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OPTIMUM TAXATION AND SHADOW PRICING IN A DEVELOPING ECONOMY

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Comments Welcome

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Abstract

This paper analyses the determinants of optimal tax and investment policies in a developing country and investigates how those policies should respond to a sustained rise in the price of imported intermediate inputs such as oil. The effect on the terms of trade between industry and agriculture are also reported. The paper breaks new ground by combining in one model the tax and investment policies considered in the literature on shadow pricing and incentives in the presence of tax restrictions with the notion of labour market imperfection central to contributions on the dual economy. This general model is then subjected to external shocks.

A simple general equilibrium model is constructed and its parameters are chosen so that it can approximately replicate the observed economic data for a particular developing country. Optimal policies are first derived analytically, providing rules for organizing production, setting taxes and offsetting labour market distortions. They are then computed under alternative assumptions about government objectives, its ability to tax transactions within the agricultural sector of a developing economy and the environment in which government policies must operate. While many important insights can be obtained from simple static models, an analysis of investment policy also requires the computation of optimal intertemporal policies. This paper presents optimal one-period policies as well as optimal steady-state policies.

The numerical analysis demonstrates that the optimal tax structure depends crucially on the extent to which agriculture can be taxed and on the assumptions made about the distribution of land ownership among rural and urban residents. It turns out that, whenever tax restrictions are present, increased prices of imported energy reduce the size of government subsidies. There is also a suggestion that increases in the energy price should lead to a reduction in the optimal rate of investment.

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1. Introduction

The purpose of this paper is to explore the relationships between taxation, tariff and investment policies and such key prices as shadow wage rates and accounting rates of interest used in the evaluation of alternative policies and projects in a developing economy. The exercise makes precise the crucial role of assumptions about the domain of government control, the nature of rural to urban migration, the pattern of property rights and trade possibilities open to the economy. The effects of varying these assumptions on optimal public policies and associated shadow prices are then examined both analytically as well as numerically. This can help isolate those considerations to which the results are particularly sensitive and thus point to areas which should receive priority in empirical work.

The motivation underlying this work is the following. Cost-benefit analysis evaluates changes in the economy at shadow prices which represent social opportunity costs. Of the numerous reasons which cause shadow prices to diverge from market prices, two may be cited. First, there are numerous distortions in developing countries, especially with respect to the operation of labour markets. Second, the difficulty of raising resources for investment makes the latter socially more valuable than consumption which accordingly receives a lower weight in cost-benefit calculations. One of the objectives of this work therefore is to examine the relation between market prices and shadow prices in a model which includes labour market failure as well as tax restrictions which can prevent the government from mobilizing resources except in an increasingly distortionary way. 1/

Another objective of this work is provided by the need to discover how optimal policies and shadow prices in a labour market-distorted, tax-restricted developing economy respond to a rise in the price of imported intermediate inputs such as oil. The reason for studying the impact of increased oil prices is clear enough. Many developing countries have suffered considerably from the increased oil prices of the 1970s and early 1980s. It is then interesting to ask how taxes and shadow prices that can be used for decentralized evaluation of policies and projects should respond to such a change in the price of oil.

A fully satisfactory treatment of these questions requires the formulation and empirical implementation of an intertemporal model of an archetypal developing economy. The intellectual antecedents of the present work are: (1) the basic model of optimum taxation and public production due to Diamond and Mirrless (1971), as extended to tax-restricted economies by Stiglitz and Dasgupta (1971), Munk (1980) and Heady and Mitra (1982) and (2) the model of optimal development in a dual economy due to Dixit (1968), Stern (1972) and Newbery (1972). The approach breaks new ground by combining in one model the tax and investment policies considered in the first area with the notion of dualism that is central to the second. This leads to a quantitative framework for studying the main questions of cost benefit analysis as

1/ This is not to deny the practical importance of policy-imposed distortions in trade regimes and other incentive systems. The focus of the present study is the behavior of an optimizing government in the presence of "natural" distortions.

developed in UNIDO (1972) and Little and Mirrlees (1974), and helps make clear that the context for that work is an optimizing government with limited tools at its disposal.

The general model so developed is then subjected to external shocks such as a deterioration in the terms of trade. Empirical evidence on the magnitude of such shocks and the adjustment made by developing countries is analyzed in Balassa (1981) and, using a macroeconomic modelling approach, by Mitra (1983). Numerical simulation of the effects of shocks in applied general equilibrium models of archetypal developing countries has been undertaken by Dervis, de Melo and Robinson (1982). But in doing so within a framework which captures some of the institutional structure of a developing country, the work breaks new ground by providing a unified perspective on the entire range of tax, tariff and production adjustments which are desirable in response to external shocks.

