Development Research Center

Discussion Papers

No. 15

UNEMPLOYMENT, LABOR MARKET SEGMENTATION, AND THE RETURNS TO EDUCATED LABOR

by

Sebastian Piñera and Marcelo Selowsky

NOTE: Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publication to Discussion Papers should be cleared with the author(s) to protect the tentative character of these papers. The papers express the views of the author and should not be interpreted to reflect those of the World Bank.
The following question is addressed: What is the (shadow) marginal product of labor classified by education and its relation to observed market wages when (a), Educated and non educated labor are different inputs in the production function of the urban economy (b), Unorganized or free entry labor markets for each skill coexist with restricted-entry labor markets with wages above clearing levels (c), We observe unemployment of both types of labor.

By specifying the unemployment behaviour and a production function for the free entry sector, the marginal product of labor by education is derived. The discrepancy between such marginal product and the free entry wage stems from the fact that (a) An extra worker in the labor force can induce additional employment, of that type of labor, in an amount larger or smaller than one. (b) It can also affect the employment of the other skill to the extent both labors are complements or substitutes.

An empirical evaluation of the above discrepancy is then evaluated under two extreme typologies of urban labor markets; such typologies are defined according to the relative size of the restricted entry labor market, its "education intensity", and the unemployment rate. Our findings are that such discrepancy is fairly constant across typologies and basically a function of the demand elasticity for labor in the free entry sector. For a wide range of such elasticity observed wages in the unprotected sector represent an upper bound for the true contribution of each labor. Nevertheless those wages can be appropriately used for assessing the true relative contribution of labor classified by schooling.
I. INTRODUCTION

Assume we are evaluating an educational project which will produce laborers with a given level of education in an urban economy where: (a), unemployment of that type of labor is being observed, (b) we observe a spectrum of wage earnings for that particular labor, such spectrum being characterized by strong differences among wage rates.

This paper addresses the following question: which one of these wages (or combination of them) should we use to compute the marginal product of that labor when he enters the labor force?.

The above scenario is certainly true for the urban economy of a large variety of developing countries and represents a challenge for those evaluating, educational projects in such an environment. Intuitive suggestions on what ought to be the proper wage to be used as the marginal product of an extra worker have been the following:

1/ It is important to notice that this issue is independent and different from the one being discussed in the present debate concerning the "value added" of education, the "screening hypothesis" being one of such arguments.

It has been argued that, even if one accepts the notion that wage differentials by schooling reflect productivity differences, these differentials do not necessarily represent a positive value added of education from the point of view of the production function. The screening hypothesis is perhaps the best known of these arguments: "It suggests that inter-educational earnings differentials, even when standardized for differences due to non-educational factors, reflect no direct productivity - enhancing effects of education but only its effects as a device for signaling pre-existing ability differences" (Layard and Psacharopoulos 1974).

The "value added" arguments, those embodying the hypothesis that wage differentials overstate the true contribution of education can, in principle, be tested. They are empirical rather than theoretical considerations. They basically represent a "missing variable bias" argument: If one had data on all background variables positively colinear with education and having an independent effect on wages (including pre-school levels of ability to take care of the screening hypothesis) one could arrive to an estimate of the true value added of schooling.
(a) The marginal product ought to be zero as long as unemployment of that labor is being observed. The implicit hypothesis is that unemployment is involuntary, i.e., at the wages prevailing in each labor market for that labor there is an excess supply of labor. Under this circumstances an extra worker either becomes unemployed or, by finding a vacancy, it substitutes the employment of a presently unemployed laborer.

(b) It is argued that, if a free entry labor market does exist (where the wage performs a clearing mechanism), the relevant wage should not be zero but the one prevailing in such a market. The acceptance of free entry labor markets precludes obviously the notion of involuntary unemployment. A new type of unemployment behaviour must therefore be specified.

It is important to notice that, in this view, the marginal product of an extra laborer is the free entry (labor market) wage irrespective of the final employment status of that particular worker. If he is lucky and fills a highly paid vacancy in a restricted market (where the wage does not clear the market) his net contribution is still the free entry wage: he has simply substituted a colleague working in the free entry sector. If he decides to become unemployed it means he implicitly induces a presently unemployed worker into a free entry job: this argument implicitly assumes that an additional worker in the labor force leaves the volume of unemployment constant.

(c) The marginal product ought to be a weighted average of the wage earnings of that type of labor in each labor market (including zero for the unemployed), where the weights are the fraction of laborers in each of them. This is equivalent to compute a weighted average of the wages of employed workers corrected by the employment rate.

This technique assumes that extra workers in the economy will be allocated among different markets in the same proportion than the existing labor force.
The view that free entry labor markets do not exist (the only alternative to employment in the formal-restricted markets is unemployment) does not seem to properly characterize a variety of urban economies. This becomes particularly true if self employment is an important employment option.

The notion that free entry markets coexist with restricted markets—the idea behind (b)—has becomes an acceptable working hypothesis in dealing with urban labor markets. What we do want to question is the proposition that in such scenario the free entry wage does represent the marginal product of labor, i.e., an extra worker in the labor force induces additional employment in the free entry sector by one worker.

Our contention is that this last proposition is not at all obvious. It assumes a particular unemployment behaviour that results in a constant volume of extra unemployment in spite of an extra worker in the labor force. The purpose of this paper is to highlight this fact by (a), explicitly specifying such behaviour and then, (b) deriving the contribution to output of an extra worker stemming from such specification.

II. SEGMENTED LABOR MARKETS AND THE EXISTENCE OF VOLUNTARY UNEMPLOYMENT

For the purpose of the analysis we will postulate the following typology of labor markets:

For each type of labor classified by schooling (for simplicity we can refer to "uneducated" and "educated" laborers) we assume there exists basically two labor markets, with different wages prevailing in each of them. One, where there is free entry (the "unprotected" or "unorganized" market for some authors) and where
the wage performs a clearing role; this market includes hired labor as well as self employment as long as the free entry assumption holds.

