeping interest rates at levels equivalent to international rates, but to delink domestic assets markets, at least partially, from the home economy. The means would be a special, separate exchange rate for financial transactions. Free capital mobility
at a fixed or flexible special rate, separate from commercial transactions, would
be a way of separating equation 1 from equations 2 and 3. Having more than
one exchange rate might make it easier to live with the effects of capital market
integration on the exchange rate and the economy.

An alternative is to opt against international integration of asset markets by
instituting formal capital control. This may take the form of a prohibition of
foreign asset holding by residents. The difficulty is to make that prohibition
stick: black markets will emerge, or capital flows will take place implicitly
through under invoicing of exports or over invoicing of imports in current ac-
tount transactions. In response, the government may be tempted to quasi-legal-
ze (this is a peculiarly Latin notion) parallel markets for foreign exchange or
create domestic equivalents in the form of a dollar-denominated government-
issued security or dollar deposits. The effectiveness of capital controls deter-
nines here how successfully a government can split markets and isolate the
home economy.

A two-way classification helps distinguish the possibilities. First, the rate for
asset market transactions may be managed (fixed as a special case) or freely
determined. Second, access to the exchange market for capital account transac-
tions may be restricted or completely open. Institutional arrangements fall some-
where within these possible ranges. For example, Mexico in 1983–84 had a
heavily managed asset transaction rate with unrestricted access to that market,
while Venezuela in the same period also had unrestricted access but considerably
less intervention in the rate. Brazil completely restricts access to the official
market where the rate is managed. Even Brazil’s black market has a somewhat
managed rate and an implicit restriction of access by corporations. The remain-
der of this article examines some of these systems to see what particular prob-
lems they solve and what problems they create.

II. Dual Exchange Rates

This section discusses systems in which a significant part of commercial trans-
actions is conducted at a uniform fixed rate, while capital account and selected
commercial transactions are conducted at another free or managed rate. The
fact that the foreign exchange market is opened to capital account transactions
establishes an immediate linkage between financial markets (and expectations
therein) and the exchange rate or the level of intervention. By separating finan-
cial transactions from commercial transactions, the authorities attempt to main-
tain the advantages of a managed, stable exchange rate for commercial
transactions that is not upset by volatility in international capital flows.

Dual rates are typically established by countries that feel they cannot or do not
wish to prohibit capital account transactions altogether. In circumstances where
the macroeconomy is highly unstable, capital flows will be very volatile and
potentially massive. If foreign exchange reserves are limited, a country has essen-
tially two choices. It can set a uniform rate that is so undervalued that there can only be an expectation of appreciation and hence no threat of capital flight. Or alternatively, the rates can be split so that the capital account rate can depreciate to whatever level required to make the public willing to hold the existing stock of domestic assets.

Each alternative has serious drawbacks: the overdepreciation of a uniform rate represents a shock to real wages and inflation. It poses the question why real wages should be cut merely to stabilize the expectations of wealth holders. But the free rate on capital account also raises questions. Will it distort allocation, as some commercial transactions slip into the free market? Will it be stable in the absence of intervention? Will there ultimately be exchange rate unification?

Figure 1 makes some of these issues more concrete by showing the three different Mexican exchange rates in effect from 1982 to 1984. Figure 2 shows the premium of the New York rate over the controlled rate in Mexico. The huge differential up to January 1983 corresponds to the early experimentation with exchange control of various kinds. Since then, the levels and differentials have been established in a manner such that the Central Bank increasingly has come into a position of managing the two rates, and subsequently the differential has been moderate.

Venezuela's case, by contrast, did not show a settling down (Figure 3). Following a long tradition of fixed exchange rates, in March 1983 the government abandoned pegging the rate except for essential imports. Specifically, capital movements were to be conducted in a free foreign exchange market. The premium of the capital over the fixed rate reached a level, using monthly averages, of more than 260 percent (Figure 4). The volatility of the free rates and the extent of discrepancies between free and controlled rates posed important issues for resource allocation and macroeconomic policy.

**A Model of the Dual Market**

This section presents a sequence of models of dual markets that build up progressively the key linkages between asset markets and the macroeconomy. It starts with a model that assumes full employment, a constant commercial exchange rate, purchasing power parity, rational expectations, and only two assets: domestic money and foreign nominal interest-earning assets.¹

In the asset market, because domestic currency earns no return, the desired ratio of money to foreign assets, \( M/eK \), depends on the rate of return on foreign assets. This rate of return is the sum of the asset's interest rate plus the additional domestic currency value of the foreign exchange earned as the domestic currency depreciates. This expected rate of depreciation is written \( \overline{e}/e \), where \( e \) represents the unregulated capital account exchange rate. \( K \) denotes the stock of foreign

¹ For more detailed discussion and derivations, see the appendix.
Figure 1. *Mexican Exchange Rates*

Note: The New York rate is the cable transfer quote rate.
The domestic market rate applies to capital accounts, tourism, and other transactions not covered by the controlled rate.
The controlled rate applies to essential imports, most exports, and debt service.

Figure 2. *The Mexican Free Market Premium on Exchange Rates*

Sources for figures 1 and 2: Banco de Mexico and U.S. Federal Reserve Bank, *Federal Reserve Bulletin.*
Figure 3. *Exchange Rates in Venezuela*

![Graph showing exchange rates in Venezuela with market rate and controlled rate.]

**Note:** The market rate applies to capital transactions and all trade except essential imports. The controlled rate applies to essential imports only.