The methodology employed here is similar to that in Heady and Mitra (1982). A general equilibrium model which is simple enough to capture the issues outlined above is constructed and a number of analytical results are derived. The analytical approach provides useful characterizations of certain features of a solution and serves as a valuable check on numerical calculations. But the resulting formulae are frequently too complicated to indicate the magnitude of the wedges between market prices and shadow prices and the sensitivity of those results to underlying assumptions about government objectives, the degree of control over policy instruments and the environment in which those policies operate. To get a feel for these issues, the paper adopts a numerical approach. Parameter values are chosen so that the model can approximately replicate observed economic data for a particular country and optimal policies are then computed under a large number of different assumptions.

The computation of intertemporal policies in this type of model, with several sectors and limited government control, is entirely new and has proved to be difficult and costly in computer time. To make the program of work and the presentation manageable, this paper concentrates on the static model and on a special kind of dynamic model, viz., steady states. Many important insights can be obtained from studying these cases. Results from intertemporal analysis, though available, are at a preliminary stage, and it is hoped to present that work separately elsewhere.

The rest of the paper is organized as follows. Section 2 outlines the basic model. Section 3 discusses those results that can be obtained analytically. Section 4 uses the model to solve for optimal static policies. Section 5 looks at the dynamic aspects of policy by considering the model in steady-states. Section 6 summarizes the main results. The appendix gives a formal statement of the model and derivations of the analytical results discussed in Section 3.

2. The Model

Production and Trade

The economy is divided into a rural sector and an urban sector. The rural sector produces food, which is a traded good, using land and labour, both of which are factors specific to the sector, and fertilizer, which is purchased from the urban sector and the rest of the world. The urban sector produces three goods: a traded consumer good (clothing), a nontraded consumer good (services) and an intermediate good (fertilizer), using capital, urban labour and imported energy. These factors are assumed to be fully mobile among the three urban-based industries. As there is no domestic capital goods industry, any investment takes the form of buying capital goods from abroad. The country is assumed to be unable to borrow or lend internationally. Thus, investment is paid for by a balance of trade surplus on all other goods. It is assumed that food, clothing, fertilizer and energy are internationally traded at fixed world prices. With the exception of energy, which must be imported, the direction of trade is determined endogenously in the model.

Consumption

There is a constant population, and people in the economy are assumed to be identical, except that some live and work in the rural sector and the rest live and work in the urban area. They are assumed to choose their hours of work and consumption of food, clothing and services by maximizing their utility subject to the going prices and other incomes that they have.

Migration

Individuals have the choice of migrating from one sector to the other. This raises the issue of what assumption is appropriate regarding a rural to urban migrant's rights to income from land. The paper presents two polar alternatives. Model 1 assumes that people who migrate from the rural sector give up their rights to land, while migrants into the rural sector acquire rights to land. Thus, land is divided equally among all rural residents. Model 2 assumes that the ownership of land and the income accruing from it are divided equally among the entire population. It is recognized that these alternatives embody different assumptions about the property rights enjoyed by potential migrants. However, empirical evidence on this matter is regrettably scarce in developing countries, so that the present approach to this question must be taxonomic. Sensitivity of the results to these alternative specifications will be illustrated through numerical analysis.

It is assumed that everyone has the same utility function in terms of leisure and consumption goods. However, it is also assumed that people dislike living in urban areas so that their utility is reduced by a given proportion. Thus, if a particular consumption and leisure choice would result in a utility of V_M for a rural resident, it would produce a reduced utility of θV_M (where $\theta < 1$) for an urban resident. A migration equilibrium occurs when the utility of rural residents, V_A, is equal to the reduced utility of urban residents, θV_M.

Ownership

Rural and urban labour are owned by the inhabitants of the respective sectors. The ownership of land is also private but its division across sectors depends on which of the two models of migration is considered. All capital stock is owned by the government, although nothing would be altered if it were privately owned but subject to a 100% profits tax.