A second market (or the "protected" market) where, at the wage prevailing in that market, there is a clear excess supply of that labor, i.e., the wage is larger than the one prevailing in the free entry market. The reasons can be several: Minimum wage and social security legislation is implemented in such sector (not being true in the informal sector), the degree of unionization is larger (usually correlated with the size of the firm), multinational firms paying wages clearly above the employment alternatives of its employees due to "image reasons, etc. As Warberger clearly points out: "Protected jobs can readily be identified because so many people want them. Companies paying wages higher than market levels for equivalent skills and working conditions tend to have very low labor turnover and long lists of applicants waiting for an opening to arise."

We will further assume that the free entry wage for educated workers is higher than the wage for non educated workers in the protected sector. This means that there is no incentives for educated workers to compete with non educated workers for their "protected" Jobs. This assumption, although it can change the magnitude of our conclusions, does not change the basic nature of the issues we want to highlight. ¹/

The existence of free entry labor markets implies that observed unemployment must be of a voluntary nature and a result of a process of job search. The idea behind is that a worker increases the probability of obtaining a job in the protected sector by being unemployed and investing in search. Unemployment becomes

¹/ We plan to relax this assumption in a future extension of the paper.
the result of a process of choice where the cost are the present foregone earnings in the free entry sector and the benefits the present value of a higher probability of finding a job in the protected sector.

The notion that urban unemployment - in the presence of unprotected markets - can be viewed as the result of a process of job search appears to be a fruitful scenario to analyze several questions concerning the urban economy. Such notion has been extensively used in the work of several authors; among them the work by Holt (1970), Mortensen (1970), Harberger (1971), Fields (1975) and Eaton and Neher (1975).

In this paper we use the same basic scenario to derive the contribution to output of educated and non-educated workers and its discrepancy from the observed unprotected wage. Such discrepancy stems from the additional employment of both skills induced by an extra worker of a given skill in the labor force. Such induced employment effect is the result of two set of forces: First, the extra worker changes the probability of his colleagues of finding a protected job. Second, to the extent there are diminishing returns and factor complementarity or substitutability, that extra worker changes the equilibrium wages of both types of labor. Such change has a further employment effect by changing the relative payoff of the unemployment option.

In section III we first spell out an employment-unemployment strategy characterizing the behaviour of workers outside the protected sector. In Section IV the above behaviour is integrated into a production function framework characterizing the unprotected sector. The equilibrium conditions of such framework are then differentiated with respect to the endowment of both labor factors so as to derive their marginal contribution to the economy. Finally, in Section V, we attempt an empirical evaluation of the marginal contributions derived before.
For such evaluation we assume two extreme typologies of urban economies: one where we observe a relatively small protected sector and a rather high unemployment rate and a second typology of a rather large protected sector and lower unemployment rates.
III. THE EMPLOYMENT-UNEMPLOYMENT DECISION MAKING

1. The expected flow of earnings out of alternative employment plans

Let us suppose that in any particular period the workers outside the protected sector compare the following set of employment plans:

(a) Plan 1st: To become unemployed during the present period so as to increase - due to search activities - the probability of obtaining a job in the protected sector during next period. If in that period no job in the protected sector is found the plan - as seen from today - does not consider additional periods of unemployment. The worker plans to enter the free entry sector and remain there with lower probabilities of getting a job in the protected sector. 1/

We assume that unemployment today - although it increases the probability of getting a job in the protected sector next period - does not affect that probability once he enters again the free entry sector.

(b) Plan 2nd: To accept employment in the free entry sector and remain there unless one succeeds in getting a job in the protected sector.

We will define therefore:

\[ \Pi_T = \text{expected probability of getting a protected job in any future period } T \] after spending period \( T-1 \) in the free entry sector, where \( T = 1, 2 \ldots \)

\[ \beta = \text{number of times the above probability increases when the worker remains unemployed in period } T-1 \text{ devoting all of his time to search.} \]

\[ \beta \Pi_{T-1} = \text{expected probability of getting next year a job in the protected sector} \]

if one decides for unemployment during the present period, where \( \beta > 1 \).

\[ W_0, W_O = \text{present wage in the protected and free entry sector.} \]

\[ W_T, W_T = \text{expected wage in the protected and free entry sector in any future period } T , \text{ where } T = 1, 2 \ldots \]

1/ Although workers may have an implicit discount rate, we could assume that the lack of capital markets prevents them from considering in their plans additional periods of unemployment.
Table I shows the path of expected earnings implicit in both plans under the assumption that \( \Pi \) remains constant over time and equal to \( \Pi_1 \).

In equilibrium the expected present value of both options must be equal namely:

\[
\sum_{T=1}^{\infty} \frac{\hat{W}_T - (\hat{W}_T - W_T) (1-\Pi_1)}{(1+r)^T} (1-\Pi_1) T = W_0 + \sum_{T=1}^{\infty} \frac{\hat{W}_T - (\hat{W}_T - W_T) (1-\Pi_1)}{(1+r)^T} (1-\Pi_1) T
\]

The left hand side of expression (1) represents the expected present value of the first plan; the right hand side the value of the second plan; \( r \) represents the discount rate. Rearranging terms we can write:

\[
-1 + \frac{(\beta-1)\Pi_1}{(1-\Pi_1)} \sum_{T=1}^{\infty} \frac{(1-\Pi_1)^T}{(1+r)} \left( \frac{\hat{W}_T - W_T}{W_0} \right) = 0
\]

Defining \( \delta = (W - W_0)/W_0 \) as the present wage differential between both sectors and \( g \) as the expected growth rate of wages we can write

\[
\frac{\delta \Pi_1 (\beta-1)}{(1-\Pi_1)} \sum_{T=1}^{\infty} \frac{(1-\Pi_1) (1+g)^T}{(1+r)^T} = 1
\]

\[
\frac{\delta \Pi_1 (\beta-1)(1+g)}{(1+r)- (1-\Pi_1)(1+g)} = 1
\]

\[
\Pi_1 (1+g) = (\frac{\delta}{g}) \text{ where } \delta = (\beta-1)\delta - 1
\]