Figure 4. *The Venezuelan Free Market Premium on Exchange Rates*

![Graph showing the Venezuelan free market premium on exchange rates.]

assets and $eK$ their value in home currency. The desired ratio of money to foreign assets is determined as:

\[
\frac{M}{eK} = L(i^* + \dot{e}/e), \quad L' < 0
\]

or, inverting the equation,

\[
\frac{\dot{e}}{e} = h\left(\frac{M}{eK}\right) - i^*, \quad h(\cdot) = L^{-1}, \quad h' < 0
\]

We focus here on domestic events rather than on the effects on the home economy of foreign interest earnings. But the simplification is one of convenience; it does not fundamentally alter the analysis. Since we are not focusing on foreign asset accumulation, we assume the $i^* = 0$. Hence the value of $K$ remains constant over time except for government intervention, and $\dot{e}/e$ simply equals $h(M/eK)$.

In this model, private savings is composed of both a stock adjustment and a flow component: 

\[
S = \nu(w - m - k) + \lambda m + (\lambda - \dot{e}/e)k.
\]

The first component is some proportion, $\nu$, of the excess of targeted real asset holdings, $w$, over actual real asset holdings, $(M + eK)/x$ where $x$ denotes the exchange rate for commercial transactions. The second portion is an adjustment made for anticipated capital losses (or gains) on real balances and foreign assets, $[\lambda m + (\lambda - \dot{e}/e)k]$ with $\lambda$ being the rate of depreciation of the commercial rate, $x/\hat{x}$. Capital losses are effectively inflation taxes, which reduce the value of nominal money stocks and thus increase the nominal savings needed to meet the real savings target. The impact of an increase in inflation over capital account depreciation ($\lambda > \dot{e}/e$) will similarly raise nominal savings, while a net depreciation will increase the domestic currency value of foreign assets and thereby reduce the desired rate of savings. Thus, in a steady state, the stock of actual savings may be on target, but individuals will continue to save to compensate for the reduction in real value of savings stocks caused by inflation.

By assuming that taxes and investment are zero, the traditional national income accounts can be revised so that the trade balance, $B$, will equal private savings, $S$, less real government spending, $G$, which is financed by domestic credit creation. If $P$ denotes domestic prices and $P^*$ foreign prices, given purchasing power parity (PPP), $P = x \cdot P^*$. Setting $P^* = 1$ gives $P = x$, which can be applied here without loss of generality. These assumptions, and the composition of private savings described above, are the basis of the following expression for the real trade balance:

\[
B = S - G = \nu(w - m - k) + \lambda m + (\lambda - \dot{e}/e)k - G
\]

where

\[
m = M/x, \quad \text{and} \quad k = eK/x
\]

2. Alternative models have been presented (de Macedo 1983, Lizondo 1984, Flood 1978) in which external asset accumulation plays a significant part.
The rate of increase in real domestic currency balances, $\dot{M}/x$, is determined by real government spending, $G$ plus the real trade surplus, $B$. Thus the change in the real money stock can then be written as:

$$\dot{m} = v(w - m - k) + [\lambda - h(m/k)]k$$

The other dynamic equation of the model describes the evolution of the real capital account rate, $q = e/x$ (given PPP, deflated here by $x$). Equivalently, the premium of the capital account rate over the commercial rate is written as:

$$\frac{\dot{q}}{q} = \frac{\dot{e}}{e} - \dot{x}/x = h(m/k) - \lambda$$

Figure 5 shows the schedules along which, respectively, real balances are constant ($\dot{m}/0$) and the real capital account rate is constant. The schedule for a constant capital account rate ($q = 0$) is positive because the premium of the capital to commercial account rates must increase to induce people to increase their real money balances, $m$. An increase in the premium now will preclude expectations of further near-future devaluations. Thus investors will not expect an increase in the value of foreign assets in the near future and will be content to hold the larger money balances. The $\dot{m} = 0$ schedule is drawn with a negative slope, although this need not be the case. The arrows indicate the dynamics and, as is conventional in perfect foresight models of this structure, there is a unique

---

Figure 5. Asset Market Adjustment with Dual Exchange Rates

- $m = M/x$
- Real money balances
- $m^*$
- $m_0$
- $k, q^*$
- $q = e/x$
- $\dot{q} = 0$
- $\dot{m} = 0$

Note:
- Real capital account exchange rate (market-determined)
- $k = eK/x = $ home currency value of foreign assets for a given $m$
- $J$ = stable asset market adjustment path for a given rate of depreciation of the commercial account exchange rate, $\lambda$
stable trajectory, $JJ$. From any initial stock of real balances, say $m_0$, the economy converges to the long-run equilibrium at $E$ along the path $JJ$.\(^3\)

The model is closed by specification of the rate of depreciation of the commercial rate. It is assumed that the commercial exchange rate depreciates at a rate, $\lambda$, that is sufficient, in the steady state, to generate the inflation tax revenue with which to finance the given level of real government spending.

Given any initial real money stock such as $m_0$, there is a unique equilibrium on $JJ$ and hence a specific value of foreign assets, $q_0K_0$. With a given value $K_0$, there is a unique capital account rate at which the asset markets clear. Over time, the system evolves to the steady state equilibrium at $E$. If real money balances initially are low as at $m_0$ the path is characterized by rising real money balances and a rising real value of foreign assets or an increasing premium of the capital account rate relative to the commercial rate, $e/x$. Thus if assets are initially low, savings will be high and there will be trade surpluses that cause the real money stock to rise as the central bank intervenes to sustain the commercial rate. At the same time, the real value of foreign assets is rising because of capital gains.