Tax Restrictions

All private sector agents behave competitively in as much as they take market prices as given. It is in principle possible to allow for differences in prices between producers and consumers as well as between rural and urban prices. In that event, there are four different sets of prices: urban consumer prices (q), urban producer prices (m), rural consumer prices (r) and rural producer prices (p). However, given the institutional structure of the economy, it may be virtually impossible to vary those prices independently. It is reasonable to impose the restriction that the government cannot drive a wedge between the price at which a rural producer sells food and that at which a rural consumer buys it. Nor, similarly, can the government be interposed between a rural agent in its manifestation as a supplier of labour and land and its manifestation as a demander of those inputs into food production. To assume the contrary would be to allow taxation on intra-household trades. For this reason, it is supposed that $r = p$, i.e., that it is not possible to tax or subsidize internal transactions within the rural sector. Taxes or subsidies can and will however be levied on the sector's transactions with the rest of the economy and the outside world, viz., on the marketed surplus of food and the purchase of clothing, utilities and fertilizer. This limitation to taxing only net trades means that, from a public finance point of view, there is no longer any significance to the distinction between production and consumption in rural areas. There is no loss in simply regarding a farm as a consumer that purchases clothing, services and fertilizer which it finances by selling food.

In addition to the above restriction , which is less an assumption than a virtual necessity dictated by the institutional structure of the problem, it is assumed that there are certain commodity specific tax restrictions. This applies to agricultural output, A, and clothing, C. It is supposed that the ratio of the consumer prices of goods A and C in the rural and urban sectors are equal. The restrictions may be written:

$$q_A = \frac{r_A}{1+\psi}$$

$$q_C = \frac{r_C}{1+\psi}$$

Thus, a consumer from one sector cannot get more favourable terms on these commodities by pretending to be a consumer from the other sector. The assumption is intended to reflect the practical difficulties of implementing dual pricing policies in situations where possibilities of arbitrage are present. It does not apply to services which can often be provided in a discriminatory manner. For example, rural electricity can be sold at a different price from urban electricity.

It is worth noting here and elsewhere that a subsidy is produced by a negative tax on a good an agent buys or a positive tax on a good an agent sells.

Choice of Numeraire

The calculation of taxes and subsidies requires a numeraire to be chosen and it is therefore worth noting that the particular tax rates presented depend critically on that choice. The specification of migrants' rights to land income has implications for the choice of numeraire. In both models, decisions by rural consumers depend solely on rural consumer prices, while decisions by rural (resp. urban) producers depend solely on rural (resp. urban) producer prices. The situation is however different for urban consumers. With all connections to the rural sector severed in Model 1, their economic decisions depend solely on urban consumer prices. This ceases to be true in Model 2, where the retention of rights to land income causes the consumer price of land, in addition to urban consumer prices, to affect their decisions.

Both models permit producer prices in each sector to be normalized independently. Furthermore, Model 1 allows consumer-prices in each sector to be normalized independently as well. In the results presented below, it will be usual to choose rural food as the numeraire for rural consumers as well as for producers. Urban food will be selected as the numeraire for urban consumers. The issue is unimportant for urban producers whom, under the assumptions of the model, it will turn out never to be optimal either to tax or subsidize. Since rural and urban consumer prices both include the consumer price of land in Model 2, they can no longer be normalized independently. It will therefore be usual, when presenting results for Model 2, to choose land as the numeraire for both sets of consumers and, to make the results more comparable, for rural producers as well. Once again, the issue is of no consequence for urban producers.

Shadow Taxes and Subsidies

In models such as these, it will turn out that there is yet another set of prices, viz., shadow prices at which the government calculates the costs and benefits of any proposed policy change. Since shadow prices represent social opportunity costs, it is natural to think of wedges between consumer (resp. producer) prices and shadow prices as capturing the degree of intervention in consumer (resp. producer) decisions. Those wedges may be referred to as "shadow taxes," although for brevity they will be referred to as taxes, unless explicitly indicated otherwise. As an example, the shadow tax rate on an urban consumer good is defined as

$$\left[\frac{\text{Urban consumer price of the good / Shadow price of the good}}{\text{Urban consumer price of numeraire / Shadow price of numeraire}} - 1\right] \text{X } 100\%$$

The definition of shadow prices will become clear once the overall optimization problem has been described.