It is important to notice again that, except for \( \delta \), all of the parameters entering expression (5) represent expected magnitudes; they do not represent actual or effective parameters.
<table>
<thead>
<tr>
<th>Type of Plan</th>
<th>$Q$ (Present)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Total expected earning period $T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choice of unemployment in the present; if bad luck return to the free entry sector.</td>
<td>0</td>
<td>$\beta \pi_1 \hat{W}_1$</td>
<td>$\beta \pi_1 \hat{W}_2$</td>
<td>$\beta \pi_1 \hat{W}_3$</td>
<td>$\hat{W}_T - (\hat{W}_T - W)(1-\beta \pi_1)(1-\pi_1)^T$</td>
</tr>
<tr>
<td>To stay in the free entry sector until one succeeds in obtaining a protected job.</td>
<td>$W_0$</td>
<td>$\pi_1 \hat{W}_1$</td>
<td>$\pi_1 \hat{W}_2$</td>
<td>$\pi_1 \hat{W}_3$</td>
<td>$\hat{W}_T - (\hat{W}_T - W)(1-\pi_1)^T$</td>
</tr>
</tbody>
</table>
Two assumptions have been made in arriving to that expression: first, that the expected probabilities entering the plan remain constant and equal to today's expected probability of finding next period a (protected) job. Second, expectations about equal growth rates for the protected and unprotected wage enter the plan. Both assumptions are not independent; as we will analyze shortly a rational behavioral implies that the second assumption is required for the first one.

2. The relation between expected and effective probabilities.

We will define as $P$ the present effective probability of getting next period a vacancy in the protected sector. Such probability is the relevant for a worker that spends this period working in the unprotected sector. Such probability is equal to:

$$P = \frac{V}{S}$$  \hspace{1cm} (6)

Where $V$ are the vacancies to be open next period in the protected sector - out of employment rotation and net employment growth in that sector. $S$ represents today's equivalent searchers for such jobs, namely:

$$S = L + \beta U$$  \hspace{1cm} (7)

Where $L$ are today's employed workers in the unprotected sector and $U$ the present amount of (voluntary) unemployed workers investing fully in search.

We are now able to connect the expected probabilities with presently observed magnitudes. We will postulate that today's expected probability is equal to the present effective probability $P$, namely:

$$\Pi_1 = P$$  \hspace{1cm} (8)

$$\Pi_1 = \frac{V}{L + 2U} = \frac{(V/L)}{u(1-u)}$$  \hspace{1cm} (9)

where $u$, the unemployment rate is equal to $U/L$ and
\[ \lambda = 1 + U \text{ is the "out of the protected sector labor force".} \] 
Defining \( L_p \) as the employment in the protected sector, \( \lambda \) the rate of openings in that employment \( (\lambda = V/L_p) \), and \( L_p/L \) as the relative "size" of that sector we can write:

\[ \Pi_1 = \frac{\lambda (L_p/L)}{\nu (\beta - 1) + 1} \]

We are now able to identify the type of perception or expectations about the future that is implicit behind expecting a constant \( \Pi \) and equal to \( \Pi_1 \).

First, it implies an expectation of a constant rate of openings and a constant relative size of the protected sector over time. Second, it implies and expectation of a constant rate of unemployment. Such expectation requires, according to equations (5), a constant value of \( \delta \), i.e., expecting a constant and equal growth rate for the protected and unprotected wage over time.

The above considerations can be summarized by stating that a constant expected value of \( \Pi = \Pi_1 \) implies a "steady state expectation" on the part of the labor force.

IV. THE MODEL

1. Assume the aggregate production function in the free entry sector can be described as:

\[ X = F [N, E, K] \]

where \( N \) and \( E \) represent uneducated and educated labor and \( K \) an index of non-labor inputs. The level of employment in such sector is determined where the marginal product of labor is equal to the wage rate \( W \).

\[ F_N (N, E, K) = W_N \]

\[ F_E (N, E, K) = W_E \]
Factor endowment of the labor force in that sector is defined as:

\[(14) \quad E + E_U = \bar{E} \]
\[(15) \quad N + N_U = \bar{N} \]
\[(16) \quad \bar{E} + \bar{N} = \bar{L} \]

Where $E_U$ and $N_U$ represent the amount of unemployed workers of both skills and $E + N = L$ represents the total labor force outside the protected sector.

An equilibrium level of (voluntary) unemployment requires that the present value of both employment plans must be equal.

\[(17) \quad (r - g_N) = p_N (1 + g_N) a_N \quad a_N = (g_N - 1) \delta_N \]
\[(18) \quad (r - g_E) = p_E (1 + g_E) a_E \quad a_E = (g_E - 1) \delta_E \]

where:

$P_N$, $P_E$ = the probability of getting next period a job in the protected sector if this period has been spent working in the free entry sector.

$\beta_N$, $\beta_E$ = the number of times the above probability increases when the worker remains unemployed during this period and devotes all of his time to search.

$g_N$, $g_E$ = today's wage differential between the protected and unprotected sector, for both types of skills respectively.

$\delta_N$, $\delta_E$ = growth rate in wages; for each type of labor this rate is equal for the unprotected and protected wage.

The probabilities $P_N$ and $P_E$ can be written as:

\[(19) \quad P_N = \frac{V_N}{S_N} = \frac{V_N}{p_N N_U + N} = \frac{V_N}{\bar{N} + (1 - \bar{N})N} \]

1/ Notice that from now on the parameters defined in Section 111 have a subscript $N$ or $E$ reflecting its "skill specific" characteristic.
where \( S_N \) and \( S_E \) are the "equivalent searchers" for the protected sector vacancies, for both types of labor respectively.

2. We are interested in evaluating the total contribution to output of an additional worker (of each skill) that enters the labor force, namely \( \bar{N} \) and \( dE \). Such contribution can be different to their respective wage in the unprotected sector \((F_N \text{ and } F_E)\) as a result of two reasons:

(a) one unit of extra labor of a particular type may induce an increase in employment in that type of labor in an amount larger or smaller than one. It will depend on the changes in the net expected gains of being unemployed induced by that additional unit of labor in the economy. The mechanism is the change in the probability of obtaining a protected job induced by that additional labor as well as the change in the equilibrium wage in the unprotected sector.