In the steady state, trade is balanced and the real money stock is constant ($\dot{m} = 0$). The premium of the capital account rate is constant ($\dot{q} = 0$), as the rate depreciates at the same pace as the commercial rate. The seignorage supported by depreciation finances real government spending. The equilibrium dynamics in figure 5 are shown for a given rate of depreciation of the commercial rate, $\lambda$. It is interesting now to ask how an increase in the rate of depreciation will affect the premium.

It is readily shown that with money demand inelastic with respect to the rate of inflation, an increase in government spending requires an increase in the rate of depreciation to yield the required increase in the inflation tax revenue. Increased rates of depreciation of the commercial exchange rate will immediately bring about a depreciation of the level of the capital account rate or an increase in the premium. Even in the steady state, the premium will increase.

In the steady state, the capital and commercial rates depreciate at the same rate. An increase in the rate of depreciation of the capital account rate shifts asset holders from money to foreign assets. Given the fixed supply of foreign assets, $K$, only an increase in the premium can bring about the rebalancing of portfolios. The increase in the premium caused by an increased rate of crawl of the commercial rate was demonstrated by Lizondo (1984).

**Expectations**

Having sketched the effect of a current increase in the rate of government spending, the impact of a shift in expectations is now examined. Starting in the steady state, the public anticipates that the government will increase real spending, deficit finance, and depreciation at some known future date. What is the

---

3. For a comprehensive explanation of the dynamics of phase diagrams, see Sheffrin (1983).
path of adjustment to this disturbance? This is an interesting question if one wants to explain the large fluctuations in the data for the dual market premium.

Figure 6 shows the initial equilibrium at point E. As shown in the appendix, an increased steady state rate of depreciation shifts the schedules. The \( \hat{q} = 0 \) rotates clockwise, and the \( \hat{m} = 0 \) schedule shifts out and to the right. Only a larger real premium will stabilize relative depreciation rates (\( \hat{q} = 0 \)) for any given value of the money stock.

Now consider the adjustment process. At the moment the expectation of higher future government spending develops, there is an immediate portfolio shift from money to foreign assets, which leads to a jump in the premium from point E to a point like A. The extent of this instantaneous depreciation depends on (among other things) how proximate in time the shift in monetary policy is. If it were almost immediate, the jump would be virtually all the distance to \( J'J' \). At point A, despite expectations of a new rate, the dynamics are still governed by the initial monetary policy and thus, with the high level of the premium, the value of foreign assets is high relative to real balances. This can only be an equilibrium if the rate of depreciation of the capital account rate has risen and hence is now higher than that of the commercial rate. Accordingly, the system moves in the direction of point B with the capital rate increasing and real balances declining. In the perfect foresight model, the economy arrives at B.

Figure 6. Adjustment to an Anticipated Increase in Government Spending

Note:
- \( E \) = initial equilibrium
- \( A \) = instantaneous depreciation of capital account rate in response to anticipated commercial rate devaluation
- \( B \) = value of \( m \) and \( q \) when new commercial rate is implemented
- \( E' \) = new equilibrium with depreciated commercial rate
- \( J'J' \) = adjustment path under new commercial rate
precisely at the time when the more rapid rate of depreciation of the commercial rate is implemented officially. From there on, the movement is along \( J'J' \), with some decline in the premium. The new steady state is at point \( E' \) with an increased steady state premium \( \tilde{q}' \).

Note that the rate of depreciation of the capital account rate can be determined from the conditions of monetary equilibrium as shown in equation 4a: \( \frac{\dot{e}}{e} = h(M/ek), \ h' < 0 \). With a declining ratio of real balances to external assets along the path \( ABE' \), the rate of depreciation of the capital account rate must be rising throughout, although the increase is less than that of the commercial rate after the new rate is implemented at point \( B \). Figure 7 shows the path of the rate of depreciation of the capital account rate over time. The times \( T_0 \) and \( T' \) correspond to the initial shift in expectations and the implementation of the new policy (point \( B \)). Up to time \( T_0 \), we have equal rates of depreciation: \( \frac{\dot{e}}{e} = \lambda \). Then, at the time expectations shift, there is a jump in the premium and in the rate of depreciation of the capital account rate, \( \frac{\dot{e}}{e} \). Since the rate of depreciation of the commercial rate is still unchanged, the premium is further appreciating until time \( T' \). Now the commercial rate depreciates at the higher rate \( \lambda' \) in excess of the capital account rate, which implies some real appreciation. Over time, the two rates converge to depreciate at the same pace. The increased level of the premium up to the time of the commercial account devaluation, shown

Figure 7. Expectation and Implementation of Commercial Account Rate Depreciation: An Increased Rate of Crawl

![Diagram](image)

Note: Letters in parentheses refer to points in figure 6.

- \( C > D \): increased premium in \( T_0 \) to \( T' \) is greater than the excess of the commercial over the capital rates in \( T' \) to \( T'' \).
- \( T_0 \): initial expectations of depreciation of commercial rate, \( \lambda \) at time \( T' \).
- \( T' \): point of actual government devaluation of commercial rate.
by area C in figure 7, is larger than the amount by which the commercial account rate depreciation exceeds that of the capital account rate after the devaluation, shown here in area D.

It is interesting to comment also on the trade balance in the adjustment process. As long as monetary policy is unchanged, following the shift in expectations, there is a trade deficit. The deficit arises because the increase in the premium raises wealth relative to target and the real capital gains lead to dissaving. Furthermore, with declining real money balances, seignorage starts falling short of the initial level of spending, and the trade deficit thus further deteriorates. The expectation of a shift in policy will therefore lead to trade deficits and potential difficulties in sustaining the path of depreciation.

