[03.02]

Estimating Government Stock of Fixed Capital

Derek Blades

7th Technical Advisory Group Meeting
September 17-18, 2012
Washington DC
Introduction

If productivity adjustments are made for government in ICP 2011, they will probably require an estimate of the government capital stock. For ICP 2005 the relative sizes of government capital stocks in different countries was (initially) assumed to be proportional to each country’s total constant price GFCF cumulated over the last 10 years. Clearly that is a crude estimate of the relative size of the fixed capital assets held by governments in the different countries.

The short-cut method outlined below should give a better estimate of government capital stocks. It could be used by the countries themselves or by the global or regional offices. It only requires information on government GFCF for a single years and plausible assumptions about depreciation rates and past growth rates of government GFCF.

Net capital stock model

The net capital stock at the beginning of the benchmark year 0 \( (K^t_0) \) is approximately equal to the sum of the assets \( (I^t) \) that were installed in earlier years and that are still in use. The market value of these assets is assumed to decline each year by a constant rate \( (\delta) \) through obsolescence and wear and tear. Equation (1) expresses this relationship:

\[
K^t_0 \approx I^{t_0-1} + I^{t_0-2} (1 - \delta) + I^{t_0-3} (1 - \delta)^2 + \ldots
\]  (1)

Suppose now that GFCF grows each year in real terms by a constant rate \( \theta \) so that \( I^{t_0-2} = I^{t_0-1}/(1 + \theta) \), and \( I^{t_0-3} = I^{t_0-1}/(1 + \theta)^2 \), etc., etc.

Then the net capital stock at the beginning of the benchmark year can be written as:

\[
K^t_0 \approx I^{t_0-1} \left\{ 1 + [(1 - \delta)/(1 + \theta)] + [(1 - \delta)/(1 + \theta)]^2 \ldots \right\}
\]  (2)

Equation (2) is a geometric series with \( \frac{1-\delta}{1+\theta} \) as the common ratio. Summing to infinity, equation (2) becomes:

\[
\text{Sum to infinity of } \left\{ 1 + [(1 - \delta)/(1 + \theta)] + [(1 - \delta)/(1 + \theta)]^2 \ldots \right\} = \frac{1}{(1 - \delta)(1 + \theta)}
\]  (3)

---

1 After consulting with RABs, countries were assigned to 3 groups each with a standard stock/GDP ratio.
2 This is an approximation to \( K^t_0 \) because \( I^{t_0-1} \) will also have depreciated by the beginning of the benchmark year except in the unlikely event that all \( I^{t_0-1} \) occurred on the last day of the year.
3 The sum to \( n \) of a geometric series of the form \( a + ar + ar^2 + \ldots + ar^n \) is \( a(1 - r^n)/(1 - r) \). As \( n \to \infty \), \( r^n \) approaches zero if \( r < 1 \). The sum to infinity then becomes \( a(1 - r)/(1 - r) \). Here \( r = (1 - \delta)/(1 + \theta) \) which must always be less than unity whatever the (positive) values of \( \delta \) and \( \theta \).
In words, equation (3) shows that the net capital stock of the benchmark year \( K^{t0} \) is approximately equal to the GFCF of the benchmark year \( I^{t0} \) divided by the sum of the rate of depreciation \( \delta \) and the average real growth rate of government GFCF \( \theta \).

**Depreciation rate**

Clearly \( \delta \) will vary depending on the type of asset so we need to break down government GFCF into as many categories as possible. At the minimum it would be important to separate machinery and equipment from civil engineering and buildings.

The depreciation term \( \delta \) is calculated as \( d/L \) where \( L \) is the expected service life of the asset and \( d \) is the depreciation factor which is usually set between 1.0 and 2.0. If \( d \) is set at 2.0 depreciation is described as double declining. Here we set \( d \) at 1.6.