(b) a cross effect on the employment of the other type of labor. Additional employment of a particular type of labor changes the productivity of the other labor category and therefore changes the net pay off of unemployment for that labor.

The above effects can be summarized as:

\[
(20) \quad \frac{d \bar{N}}{dE} = \frac{V_E}{S_E} = \frac{V_E}{\beta_E E_U + E} = \frac{V_E}{\beta_E E + (1-\beta_E)E}
\]

where \( \frac{dN}{dN} \) and \( \frac{dE}{dE} \) are the "own employment effect" of an extra labor of each type and \( \frac{dN}{dE} \) and \( \frac{dE}{dE} \) represent their "cross employment effect".
Given that \( W_E \) and \( W_N \) are observable market data, the evaluation of \( \frac{dx}{dN} \) and \( \frac{dx}{dE} \) requires knowledge on the "employment terms" described above.

3. Substituting (19) into (17) and (20) into (18) and differentiating with respect to \( \bar{N} \) and \( \bar{E} \) we get the following expression for the "own employment" terms:

\[
\begin{align*}
(23) \quad \frac{dN}{d\bar{N}} &= \left[ \frac{\beta_N}{\beta_N - 1} \right] \left[ \left( \frac{1}{1 - \Lambda_N / \eta_{NN}} \right) \left( 1 + \frac{\Lambda_N \Lambda_E / \eta_{NE} \eta_{EN}}{\Delta} \right) \right] \\
(24) \quad \frac{dE}{d\bar{E}} &= \left[ \frac{\beta_E}{\beta_E - 1} \right] \left[ \left( \frac{1}{1 - \Lambda_E / \eta_{EE}} \right) \left( 1 + \frac{\Lambda_N \Lambda_E / \eta_{NE} \eta_{EN}}{\Delta} \right) \right]
\end{align*}
\]

where:

\[
\Lambda_N = \left( \frac{S_N}{N} \right) \left( \frac{\delta_N + 1}{a_N} \right) > 0 \quad \Lambda_E = \left( \frac{S_E}{E} \right) \left( \frac{\delta_E + 1}{a_E} \right) > 0
\]

\[
a_N > 0 \quad a_E > 0
\]

and

\[
\Delta = \left[ \frac{\Lambda_E \Lambda_N}{\eta_{EE}} \left( 1 - \frac{\Lambda_N}{\eta_{NN}} \right) - \frac{\Lambda_E \Lambda_N}{\eta_{EN} \eta_{NE}} \right]
\]

The parameters \( \eta_{EE} \) and \( \eta_{NN} \) represent the own price elasticity of demand for each type of labor in the unprotected sector; \( \eta_{ij} \) \((i \neq j)\) represents the inverse of the (cross) elasticity of the marginal product of labor \( i \) with respect to the employment of labor \( j \). Therefore:

1/ From the equilibrium condition (5) we can see that a will be positive as long as \( (r-g) \) is positive. Such condition very probably characterizes most empirical situations; at the same time it is also a sufficient condition of convergance for the series being used to transform (3) into (4).
(25) \[ \eta_{ij} < 0 \]

(26) \[
\begin{cases}
\eta_{ij} = 0, & \text{both labor factors are independent.} \\
\eta_{ij} > 0, & \text{both types of labor are complements.} \\
\eta_{ij} < 0, & \text{both types of labor are substitutes.}
\end{cases}
\]

Both expressions (23) and (26) are positive if \( \Delta > 0 \), this condition being fulfilled by any concave production function.\(^1\)

A more intuitive form of examining expressions (23) and (24) is to interpret their terms as:

\[
\begin{bmatrix}
\text{Total own employment effect} \\
\text{effect}
\end{bmatrix} = \begin{bmatrix}
\text{Probability effect} \\
\text{effect}
\end{bmatrix} \begin{bmatrix}
\text{(Own wage effect)} \\
\text{(Cross wage effect)} \\
\text{Total wage effect}
\end{bmatrix}
\]

The own employment effect consists of three multiplicative effects:

(a) A probability effect which is positive and larger than one, i.e., one additional worker in the labor force induces an increase in employment in that type of labor by an amount larger than one.

This effect operates when an additional worker in the labor force increases the number of "protected job" searchers and lowers the probability of getting such a job. The change in the probability induces additional labor into accepting a job in the unprotected sector.

\(^1\) A sufficient condition for \( \Delta > 0 \) is the following relationship among the second derivatives of the production function:

\[
F_{EN}^2 \leq F_{EE} F_{NN}
\]
An explanation for this effect can be derived if we assume the "total wage effect", correcting the "probability effect", to be absent. This will be true if \( \eta_{ii} = \infty \) (additional employment does not drive down the wage rate) and if \( \eta_{ij} = \infty \), the productivity of one type of labor is independent of the quantity of the other one. If the total wage effect is absent, additional employment does not affect the equilibrium wage rate in the unprotected sector, i.e., \( \delta \) (the wage differential) and the value of a remain constant.

An intuitive explanation of the probability effect can now be derived by examining the equilibrium equations (17) and (18). If the value of \( \lambda \) remains constant, preservation of the employment-unemployment equilibrium conditions implies that an extra labor must leave the probability \( P \) invariant; this also means that \( S \) must remain constant in face of a new worker in the labor force.

By examining as an example the value of \( S_N = \beta_N \bar{N} + (1-\beta_N)N \), we can evaluate the required change in employment \( \Delta S \) which is needed to leave \( S_N \) invariant \((\Delta S_N = 0)\) when \( \bar{N} \) changes by \( \Delta \bar{N} \).

\[
\begin{align*}
(27) \quad \Delta S_N &= \beta_N \Delta \bar{N} + (1-\beta_N) \Delta N = 0 \\
(28) \quad \frac{\Delta N}{\Delta \bar{N}} &= \frac{\beta_N}{\beta_N - 1}
\end{align*}
\]

therefore an extra member of the labor force will increase employment in a number bigger than one given our assumption of \( \beta_{ii} > 1 \).