Increased real government spending and deficits financed by money creation have been considered here. The increase in spending can be viewed as either spending on goods and services or as transfers abroad by the government, such as government debt service. In this broader interpretation, the exercise is of interest because it suggests that any disturbance that leads to an increased deficit will provoke an increase in the premium. Moreover, since real government spending in excess of real tax collection is being examined, one can also think of the exercise as a loss of real government revenues caused by reduced taxes or a loss of external resources. From this perspective, for an oil-exporting nation the expectation of a decline in real oil prices, for example, would increase the state enterprise and government budget deficits, imply deficit finance, and hence force a depreciation of the capital account rate. This interpretation is suggestive of the case of Mexico in 1985.

Consider now the effect of intervention. Assume the central bank sells foreign assets or foreign exchange in exchange for domestic money. The impact of such an intervention can be decomposed into two separate effects. The private sector now holds increased foreign assets at each level of the premium. With unchanged real balances, the premium would immediately decline to move the economy back to portfolio balance at point \( E \) in figure 5 above. But real money balances in private hands have declined as part of the open market operation, just as they would have in the case of a devaluation.

The decline in real balances takes the economy to the southwest of point \( E \), on \( JJ \), in figure 5. Accordingly, the premium declines proportionately more than the increase in foreign assets. Intervention in the capital account market thus is effective in depressing the premium. Interestingly, it also gives rise to a trade surplus via the wealth effects of the decline in the premium.

**Extensions of the Model**

The basic model has served to show the linkage between financial policies and the premium in the dual market. But the analysis needs extension if some macroeconomic complications coming from dual markets are to be seen. So far, the dual rate exerts effects only on the value of wealth and hence on income and
spending. But in fact the more important channels operate presumably via relative prices and domestic interest rates. These, too, are linked to the free rate, and the important point to recognize is that financial disturbances have macroeconomic effects via the free exchange rate. Furthermore, these effects often occur as a result of expectations.

We now consider the case where some goods—nonessential imports and nontraditional exports—are traded along with capital account transactions in the free market. Essential imports, say food and materials, and traditional exports are traded at a (generally overvalued) fixed rate. Since part of the goods now are traded at the free rate, the aggregate price level is influenced by both the commercial and the free rates. Moreover, the premium of the free rate now sets the relative price of those goods entering via the free market. Instability of asset demands, policies, and expectations now introduces instability in the price level and in relative prices.

Moreover, if the dual rate regime is chosen to defend the foreign exchange reserves, this result may not in fact be achieved. Financial disturbances that lead to an increase in the premium draw production resources into the premium market while inducing consumers to substitute toward the controlled market. A rise in the premium associated with a “flight from domestic money” will still lead to reserve depletion, except that it now takes place via the enlarged trade deficit at the regulated rate rather than via actual capital flows.

It is important to recognize that now the free market no longer involves finding the price at which an existing stock of foreign assets, \( K \), is held. The market now can generate an accumulation or decumulation of foreign assets via current flows. Specifically, we look at the possibility of trade being diverted from the official market to the free market. The central bank faces larger trade deficits and loses reserves, while in the free market, a trade surplus leads to accumulation of foreign assets. One can think of the implications of shifting transactions to the free market as legalizing the capital flight involved in the underinvoicing of exports or in the import smuggling financed by underinvoiced exports.

To demonstrate these results, most of the previous model’s structure is maintained. The specification of asset markets remains unchanged. But now the markets for the two classes of goods need to be separated, while the assumption of given world prices and PPP at the relevant exchange rate for each good is maintained. Let the given foreign prices of all goods be unity, so that \( e \) and \( x \) denote the prices of goods that trade in the home country at the official and free rate, respectively. The aggregate price level, \( P \), is now an expenditure-weighted function of these two prices.\(^4\)

\[
P = P(e, x)
\]

In the previous equations, \( P \) now replaces \( x \) as the deflator for assets.

\(^4\) The premium and the relative price of goods trading in the free market continue to be denoted by \( q \), where \( q = e/x \).
To simplify matters, depreciation of the regulated rate is dispensed with, so that $\lambda = 0$. Because the free market now involves not only stocks but also flows, the trade balances for the official and the free market need to be specified separately. Let $B$ and $V$ now denote trade balances at international prices of the regulated and free markets, respectively.

(9) \[ B = B(\alpha, q, a, \hat{e}/e, G), \quad V = V(\alpha, q, a, \hat{e}/e) \]

where $\alpha$ denotes the fraction of goods traded at the fixed exchange rate and $a$ is actual wealth.

An increase in the free market rate relative to the fixed official rate will deteriorate the official trade balance, $B$, and improve $V$, the trade balance in the free market. The reason is that consumers will substitute toward the now relatively cheaper imported goods traded at the official rate while producers will move resources out of production for the official market and into activities that benefit from the free rate. This substitution is one of the most important features of a dual rate regime once commercial transactions are included in the free market.