Social Valuation

In the static version of the model, the government maximizes a social welfare function of the form:

$$W = \frac{1}{\mu}\,[L_A\,(V_A)^{\mu} + L_M(\theta V_M)^{\mu}];\ \mu \neq 0$$

$$= L_A \log V_A + L_M \log \theta V_M;\ \mu = 0$$

where L_A is the number of people in the rural sector L_M is the number of people in the urban sector. V_A is the utility of rural residents, θV_M is the reduced utility of urban residents. μ is a parameter which reflects the government's concern for inequality, ranging from $\mu = 1$ (pure utilitarianism) to $\mu = -\infty$ ("Rawlsian" concern with the welfare of the worst-off individual). It will be noticed that where the migration constraint is binding, so that $V_A = \theta V_M$, the above reduces to

$$W = \frac{1}{\mu}\,L\,(V_A)^{\mu}\quad;\quad \mu \neq 0$$

$$= L \log V_A\quad;\quad \mu = 0$$

where $L = L_A + L_M$, the total population in the economy.

The Optimization Problem

The government chooses consumer and producer prices in each sector to maximize social welfare subject to (1) equality of demand and supply in markets for all goods and factors; (2) overall balance of payments equilibrium and (3) restrictions on possibilities of taxation. The shadow price of a good, which emerges from a solution to this problem, reflects the addition to the economy's social welfare arising from the availability of an extra unit of that good. The requirement of equilibrium on the balance of payments, combined with the assumption that all consumers are on their budget constraints, implies that the government budget must balance, i.e, that the sum of tax revenue and returns to publicly owned capital must equal the sum of subsidies and the value of public consumption.

Dynamic Model

The dynamic version of the model is the same, except that the government can divert some of its revenues from subsidies to investment that will increase next period's capital stock:

$$K_{t+1} = K_t + I_t$$

where K_t is capital stock in period t

I_t is investment in period t .

As mentioned above, such investment requires the importation of investment goods and is thus a charge on the balance of payments.

The government is assumed to discount future utility more heavily, the farther removed it is from the present. The intertemporal objective function is thus:

$$\sum_{t=0}^{\infty} (1 + \delta)^{-t} \frac{1}{\mu} [L_{At}(V_{At})^{\mu} + L_{Mt}(\theta V_{Mt})^{\mu}]$$

for $\mu \neq 0$. The usual modification is made for $\mu = 0$.

Most of this paper is devoted to results from the static model, except for Section 5 which concentrates on optimal steady states

Resource Flows: An Example

Before proceeding to a discussion of the analytical and numerical results, it is useful to trace the main flows of resources in the economy in tabular form for what is described later in the paper as the base case for Model 1, where rural to urban migrants lose all rights to land income. This will also help illustrate the role of the numeraire in the presentation of the results. An entry in row i and column j of Table 2.1 represents a flow from the agent designated on top of the column to the agent designated at the beginning of the row. Thus the entry 0.266 in (row 3, column 1a) represents the expenditure of rural households on own food consumption. A row total shows total incomings for the agent in that row, while a column total signifies total outgoings for the agent in that column. In drawing up the table, it was decided to adopt the following normalization: each set of producer prices, rural and urban, each set of consumer prices, rural and urban and the set of shadow prices are normalized to add to unity. The particular normalization chosen affects the sign and magnitude of the entries in the table, as will become clear.

It is convenient to begin by focussing on public activities in the economy. The public sector is subdivided into commodity-specific procurement agencies who are entrusted with buying commodities from different sectors and the rest of the world at different prices. Thus, for example, the food procurement agency which may, in the developing country context, be thought of as a marketing board, pays agricultural producers 0.836 (row 4a, column 1c) to get marketed surplus at producer prices. Row 1c indicates that the agency sells food to urban households at urban consumer prices (0.279 at column (1b) and exports food at shadow prices (0.195 at column 5). These numbers also show that the agency's expenditures (0.836) exceed income (0.474), so that its losses on food trade must be financed by profits elsewhere in the public sector. By contrast, for example, the fertilizer procurement agency imports at shadow prices (0.018 at row 5, column 1f and sells to farmers at producer prices (0.043 at row 1f, column 4a), realizing a profit on its operations. The activities of the energy agency and the services agency (think of the

Table 3.1

Social Accounting Matrix: Base Case with Migrants Losing Rights to Land

(Sum of urban consumer prices = sum of urban producer prices
= sum of rural consumer prices = sum of rural producer prices
= sum of shadow prices = 1)