\( \delta \) A wage effect that corrects the above probability effect. Such effect can be subdivided into an "own wage effect" and a "cross wage effect".

If \( \eta_{ii} \) is less than infinite, additional employment will drive down the equilibrium wage rate in the unprotected sector; such change increases the incentives to remain unemployed and offsets some of the positive effect induced by the probability effect. Such offsetting effect - that corrects the probability effect - is the one defined as the "own wage effect". The correction factor is
always lower than one and becomes smaller the less elastic is the demand for that type of labor.

The "cross wage effect" is positive and larger than one, i.e., it reinforces the positive contribution of the probability effect. Notice that this corrective factor increases the employment terms of labor i, independently of labor i and j being complements or substitutes (the product \( n_{ij} \) \( \beta_i \) is always positive). If both are complements additional employment of labor i increases the productivity of labor j and therefore the employment of labor j; this increased employment tends to increase the (productivity) wage of labor i and therefore its employment. If they are substitutes additional employment of i tends to reduce the productivity and employment of labor j. Such reduction in employment of j increases the (productivity) wage of i and reinforces the employment effect.

Finally it is important to notice that the correction factor represented by the "cross wage effect" is identical for both types of labor.

4. Let us now explore the cross employment effects \( \frac{dE}{dN} \) and \( \frac{dN}{dE} \). Differentiating (17) and (18):

\[
\frac{dE}{dN} = \left( \frac{n_{ij}}{\hat{N} - 1} \right) \left( \frac{\Lambda E}{\Delta n_{ij}} \right) \left( \frac{E}{N} \right)
\]

\( > 0 \), complements

\( < 0 \), substitutes

\[
\frac{dN}{dE} = \left( \frac{n_{ij}}{\hat{N} - 1} \right) \left( \frac{\Lambda E}{\Delta n_{ij}} \right) \left( \frac{N_{ij}}{E} \right)
\]

\( > 0 \), complements

\( < 0 \), substitutes

The sign of the cross employment effects depends solely on the sign of \( n_{ij} \), i.e., on both types of labor being complements or substitutes.
5. The employment effects described before can also be shown graphically by deriving a supply of work schedule for both types of labor. Let us derive such supply schedule for one type of labor, let us say labor $N$.

From the behavioural (equilibrium) condition (17) we can solve for $S$ and get:

$$ N^S = \left( \frac{g_N (1+g_N) V}{(r-g_N)} \right) \frac{S}{N} - \left( \frac{V_N (1+g_N) \hat{W}_N}{r-g_N} \right) \frac{1}{W_N} $$

where $N^S$ must be interpreted as the amount of uneducated laborers willing to accept employment in the unprotected sector at a wage $\hat{W}_N$, given the total stock $N$ and the protected wage $\hat{W}_N$. 

The demand for such labor can be derived from the wage determination equation (12) and can be written as:

$$ N^d = N^d (\bar{K}, E, \hat{W}_N) $$

where $\bar{K}$ is the capital stock and $E$ the level of employment of educated labor.

Figure 1 shows the schedules for $N^S$ and $N^d$; $N^S$ is drawn given $\hat{W}_N$ and $S$, the protected sector wage and the stock of labor $\bar{S}$. $N^d$ is drawn given $\bar{K}$ and $E$.

---

1/ This supply function is defined for the range $0 < \hat{W}_N < \hat{W}_N$. In the limit we get:

- $\lim \hat{W}_N = \hat{W}_N$
- $\lim \hat{W}_N = 0$
- $\lim \hat{W}_N \cdot \hat{W}_N = \hat{W}_N \cdot 0$
We can now show the three effects described before determining the change in employment (AN) induced by an extra labor entering the labor force (ΔN).

The horizontal shift of the supply curve in face of an additional laborer ΔN is equal to the pure probability effect. Such effect is equal to $\frac{\beta_N}{\beta_N - 1} \Delta N$, the change in employment that would have taken place had the wage rate remained
constant. If the demand for labor is not completely elastic the wage rate $w_N$ will tend to decline having a negative effect on employment. This is the "own wage effect", described by the second arrow. The net result of both effects must increase employment if the demand for labor has some elasticity.

Finally we came to the third effect, or the cross wage effect, described by the third arrow. An increase in the employment of $N$ will increase the employment of $E$ if they are complements and will decrease it if they are substitutes. In either case the (productivity) demand for labor $N$ will increase with a positive contribution to the employment of that labor.

Notice that figure 1 has been drawn in such a way that the sum of both wage effects is negative, i.e., the own (negative) wage effect is stronger than the (positive) cross wage effect. This does not have to be necessarily so; if the cross wage effect is larger than the own wage effect the net contribution of the total wage effect would have been positive, reinforcing the positive probability effect.

From the above we can conclude that if the "cross wage effect" is equal or larger to the "own wage effect" (a zero or positive "total wage effect") an extra worker in the labor force will induce an increase in employment, of that type of labor, in more than one laborer.

6. We want now to explore the sign of the marginal contribution to output of both factors and their relationship with the observed wages in the unprotected sector.

Substituting (23), (24), (29) and (30) into (21) and (22) we get:

$$
\frac{dX}{dE} = \left[ \frac{\beta_E}{\beta_E-1} \right] \frac{1}{\Delta} \left[ \left( 1 - \frac{\Lambda_N}{\eta_{NN}} \right) w_E + \left( \frac{\Lambda_N}{\eta_{NE}} \right) w_N \right]
$$
The contribution of the own employment effect is always positive. The contribution of the cross employment effect will be positive if both labor factors are complements. In this case the total contribution is unambiguously positive for both types of labor.

If both factors are (technologically) substitutes the contribution of the cross employment effect is negative. In this case a sufficient condition for having a positive total contribution, meaning a dominant (positive) contribution of the own employment effect, is:

\[
\frac{\frac{F_{NN}}{W_N}}{\frac{F_{NE}}{W_E}} < 1
\]

for the case of labor E

\[
\frac{\frac{F_{EE}}{W_E}}{\frac{F_{EN}}{W_N}} < 1
\]

for the case of labor N

these conditions are fulfilled by any type of one or two stage CES production function among the three factors (see Appendix A).