Formally, the model is now more complex because the real money stock, the stock of foreign assets, and the premium each must be tracked; hence, a simple phase diagram can no longer help. But one can still get a lot of answers by just looking at comparative steady states, since in long-run equilibrium, actual wealth, $a$, equals planned real wealth, $w$, and the depreciation of the unregulated exchange rate is zero: $\hat{e}/e = 0$. Thus, as shown in the appendix, the steady state is defined by the following equations:

(10) \[ \tilde{V}(\alpha, q, w, 0) = 0 \]

(11) \[ a = m + K = w \]

(12) \[ m/ P = L(0) \]

In equations 10 and 11, assets are now deflated by the new expenditure-weighted price level, $P$, rather than the regulated commercial rate, $x$. The price level, reflecting the free and official exchange rates, respectively, is given the simple form, $P = e^{l-\alpha} x^\alpha$. Substituting equation 11 into 12 now yields the relation between the stock of foreign assets and the premium:

(11') \[ m + K e P = w \]

\[ m + \frac{K e}{e^{l-\alpha} x^\alpha} = w \]

\[ m + \frac{K e^\alpha}{x^\alpha} = w, \text{ or} \]
\( w = m + Kq^a \)  

(11a)

\[
\frac{m}{eK/e^1-x^a} = L(0)
\]

\[
\frac{m}{e^aK/x^a} = L(0), \text{ or}
\]

(12')

Substituting 12a into 11a gives:

\[
w = [L(0) q^a] + Kq^a
\]

(13)

Hence for a given target level of wealth, in figure 8 the downward-sloping schedule \( \omega \omega \) captures both portfolio preferences and saving behavior. Trade balance in the free market, equation 10, is also shown as a horizontal schedule. The steady state equilibrium for a given \( w \) and \( \alpha \) is at point E.

Consider now the following policy measure. The government moves some export activity that previously was conducted at the official rate into the free market. The effect of a shift of goods to the free market implies a reduction in the parameter \( \alpha \) and a shift of the \( \omega \omega \) schedule given by:

\[
dK/\alpha = -\alpha K/g.
\]

Figure 8 shows the effect as a downward shift of the \( \tilde{V} = 0 \) curve. The increased export surplus of the free market raises the stock of foreign assets, \( K \), and must be compensated by a reduced premium so that the schedule for balance in the free market, \( \tilde{V} = 0 \), drops to the position indicated by the dashed lines. (To obtain a downward shift, it must be assumed that \( \partial \tilde{V}/\partial q > 0 \), so that substitution effects dominate the potentially opposing effect of increased savings undertaken to offset a decline in real wealth caused by the drop in the premium.) Across steady states, the premium declines and the stock of foreign assets rises.

From equations 11 and 12, it can be shown that across steady states, real money balances are constant:

\[
m = \sigma w, \sigma = \frac{\hat{\sigma}}{\hat{\omega}} = L(0)/(1 + L(0))
\]

(14)

where \( \sigma \) is the steady state ratio of money to wealth in the absence of inflation.

Consider next the effect on the cumulative balance of payments of a change in reserves and the nominal money stock. Since real money balances are unchanged

5. When all goods trade at the commercial rate, \( x \), equation 13 reduces to \( w = qK[1 + L(0)] \).
across steady states, it is enough to look at the behavior of the price level to know how nominal money changes. The initial shift of some goods to the free market, starting from a situation where all goods are traded at the commercial rate, gives unambiguous results. Since some goods are now shifted from the lower commercial rate to the higher free rate (that is, \( q > 1 \)), the price level must rise. Unchanged real balances in combination with a higher price level then unambiguously imply a cumulative trade surplus, as more exports now sell in world markets at the depreciated exchange rate and import purchases decline in response to the higher domestic currency price. The decline in the premium also decreases wealth and thus spending, which further improves the reserves balance.

But when some goods are already traded at the free rate, a shift of yet further goods to the free market need no longer involve favorable effects on reserves. Now two offsetting effects are at work. The shift of new goods to the free market by itself again raises price levels. But the decline in the premium now lowers the prices of all goods already traded at the free rate and through that channel lowers the price level. If this latter effect is sufficiently important, the price level will fall; hence, reserves must decline in the adjustment process.

Consider next a portfolio disturbance, specifically a shift out of money reflected in a decline in the steady state ratio of money to wealth, \( \sigma \) (equation 14).
This is, of course, the kind of event against which countries seek to protect themselves with a dual rate. As real money balances fall, if the premium remains unchanged or even declines, there must be a cumulative trade deficit at the official rate. The mechanism is the following: the portfolio shift drives up immediately the premium in the free market. As a result, there are two effects on the official trade balance. First, the higher premium diverts resources directly to the production of goods traded in the free market and shifts demand toward the goods traded at the official rate. Second, the rise in the premium raises wealth, which increases spending and thereby also increases the official deficit.

Figure 9 shows the adjustment starting from an initial equilibrium at $E$. The portfolio shift leads to a rightward movement of $ww$, and hence $E'$ is the new equilibrium. If the adjustment process is asymptotic, then there will be a path $TT$ along which the economy will travel, starting with an immediate sharp rise of the premium from $E$ to $E''$. Subsequently, as foreign assets are being accumulated and money balances reduced, portfolio proportions of $M$ and $K$ come more nearly into line with preferences. The process continues until the full adjustment in portfolios has been achieved by changes in nominal money and external assets.

The unambiguous effect is that a portfolio shift is fully absorbed by a loss in reserves, even though this occurs via the regulated market rather than as capital.
flight. This conclusion is important because it shows that dual rates can break the speed of capital flight but may not be able to stop an equivalent reserve loss occurring via the impact of the premium on the trade deficit.

III. TRIPLE AND MULTIPLE RATES

The Venezuelan case in which the free market premium reached more than 260 percent of the commercial rate was mentioned above. Such a discrepancy is of course a very large distortion. This overvalued rate for some essential imports holds down their prices and thus maintains the real value of wages. But in doing so, it heavily taxes producers of goods traded at the regulated rate. The resulting tendency toward a deficit in the regulated market brings about reserve depletion and thus expectations of devaluation. The expectation then further raises the premium and reinforces the reserve losses.