		PUBLIC SECTOR								FACTORS					ACTIVITIES					
		Rural Households 1a	Urban Households 1b	Food Procurement 1c	Clothing Procurement 1d	Services Procurement 1e	Fertilizer Procurement 1f	Energy Procurement 1g	General Government 1h	Rural Labour 2a	Urban Labour 2b	Capital 2c	Land 2d	Non-Marketed Food 3	Agriculture 4a	Clothing 4b	Fertilizer 4c	Services 4d	Rest of the World 5	TOTAL
	INSTITUTIONS																			
1a	Rural households									0.529			0.43							0.959
1b	Urban households										1.213									1.214
1c	Food Procurement		0.279																0.195	0.474
1d	Clothing Procurement	0.278	0.35																	0.628
1e	Services Procurement	0.414	0.584																	0.998
1f	Fertilizer Procurement														0.063					0.063
1g	Energy Procurement															0.004	0	0.047		0.051
1h	General Government									0.077		0.657	0.063							0.796
	FACTORS																			
2a	Rural Labour														0.606					0.606
2b	Urban Labour								0.739							0.177	0	0.307		1.211
2c	Capital															0.176	0	0.481		0.657
2d	Land														0.493					0.493
3	NON-MARKETED FOOD	0.266							0.039											0.305
	ACTIVITIES																			
4a	Agriculture			0.816										0.305						1.161
4b	Clothing				0.355															0.355
4c	Fertilizer																			0
4d	Services					0.817														0.817
5	REST OF THE WORLD				0.14		0.018	0.037												0.195
	TOTAL	0.958	1.213	0.816	0.495	0.817	0.018	0.037	0.768	0.606	1.213	0.657	0.492	0.305	1.162	0.355	0	0.817	0.195	10.965

Sum = 2.991

Sum 2.99

latter as an electricity authority) can be described in an analogous way. It should be clear that entrusting procurement agencies with the task of differential pricing is a particular institutional interpretation and that the analysis of the model may be applied to other institutional settings as well.

Consider next the behavior of the private sector. Rural households earn income from labour and land (valued in row 1a, columns 2a and 2d respectively at rural consumer prices) and spend it on clothing, services and food produced on the farm (valued in column 1a, rows 1d, 1e and 3 respectively, again at rural consumer prices). Row 1b and Column 1b describe the behavior of urban households at urban consumer prices. Income equals expenditure for all households. Production activities are portrayed in 4a to 4d. Their incomes originate from selling output to the procurement agencies (with agriculture also "selling" nonmarketed food to itself) and paying commodity and factor inputs. All sectors break even at the appropriate producer prices.

Column 5 shows the economy's export earnings from food, while row 5 portrays expenditures on clothing, fertilizer and energy imports, resulting in balanced trade.

It remains to discuss the operations of general government as well as the factor accounts. The receipts of "general government" arise from the factor accounts. Public ownership of capital confers rents -- 0.657 at row 1h, column 2c. Furthermore, with the particular normalization chosen in the table, the rural consumer price of both labour and land are less than the rural producer prices, resulting in a flow of resources to the government from rural labour (0.077 at row 1h, column 2a) and from land (0.062 at row 1h, column 2d). 1/ Expenditures of general government (column 1h) may be similarly described. With this particular normalization, the urban consumer price of labour exceeds its urban producer price, resulting in a subsidy of 0.729 (at row 2b) while the rural consumer price of food falls short of its producer price, leading to a subsidy of 0.039 (at row 3) on nonmarketed food.

Finally, it will be seen from the table that income and expenditure balance at 2.99 for the public sector as a whole, although not for its constituent parts. This simply reflects overall budget balance in the economy, given that demands for all goods equal supplies, that all agents in the private sector are on their budget constraints and that the balance of trade constraint is satisfied.

3. Analytic Results

This section explains the general results that can be obtained by applying analytic methods to the static model. While a number of these results are standard in the literature, those characterizing the structure of rural taxation and rural-urban migration are new. A formal derivation is provided in the mathematical appendix.

The static model is similar to those used in other work on optimal restricted taxes and public production (Stiglitz-Dasgupta, 1971; Munk, 1980;

1/ Since the tax restriction requires the ratio of consumer to producer prices of all goods in the agricultural sector to be the same, choice of food as the untaxed numeraire would have had the effect of making these flows zero, as well as changing other flows in the table. This would have the "institutional" implication that general government is absent in the rural sector. This illustrates the sign and sensitivity of the numbers and, to some extent, the interpretation on the (arbitrary) choice of numeraire. More positively, that choice and the associated institutional specification depend on considerations not addressed in this class of models.

Heady-Mitra, 1982). The main difference is the introduction of migration and the assumptions about the distribution of land income in the present model. There is also the fact that the same good is allowed to have different prices in different sectors. However, this introduces no new point of principle as the same good in different locations can be regarded as different goods that can be transformed into each other by the government.