A priori we can not speculate whether the true contribution to output of an additional laborer will be larger or smaller than its observed wage in the unprotected sector. However we can predict that the more elastic is the demand for labor, the more complementary the technical relation among both types of labor and the smaller the premium to search the more likely that the marginal contribution will exceed the observed wage in the unprotected sector.
V. EMPIRICAL EVALUATION

1. A simplified case.

For the purpose of simplicity (and because of lack of information) we will proceed with the assumption that $F_{NE} = 0$. Such assumption can be consistent with two types of scenarios describing the unprotected sector: One where, although N and E enter the same production function, both types of labor are technologically independent. A second scenario where the unprotected sector consists of two subsectors: one employing capital and educated labor and the other capital and uneducated labor.

Under the above assumption the "employment terms" becomes:

\[ \frac{dE}{dY} = \left( \frac{\beta_E}{\beta_E - 1} \right) \left[ \frac{1}{1 - \frac{A_E}{\eta_{EE}}} \right] \]

The cross employment effects vanish ($\frac{dE}{dN} = \frac{dN}{dE} = 0$) and we can write (33) and (34), the marginal contribution to output as:

\[ \frac{dN}{dN} = \left( \frac{\beta_N}{\beta_N - 1} \right) \left[ \frac{1}{1 - \frac{A_N}{\eta_{NN}}} \right] \]

We observe that the pure probability effect is greater than one but it is multiplied by a corrective factor (the own "wage effect") smaller than one.
Hence we cannot say a priori whether the marginal contributions will be greater or smaller than the corresponding observed wages in the unprotected sector.

Before proceeding to the evaluation of these last two expressions we must assure the consistency of the parameters to be used. Recall that the general equilibrium condition, expression (5), can be written as:

\[(41) \quad P \left[ (\beta-1)(1+g)^{E} - (1+g) \right] = (r-g)\]

Arbitrarily choosing labor \(E\) and substituting for \(P_E\) we get:

\[(42) \quad \frac{e \lambda_E}{(\beta_E-1)\mu_E+1} \left[ (\beta_E-1)(1+g)^{E} - (1+\mu_E) \right] = (r-g_E)\]

where:

\[(43) \quad e = \frac{E_P}{E} \quad \text{relative employment of educated workers in the protected sector}\]

\[(44) \quad \mu_E = \frac{E_U}{E} \quad \text{unemployment of educated workers as a fraction of the labor force - of that type of labor} \quad \text{- outside the protected sector.}\]

\[(45) \quad \lambda_E = \frac{V^E}{E_P} \quad \text{vacancies as a fraction of employment of the educated laborers in the protected sector, i.e., the rate of openings.}\]

We can observe \(e, \mu_E, \text{ and } \lambda_E\) and have a pretty good notion for the values of \(\beta_E, g_E, \text{ and } r\). In order to assure that the equilibrium condition (42) holds \(\beta_E\) must be endogenously determined as a residual parameter, i.e., the implicit search premium consistent with observable data if the world behave like the model.

For this purpose we can write:

\[(46) \quad (\beta_E-1) = \frac{E (1+\mu_E) + (r-g_E)}{e \lambda_E \left( (1+g)^{E} - (1+\mu_E) \right) - (r-g_E)\mu_E}\]
The value of \( \theta_E \) so determined, automatically assures the consistency of the model and will be the value used in our estimates of expressions (39) and (40).\(^1\)

Obviously this implies the same exercise must be undertaken for labor \( E \).

2. Two typologies of the urban economy

We will distinguish two typologies of urban economies. They will basically differ in:

(a) The employment in the protected sector relatively to the total labor force outside that sector, \( \frac{L_p}{L} \). We define this ratio as the "size of the protected sector".

(b) The employment of educated workers in the protected sector relatively to the amount of educated workers outside that sector, namely \( e \). The value of \( e \) can be written, given \( \frac{L_p}{L} \), as a function of the "relative educational intensity" of the protected sector:

\[
(47) \quad e = \frac{E_p}{E} = \frac{L_p}{L} \left[ \frac{(E_p/L_p)}{(E/L)} \right]
\]

correspondingly we can define \( n \) as:

\[
(48) \quad n = \frac{N_p}{N} = \frac{L_p}{L} \left[ \frac{1 - (E_p/L_p)}{1 - (E/L)} \right]
\]

(c) The rate of "voluntary" unemployment defined as a fraction of the labor force outside the protected sector namely:

\[
(49) \quad \nu_E = \frac{E_u}{E} = \nu_E (1 + e)
\]

\[
(50) \quad \nu_N = \frac{N_u}{N} = \nu_N (1 + n)
\]

---

\(^1\) If we were to arbitrarily assign a value to \( \theta_E \), we could deduce equilibrium unemployment levels. We are doing just the opposite since \( \theta \) is an unobservable parameter.
where \( \frac{\nu^1}{N} \) and \( \mu^1 \) represent the rate defined with respect to the total labor force. 

Typology I will be defined as having relative to Typology II: (i) a small protected sector, (ii) a large relative educational intensity of the protected sector (iii) a high rate of voluntary unemployment.

The above characteristics are summarized in Table II. Typology I corresponds better to the urban situation of some of the large countries of South East Asia, namely India, Pakistan and Indonesia. Typology II reflects better the situation of some of the more industrialized countries in Latin America namely Argentina, Chile, Colombia. We think most of the other urban scenarios would fall in between both typologies.

We will use two alternative assumptions about the wage differentials between the unprotected and protected sector. One where those differentials are independent of the typology and the other one where they are "typology specific". In the later we postulate that typology I is characterized by larger wage differentials than typology II.