A typical response to the dilemma is to maintain the regulated rate for essential imports but to shift some exports toward a third market in which the rate is also fixed but at a higher level. In Venezuela, such a multiple system took the form shown in table 1.

The change in the exchange rate structure involved a real depreciation with respect to debt service, services, and imports. The relative prices of these goods and services increased as they were shifted from the lowest rate to an intermediate level involving a 25 percent depreciation. What would one expect to be the impact on the free market premium over the basic rate, \( e/x \)? The model developed here can still be used, except that it now has another parameter, \( \beta = x'/x \), the ratio of the intermediate rate to the basic rate. The \( uw \) schedule would remain unchanged. But there will be an effect on the free market trade balance.

\[
\tilde{V}(\alpha, \beta, u, q) = 0
\]

The question is whether splitting the basic rate will increase or reduce the trade surplus of the free market. There are two extreme scenarios that can be envisaged. In the first, resources are primarily transferred from the free market into production in the sector with the new higher official exchange rate, while higher prices in that sector transfer consumer demand into the free market. In

<table>
<thead>
<tr>
<th>Table 1. The Venezuelan Multiple Exchange Rate System</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exchange rate</strong> (Bolivars per $U.S. dollars)</td>
</tr>
<tr>
<td><strong>Date</strong></td>
</tr>
<tr>
<td>February 1983</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Unregulated</td>
</tr>
<tr>
<td>February 1984</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Unregulated</td>
</tr>
</tbody>
</table>
this case, increased demand and reduced supply create a deficit in the free market, $V(\bullet)$ declines at each level of $q$, and the $V(\bullet)$ schedule in figure 8 shifts upward. In addition, with unchanged real money balances, across steady states the third rate would create a cumulative trade surplus at the official rate. Conversely, at the other extreme, the shift primarily worsens the official trade balance while improving the free market surplus, and the premium declines. The price level $P(e,x,x')$ now could fall (depending on the relative weights of the different markets in total domestic trade), and there might be a cumulative loss of reserves, as can be observed from the equation for real balances written here in terms of nominal money and prices:

$$M = P(e,x,x') aw$$

The ambiguity of the effect of the policy move on reserves is, of course, critical. It suggests that an obvious move to increase efficiency—removing some items from a severely undervalued exchange rate—may in fact produce exactly the wrong results for reserves. Moreover, there is no presumption that shifting an activity from one rate to another will in fact improve welfare, as is obvious from the second-best nature of the exercise. This point is important, because it means that increasing the number of rates by shifting activities from the lowest or basic rate toward the more “realistic” free rate does not necessarily reduce distortions in the economy. It may well increase the misallocation (see Harberger 1959).

The Unification Problem

Figure 10 shows the premium of the free rate over the official exchange rate for the Dominican Republic. The official rate is constitutionally fixed at 1 peso per U.S. dollar. An increasing number of transactions are conducted at this rate, and as is apparent, the free rate has progressively moved away from the official rate. The Dominican Republic now faces a problem common in Europe after World War I: should the official exchange rate be restored as a uniform rate, which would require deflation such as the United Kingdom undertook in moving back in 1925 to the prewar parity? Or should a new uniform rate be set that takes into account the level of the free rate, as Poincaré did in 1926 in France?

It is clear that the present system is not viable because it involves huge distortions. In response to the distortions, an increasing number of transactions are shifted to the parallel rate so that the average exchange rate is depreciating over time. Table 2 shows the effects of this reallocation on the effective exchange rate (weighting the trade categories by their appropriate exchange rate).

Much the same problem, though in perhaps less clear-cut terms, arises for a country in which the free rate and the basic rate are so far apart that the resource allocation costs outweigh any macroeconomic benefits. When the dual rate has gone far out of line, the unification of rates becomes an important macroeco-

6. This may be Venezuela’s situation but is probably not Mexico’s.
The expectations about the manner in which unification will be achieved will affect both the premium (and hence the trade deficit) and also interest rates and activity. If the expectation is one of devaluation of the official rate, as must ordinarily be the case, the free market premium will already reflect that expectation and be correspondingly higher, which thereby worsens the trade deficit. Interest rates will reflect the expectation of depreciation of the free rate and thus will rise in the period ahead of the expected depreciation. Therefore, if the devaluation is delayed, real interest rates for activities tied to the official rate increase, and that of course leads to a decline in investment activity. Bankruptcy problems arise as debt service comes to absorb declining real earnings of the sector that is atrophied by the overvalued official rate.

Table 2. *Average Exchange Rates for the Dominican Republic*  
(pesos per $U.S. dollar)

<table>
<thead>
<tr>
<th>Rate</th>
<th>1982</th>
<th>1983</th>
<th>1984a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective: imports</td>
<td>1.19</td>
<td>1.31</td>
<td>2.18</td>
</tr>
<tr>
<td>Effective: exports</td>
<td>1.0</td>
<td>1.15</td>
<td>1.77</td>
</tr>
<tr>
<td>Official</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Parallel</td>
<td>1.46</td>
<td>1.61</td>
<td>2.75</td>
</tr>
</tbody>
</table>

*Source: World Bank data.*  
a. Estimate.
It is obvious then that a dual rate at a level far from the official rate must be a very transitory policy if it is to be effective. Attempts at unification cannot be avoided, and the real wage problem ultimately cannot be solved by implicit trade taxes and subsidies that infect all markets, especially forward-looking financial markets. A more sensible model is the Mexican solution, in which the dual market is used as a strictly transitory shock absorber. Figure 2 above shows the premium in the free Mexican market relative to the controlled rate. The divergence was kept small, although financial disturbances were allowed to affect the premium. But fundamental macroeconomic changes were reflected in the official rate; the premium stayed on average well below 20 percent.