As an illustration, Table 1 gives the service lives of government assets estimated by a number of European countries for years around 2000. Most of these countries managed a 7-way breakdown of government assets. The depreciation rates \( \delta \) are 1.6 divided by the service lives shown in Table 1; as an example, for residential buildings in the Czech Republic, \( \delta = 1.6/72 = 0.0222 \) –i.e. government-owned residential buildings are assumed to be depreciating at a constant rate of 2.22% each year.

| Table 1. Service lives in years of government assets in nine European countries (Years around 2000) |
|-------------------------------------------------|---------------|----------|--------|--------|--------|--------|--------|--------|--------|
| | Czech Republic | Cyprus | Estonia | Hungary | Latvia | Lithuania | Poland | Slovak Republic | Slovenia |
| Residential buildings | 72 | 53 | 67 | 55 | 70 | 83 | 67 | 62 | 77 |
| Other buildings | 55 | 53 | 50 | 50 | 50 | 68 | 53 | .. | 77 |
| Roads | 35 | 53 | .. | 42 | 30 | 40 | 33 | 77 | 50 |
| Other infrastructure | 55 | 53 | .. | 50 | .. | 40 | 42 | .. | .. |
| Computers | .. | .. | 6 | .. | .. | 7 | .. | .. | 4 |
| Transport equipment | 9 | 14 | 10 | 7 | .. | 10 | 8 | .. | 8 |
Past growth rates of government GFCF

The growth term $\theta$ is the average real annual growth in government GFCF. Assuming that we have no long time series on real GFCF for government, $\theta$ could be set equal to the long-term real growth rate of GDP.

Illustrative example

The table below gives some illustrative values for $\delta$ and $\theta$.

<table>
<thead>
<tr>
<th>Type of government asset</th>
<th>Depreciation factor ($d$)</th>
<th>Possible service life ($L$) in years</th>
<th>Possible values of $\delta$, i.e. $d/L$</th>
<th>Illustrative values of $\theta$, i.e. long-term real growth rate of government GFCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machinery and equipment</td>
<td>1.6</td>
<td>8-12</td>
<td>0.200 to 0.133</td>
<td>0.04 – 0.06</td>
</tr>
<tr>
<td>Buildings</td>
<td>1.6</td>
<td>50 - 70</td>
<td>0.032 to 0.023</td>
<td>0.04 – 0.06</td>
</tr>
<tr>
<td>Infrastructure (roads, etc.)</td>
<td>1.6</td>
<td>100 - 150</td>
<td>0.016 to 0.011</td>
<td>0.04 – 0.06</td>
</tr>
</tbody>
</table>

Suppose, for example, that we have the following information on government GFCF in the benchmark year (2011):

- GFCF in machinery and equipment is estimated at 246,000 rupiah. Using, as an illustration, the mid-points from the table for $\delta$ and $\theta$ the government capital stock of machinery and equipment will be estimated as $246,000 / (0.167 + 0.050) = 1,133,641$ rupiah.
- GFCF in roads, bridges and other infrastructure is estimated at 403,500 rupiah. Again using mid-points for $\delta$ and $\theta$ the stock of infrastructure assets will be calculated as $403,500 / (0.0135 + 0.050) = 6,354,331$ rupiah.
- Government capital stock is then $1,133,641 + 6,354,331 = 7.5$ mn Rupiah

Possible improvements
This model for calculating a capital stock could be improved in either of two ways:

- If government GFCF is volatile from year to year it may be better to take an average of GFCF for three or four years as the starting point for calculating the capital stock.
- Another possibility is to calculate the capital stock not for the benchmark year but for the earliest year for which a time series of government GFCF (at constant prices) is available. The stock calculated for this earliest year is then updated to the benchmark year by adding each year’s GFCF and multiplying each updated estimate of the stock by \((1 – \delta)\). In this way maximum use is made of the available data rather than using an estimated growth rate of \(\theta\).

**Questions**

1. Will the short-cut method outlined above provide better estimates of government capital stocks than alternative methods?
2. Are data available for most countries on government GFCF with at least a breakdown between machinery and equipment and other GFCF?
3. If yes to both of the above, who should make the estimates of government capital stocks – the countries, RCs or the GO?