---

1/ If we assume a rate of seasonal and frictional unemployment of \( \mu^* \), we can rewrite (49) and (50) as:

\[
(49') \quad \nu_E = (1+e) (\frac{\nu^1}{N} - \mu^*)
\]

\[
(50') \quad \nu_N = (1+n) (\mu^1 - \mu^*)
\]

Where the rates of voluntary unemployment are expressed in function of the observed rate and the one that can be attributed to seasonal and frictional unemployment.
TABLE 11: COMPARISON OF TWO TYPOLOGIES OF URBAN LABOR MARKETS

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Typology I</th>
<th>Typology II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Size of the protected sector</td>
<td>&quot;Small&quot;</td>
<td>&quot;Large&quot;</td>
</tr>
<tr>
<td>(L_p/L)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Relative educational intensity of the protected sector.</td>
<td>&quot;Large&quot;</td>
<td>&quot;Small&quot;</td>
</tr>
<tr>
<td>(E_p/L_p)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(E/L)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Rate of unemployment of both labor</td>
<td>&quot;High&quot;</td>
<td>&quot;Small&quot;</td>
</tr>
<tr>
<td>(u_N, u_E)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Wage differentials between the protected and unprotected sector</td>
<td>(a) Equal in both typologies</td>
<td>(b) &quot;Larger than in (a)&quot; &quot;Smaller than in (a)&quot;</td>
</tr>
<tr>
<td>(δ_N, δ_E)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. **Empirical Computations**

We first present the *implicit* value of $\varepsilon$ consistent with the "observable" parameters being used. They show the required increase in the probability of finding a protected job - by being unemployed and investing fully in search - that would generate a rate of voluntary unemployment equal to the assumed rate.

<table>
<thead>
<tr>
<th>Assumption on wage differentials</th>
<th>Typology I</th>
<th>Typology II</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Typology independent&quot;</td>
<td>$\beta_N = 15$</td>
<td>$\beta_N = 3$</td>
</tr>
<tr>
<td></td>
<td>$\beta_E = 3$</td>
<td>$\beta_E = 2$</td>
</tr>
<tr>
<td>&quot;Typology Specific&quot;</td>
<td>$\beta_N = 11$</td>
<td>$\beta_N = 4$</td>
</tr>
<tr>
<td></td>
<td>$\beta_E = 2$</td>
<td>$\beta_E = 2$</td>
</tr>
</tbody>
</table>

The values of $\varepsilon$ are larger for typology I, particularly for uneducated workers. $\varepsilon$ must be larger so as to induce a higher rate of voluntary unemployment in spite of a relatively small size of the protected sector (and therefore protected vacancies). This last element is somewhat neutralized in the case of educated workers given that the protected sector of typology I is more "education intensive".

The introduction of the "Typology Specific" wage differential assumption narrows down the range of $\varepsilon$ across typologies: for educated labor it actually makes them equal. The reason is that the new assumption increases the gain of a protected job for typology I and it diminishes it for typology II: this means that a smaller $\varepsilon$ is now sufficient to induce the same degree of voluntary

---

1/ The values were approximated to its nearest whole number.
unemployment under typology I. The reverse is true for Typology II.

Tables IV and V show the marginal contribution of labor in terms of the observed unprotected wage for that labor.

4. Conclusions

The following conclusions can be derived from the results of Tables IV, V and VI.

(a) First, given the demand elasticity for labor, the marginal contribution in terms of the unprotected wage is almost invariant to the typology or wage differential assumption. This is quite true except for the extreme case of a perfectly elastic demand for labor.

This means that, having selected $\eta$, the corrective factor that must be applied to the unprotected wage (in order to derive the true contribution to output) is quite independent of the assumptions used to define the typology.

(b) The correction factor is one for demand elasticities smaller than approximately 2.5. The smaller the elasticity the larger the discrepancy between the contribution of labor and the unprotected wage. This means that the smaller the capacity of the unprotected sector in absorbing additional employment the smaller will be the contribution of labor relatively to the unprotected wage. For an elasticity of one the true marginal product is approximately two thirds the protected wage for the case of educated workers and four fifths for the case of uneducated labor.

(c) The basic magnitude determining the profitability of investing in schooling is the relative marginal contribution of educated to non-educated labor (Table VI).

The range of variation of this ratio is substantially lower than the range of variation of each labor's marginal contribution. This ratio ranges,
for \[ n \leq 2.5, \] between four-fifths and one time the observed wage differential by education in the unprotected sector.

(d) The above considerations can be summarized as follows: the observed wages in the unprotected sector represent an upper bound for the true marginal contribution of each labor. Nevertheless, those wages can be appropriately used for the purpose of assessing the profitability of investing in education.
TABLE III: PARAMETERS USED IN THE TKO TYPOLOGIES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Typology I</th>
<th>Typology II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_P$</td>
<td>0.15</td>
<td>0.30</td>
</tr>
<tr>
<td>$E_P/L_P$</td>
<td>$\frac{0.7}{0.4} = 1.75$</td>
<td>$\frac{0.70}{0.60} = 1.16$</td>
</tr>
<tr>
<td>$E/L$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>$\mu_N$</td>
<td>0.07</td>
<td>0.04</td>
</tr>
</tbody>
</table>

\[ \delta_E = \delta_N = \begin{cases} 
(a) & \text{"Typology independent" wage differential} & 1.5 \\
(b) & \text{"Typology specific" wage differential} & 1.25 
\end{cases} \]

\[ \lambda_E = \lambda_N = 0.10 \]

\[ \kappa_E = 0.03 \]

\[ \gamma_N = 0.02 \]

\[ r = 0.10 \]

\[ \eta_{EE} = \eta_{NN} = \begin{cases} 
(a) & -\infty \\
(b) & -2.5 \\
(c) & -2.0 \\
(d) & -1.5 \\
(d) & -1.0 
\end{cases} \]

1/ Such value is the sum of the net growth in vacancies in the protected sector plus retirement and rotation.