"Black" Markets

This analysis has focused on cases in which the government sanctions an official free market for all nonpreferential transactions. In some cases, the reaction to the high premium is to exclude certain transactions altogether, in particular capital account transactions, from access to foreign exchange. As a consequence, a black market will immediately spring up and function in a manner identical to the dual market already discussed.

Figure 11 shows the premium in the Brazilian "black" or parallel market in the past few years. It show the same erratic pattern as the Venezuelan dual rate; this pattern reflects expectations about major shifts in politics and financial and official exchange rate policy.

The market brings together all unauthorized foreign exchange transactions: import smuggling and coffee export smuggling undertaken to avoid quotas and/or export taxes, unofficial military export revenue, tourism, and capital account transactions. In Dornbusch and others (1983), it is shown that the market is well-behaved: seasonal factors, interest rates, the official real exchange rate, and anticipation of major devaluations ("maxis") explain the behavior of the premium.7

Figure 12 shows the premium in Argentina in the period since Martinez de Hoz. Except for brief periods of unified exchange markets, there has always been a premium. Politics and real interest rates are the main determinants (see Dornbusch and Moura Silva [forthcoming]). The Argentinian example shows how politics can cause a free exchange rate to vary far from PPP. An example is the pre-election period in late 1983: the premium rose to more than 100 percent (in daily data) prior to the election and immediately fell by 40 percentage points on the day after the Alfonsin election. The size and movements of the black-market premium affect resource allocation and inflation and thus pose problems for macroeconomic policy.

7. In fact, even the bid-ask spreads in the black market can be well explained in terms of the theory of dealership (see Dornbusch and Pechman 1983). Interest rates and the variability of the premium that proxies the extent of news explain the size of the spread.
Figure 11. *The Parallel Market Exchange Rate Premium in Brazil*

![Graph of the Parallel Market Exchange Rate Premium in Brazil](image)

*Source: Pechman (1984).*

Figure 12. *The Parallel Exchange Rate Market Premium in Argentina*

![Graph of the Parallel Exchange Rate Market Premium in Argentina](image)

*Source: Organizacion Techint.*
The traditional view of black markets is to see them as an offshoot of the restriction of commercial transactions. But there is sufficient evidence to support the view that they are closely tied to financial markets. At any point in time, there is a given stock of foreign assets in the hands of domestic residents. Given expected returns on domestic assets, there will be a level of the premium that will establish stock equilibrium. The level of the premium in turn influences the flows into and out of the pool of foreign assets in the hands of the public. These considerations are particularly obvious in the case of Argentina. In July 1982, for example, the government chose to solve the problem of the domestic overindebtedness of firms and of the government by freezing nominal interest rates below the rate of inflation. There was an immediate shift of portfolio holders out of domestic assets into blackmarket dollars. Within a day, the premium shot up to above 100 percent. The high level of the premium in turn encouraged underinvoicing of exports, which deprived the government of foreign exchange and of revenue from export taxes and thereby worsened the financial difficulties of the public sector.

Brazil, similarly, has had periods when the black market showed a large premium, for example, at the outset of the debt crisis in late 1982. The level of the premium was so high that a peculiarly inefficient arbitrage occurred. The government allocated foreign exchange for tourists—$1,000 for every man, woman, or child. Given a premium of nearly 100 percent, mothers with ten babies (who fly free of charge) were able to plunder the central bank by flying to and from Miami to exchange dollars for pesos at the international exchange rate. Lines for passports (required to obtain foreign exchange) were for once even longer than those for food. The large premium may have cost the government as much as one billion U.S. dollars in reserves.

An interesting fact, in this context, was the decline of the Brazilian premium in 1984–85 despite large deficits and deteriorating financial conditions. Part of the reason is, of course, the extremely high real interest rate. The rate in Brazil was above 40 percent in real terms and thus more than competitive with any capital gains expected in the black market. But a further element depressing the premium may well have been the fact that the government has ceased purchasing domestic gold, which therefore flowed through the black market. The resulting
increased flow of black market dollars kept a lid on the premium and worked to stabilize expectations.

IV. Conclusions

This study of special exchange rate systems for capital account transactions points out two problematic characteristics of international financial markets for government policy. These are, first, the range of ways that successful policy implementation is precluded in an economy which is fully open to the influence of international asset markets and, second, the macroeconomic repercussions of controlled exchange rate systems which can undermine the long-run effectiveness of such schemes.

In these dual exchange rate systems, capital account transactions are conducted at a free rate while commercial trade is maintained at an overvalued exchange rate. The models of dual exchange rate systems developed here examine the outcome when a government uses credit creation and thus commercial account depreciation to fund government spending. This strategy increases capital account depreciation and thus the premium of capital over commercial account rates. When the devaluation is foreseen by asset holders, as is common, that expectation and the subsequent capital account depreciation will create a trade deficit. The increase in the premium raises wealth relative to target levels and leads to dissavings and a movement out of money balances, which fuels the trade deficit. It is clear that this form of inflationary finance is not sustainable in the long run and that it will create distortions in other macroeconomic conditions which may, by a government's own assessment, offset the benefit from the initial spending increase.

The distortionary effects of the dual rate system are substantially offset, however, if the central bank intervenes to protect the value of the currency (through sale of foreign assets). The premium declines in response to increased foreign asset holdings, and the lower premium reduces the real value of foreign assets and thus wealth. Savings will increase, which will result in decreased spending and imports and an improved trade balance. Because of declining real money balances, however, seignorage again is not sufficient to fund increased government spending.