2/ If the unprotected sector consists of two subsectors each employing one type of labor we can write $\eta_{11}$ as $\frac{\sigma}{1-a}$ where $\sigma$ is the (constant) elasticity of substitution between capital and labor and $a$ represents the share of labor.
<table>
<thead>
<tr>
<th>Labor Demand Elasticity</th>
<th>TYPOLOGY</th>
<th>Wage Differential Assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta = 1.0 )</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td></td>
<td>( 0.85 , W_N )</td>
<td>( 0.75 , W_N )</td>
</tr>
<tr>
<td></td>
<td>( 10.85 , W_N )</td>
<td>( 0.75 , W_N )</td>
</tr>
<tr>
<td></td>
<td>Typology independent</td>
<td>Typology specific</td>
</tr>
<tr>
<td>( \eta = 1.5 )</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td></td>
<td>( 0.91 , W_N )</td>
<td>( 0.87 , W_N )</td>
</tr>
<tr>
<td></td>
<td>( 0.92 , W_N )</td>
<td>( 0.86 , W_N )</td>
</tr>
<tr>
<td></td>
<td>Typology independent</td>
<td>Typology specific</td>
</tr>
<tr>
<td>( \eta = 2.0 )</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td></td>
<td>( 0.95 , W_N )</td>
<td>( 0.95 , W_N )</td>
</tr>
<tr>
<td></td>
<td>( 0.96 , W_N )</td>
<td>( 0.94 , W_N )</td>
</tr>
<tr>
<td></td>
<td>Typology independent</td>
<td>Typology specific</td>
</tr>
<tr>
<td>( \eta = 2.5 )</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td></td>
<td>( 0.97 , W_N )</td>
<td>( 1.00 , W_N )</td>
</tr>
<tr>
<td></td>
<td>( 0.98 , W_N )</td>
<td>( 0.96 , W_N )</td>
</tr>
<tr>
<td></td>
<td>Typology independent</td>
<td>Typology specific</td>
</tr>
<tr>
<td>( \eta = \infty )</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td></td>
<td>( 1.07 , W_N )</td>
<td>( 1.30 , W_N )</td>
</tr>
<tr>
<td></td>
<td>( 1.09 , W_N )</td>
<td>( 1.25 , W_N )</td>
</tr>
<tr>
<td></td>
<td>Typology independent</td>
<td>Typology specific</td>
</tr>
</tbody>
</table>

(Pure probability effect)
**TABLE V. MARGINAL CONTRIBUTION OF ONE EDUCATED WORKER IN TERMS OF THE UNPROTECTED WAGE.**

<table>
<thead>
<tr>
<th>Labor Demand Elasticity</th>
<th>TYPOLOGY</th>
<th>Wage Differential Assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>( \eta = 1.0 )</td>
<td>0.69 ( w_E )</td>
<td>0.65 ( w_E )</td>
</tr>
<tr>
<td></td>
<td>0.69 ( w_E )</td>
<td>0.65 ( w_E )</td>
</tr>
<tr>
<td>( \eta = 1.5 )</td>
<td>0.82 ( w_E )</td>
<td>0.80 ( w_E )</td>
</tr>
<tr>
<td></td>
<td>0.83 ( w_E )</td>
<td>0.79 ( w_E )</td>
</tr>
<tr>
<td>( \eta = 2.0 )</td>
<td>0.92 ( w_E )</td>
<td>0.90 ( w_E )</td>
</tr>
<tr>
<td></td>
<td>0.93 ( w_E )</td>
<td>0.89 ( w_E )</td>
</tr>
<tr>
<td>( \eta = 2.5 )</td>
<td>0.98 ( w_E )</td>
<td>1.07 ( w_E )</td>
</tr>
<tr>
<td></td>
<td>0.97 ( w_E )</td>
<td>1.00 ( w_E )</td>
</tr>
<tr>
<td>( \eta = \infty )</td>
<td>1.38 ( w_E )</td>
<td>1.50 ( w_E )</td>
</tr>
<tr>
<td>(Pure probability effect)</td>
<td>1.45 ( w_E )</td>
<td>1.41 ( w_E )</td>
</tr>
</tbody>
</table>
TABLE VI: RELATIVE MARGINAL CONTRIBUTION OF EDUCATED TO UNEDUCATED LABOR IN TERMS OF UNPROTECTED RELATIVE WAGES

<table>
<thead>
<tr>
<th>Labor Demand Elasticity</th>
<th>Typology</th>
<th>Wage Differential Assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>$\eta = 1.0$</td>
<td>$0.81 \frac{W_E}{W_N}$</td>
<td>$0.87 \frac{W_E}{W_N}$</td>
</tr>
<tr>
<td></td>
<td>$0.81 \frac{W_E}{W_N}$</td>
<td>$0.87 \frac{W_E}{W_N}$</td>
</tr>
<tr>
<td>$\eta = 1.5$</td>
<td>$0.91 \frac{W_E}{W_N}$</td>
<td>$0.92 \frac{W_E}{W_N}$</td>
</tr>
<tr>
<td></td>
<td>$0.91 \frac{W_E}{W_N}$</td>
<td>$0.92 \frac{W_E}{W_N}$</td>
</tr>
<tr>
<td>$\eta = 2.0$</td>
<td>$0.97 \frac{W_E}{W_N}$</td>
<td>$0.95 \frac{W_E}{W_N}$</td>
</tr>
<tr>
<td></td>
<td>$0.97 \frac{W_E}{W_N}$</td>
<td>$0.95 \frac{W_E}{W_N}$</td>
</tr>
<tr>
<td>$\eta = 2.5$</td>
<td>$1.01 \frac{W_E}{W_N}$</td>
<td>$1.07 \frac{W_E}{W_N}$</td>
</tr>
<tr>
<td></td>
<td>$1.02 \frac{W_E}{W_N}$</td>
<td>$0.98 \frac{W_E}{W_N}$</td>
</tr>
<tr>
<td>$\eta = \infty$</td>
<td>$1.29 \frac{W_E}{W_N}$</td>
<td>$1.15 \frac{W_E}{W_N}$</td>
</tr>
<tr>
<td>(Pure probability effect)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1.33 \frac{W_E}{W_N}$</td>
<td>$1.13 \frac{W_E}{W_N}$</td>
</tr>
</tbody>
</table>
BIBLIOGRAPHY