More severe macroeconomic distortions may be introduced when some commercial transactions are transferred to the free market or when relatively large parallel markets emerge. In this case, the free or parallel market rate directly influences domestic prices. Consumers will substitute toward the now relatively cheaper imported goods traded at the official rate, while producers will move resources out of production for the official market and into activities that benefit from the free rate. These movements will increase supply in the free market, while higher demand and a decline in supply in the fixed rate market will create a
trade deficit. The central bank faces larger trade deficits and loses reserves, while in the free market, a trade surplus leads to accumulation of foreign assets.

In this situation, there are counteracting influences, and the impact on the price level and on savings (and thus on the deficit) is not clear. The effect of multiple exchange rates on the trade balances in the various rate markets is similarly indeterminate. The impact on official reserves will depend on the rates of substitution between the new "midrange" market and the prior fixed and free markets. A partial policy move away from a severely overvalued exchange rate, as a second-best exercise, may actually increase misallocation.

It might be argued that the government is helping workers by sustaining real wages via a low official exchange rate financed by external borrowing. But the deficit is increased by the rising premium: the free market runs a surplus that is privately accumulated, while the government borrows abroad to finance "its" deficit. Those who trade at the free rate, particularly asset holders who can move relatively easily between domestic and international markets, may ultimately be the net beneficiaries of the scheme, not labor.

All these outcomes suggest that the dual rate will be most effective if it is maintained in a range close to the free rate. In this way, the system can buffer the economy from abrupt financial disturbances, but the rate must be allowed to shift in response to fundamental macroeconomic changes.
APPENDIX. THE DUAL EXCHANGE RATE MODEL

The dual exchange rate model assumes a single good, purchasing power parity at the commercial rate, \( x \), and two assets: domestic money and a foreign security (or foreign money). Throughout, \( \lambda \) denotes the rate of depreciation of the official rate and \( u \) the depreciation rate in the free market.

The portfolio balance is given by:

\[ M/eK = L(\dot{e}/e + i^o), \quad L' < 0 \]

or, solving for \( \dot{e}/e \):

\[ \dot{e}/e = b(M/eK) - i^*, \quad b' < 0 \]

where \( e \) is the capital account rate; \( M \) is domestic money; \( K \) is the stock of foreign nominal assets; and \( i^o \) is the foreign interest rate, which is assumed to be zero.

Wealth is defined as the sum of real balances and foreign assets:

\[ a = m + k \]

It is assumed that investment and taxes are zero. A given level of real government spending, \( G \), is financed by domestic credit creation. The growth in the real money stock is determined by the rate of depreciation of the commercial rate, government spending, and the trade surplus, \( B \):

\[ \frac{d(M/x)}{dt} = (G + B) - (M/x)\dot{x}/x \]

The trade surplus is determined by the discrepancy between savings and real government spending. Real savings depends on the gap between target wealth, \( w \), and actual wealth, \( a \), and on anticipated capital gains:

\[ S = v(w - a) - (eK/x)(\dot{e}/e - \dot{x}/x) + (M/x)\dot{x}/x \]

Accordingly, savings has a stock adjustment component and a component arising from the capital gains realized from external assets and the inflation losses incurred on real balances.

Noting that the trade surplus is the excess of savings over government spending,

\[ B = S - G \]

and hence, using equations 15, 16 and 17:

\[ \frac{d(M/x)}{dt} = S = (M/x)\dot{x}/x = v(w - a) \]

\[ + \quad [\lambda - b(M/eK)]k \]

where \( m = M/x \) and \( m \) is the steady state level of real balances given a policy of spending at the real rate \( g \); \( k = eK/x; q = e/x \); and \( \lambda = \dot{x}/x \) and \( \lambda \) is the given rate of depreciation of the commercial rate that satisfies the condition of steady state deficit finance:
It is assumed that there is a unique \( \lambda \) to solve equation 6 (see Bruno and Fischer 1985).

The system can be studied in terms of the two differential equations governing the evaluation of the real value of assets:

\[
\begin{align*}
\dot{m} &= v(w - m - k) + [\lambda - h(m/k)]k \\
\dot{q} &= q[h(m/k) - \lambda]
\end{align*}
\]

These two schedules and the corresponding dynamics are shown in the phase diagram in figure 5, where it is assumes that \( m = 0 \) is negatively sloped.

For any initial value of the real money stock, \( m_0 \), adjustment takes place along the stable trajectory \( ff \) to the steady state at \( E \). In the adjustment process, a trade surplus is accompanied by a real depreciation of the capital account rate.

Once commercial transactions enter the free market, the system of equations becomes:

\[
\begin{align*}
\dot{m}/m &= [v(w - a) - h(m/k)(1 - \alpha)k]/m - \alpha h(m/k) \\
\dot{K}/K &= V(q,a,\lambda)/K - (1 - \alpha)h(m/k) \\
\dot{q}/q &= h(m/k)
\end{align*}
\]

where it is assumed that \( \dot{x}/x = 0 \) and where \( \alpha \) is the share of free market goods in the deflator:

\[
P = P(e,x)
\]

In the steady state, \( h(m/k) = 0 \). Thus the steady state system simplifies to:

\[
\begin{align*}
\hat{V}[q,qK[1 + L(0)],0] &= 0 \\
m &= L(0)qK \\
w &= K[1 + L(0)]\rho; \rho = q^\alpha
\end{align*}
\]

where the last two equations imply that:

\[
m = \sigma w, \quad \sigma = L(0)/[1 + L(0)]
\]

This system is used in the text for comparative statics.

**Bibliography**