The introduction of migration means that the government must take account of the effect of its policies on intersectoral labour mobility. However, this is not difficult to incorporate as the migration mechanism is specified in terms of utility. Thus, any policy that increases the utility of urban residents will promote rural-urban migration, while any policy that increases the utility of rural residents will reduce such migration. This means that if, for example, the government wishes to promote rural-urban migration the tax policies would be the same as if it put a higher weight on urban utility and lower weight on rural utility. Of course the question of whether or not the government wishes to promote migration is a more difficult issue. However, many of the standard results to which reference will be made below simply rely on the government's wishing to maximize a weighted average of individual's utilities and the argument of this paragraph shows that migration can be incorporated simply by altering the weights.

Production Rules

Consider first the manufacturing sector of the economy. Here it is possible for the government to vary consumer prices independently of producer prices. This is so despite the existence of commodity specific tax restrictions, because the latter tie urban consumer prices to rural consumer prices, not to urban producer prices. Although lump sum taxes and subsidies are not permitted; the degree of control available in this economy to a welfare-maximizing government is sufficient to make production efficiency desirable in the manufacturing sector. This result was originally demonstrated by Diamond and Mirrlees (1971). It implies that there should be no taxes on urban producers, i.e., that urban producer prices should equal shadow prices, which in turn equal world prices for a small open economy. Another consequence of this proposition is that there should be no protection or other intervention on trades of the manufacturing sector with the rest of the world. No urban producer taxes will therefore be reported in the numerical analysis of Sections 4 and 5.

The inability to vary consumer prices independently of producer prices in the agricultural sector implies that it will be necessary to tax producers and consumers together. Since producer taxation equals consumer taxation (as the two sets of prices must be the same) 1/, Sections 4 and 5 will only report one set of taxes. The resulting wedge between rural market prices and shadow prices amounts to an argument for trade intervention in agriculture, for example, a tariff (positive or negative) on fertilizer to achieve an aim that could have been obtained more directly and without production inefficiency if the tax restrictions were not present.

Another characterization of optimum production is available here. Since all production activities, including agriculture, are assumed to be characterized by constant returns to scale, all activities that are used at an optimum have zero profits at producer prices. Notwithstanding overall

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1/ This is trivially true for clothing and services, whose rural producer prices have no economic significance, and fertilizer, whose rural consumer price has no economic significance.

production inefficiency in the economy, those activities, as shown by Diamond and Mirrlees (1976), will have zero profits at shadow prices as well. Indeed, it is this argument which leads to the celebrated Little and Mirrlees (1974) rule for shadow pricing a small country's tradeables at their world prices.

Tax Rules

Economic intuition on tax rules is conveniently developed by imagining a situation of no tax restrictions other than the unavailabilty of lump sum taxes and subsidies. In this case the standard Ramsey rule for efficient taxation applies to both urban and rural consumer taxation. Thus, a small equiproportionate intensification of consumer taxation, with constant shadow prices, would produce equal proportionate reductions in compensated demand for all urban consumer goods. A similar statement would apply to rural consumer goods, but the proportionate reduction in agriculture would be zero, as the tax would fall entirely on the inelastically supplied land. This point is confirmed in Section 4 below.

The introduction of the commodity specific tax restriction would involve modification of the above rules to assure equal relative prices in the two sectors for the goods involved. This will, as is shown in Section 4, involve a change in all tax rates.

Since no further restrictions other than the above characterize the manufacturing sector, it is clear that the Ramsey rule for efficient taxation applies to services and labour there, with a modification in the case of food and clothing which are subject to the commodity specific tax restriction.

The situation is however very different in the agricultural sector. The fact that internal transactions may not be taxed affects the structure of rural taxation greatly, not least because in the present example it prevents all taxation from being placed on land. The following result can be derived for Model 1, where all land is owned by the rural population. When only net trades can be taxed, then, in the absence of commodity specific tax restrictions, the Ramsey rule for efficient agricultural taxation requires that the equiproportionate increase in taxation, with constant shadow prices, should produce equal proportionate reductions in compensated net trades for all goods. For land and labour, which are not traded, this requires that the absolute reduction should be zero. Once again the proportionate reduction is not the same as that in the urban sector and it must be modified for those goods that are subject to the commodity specific tax restriction. This result has not been derived before in the published literature. However, it does not hold for Model 2 where land is owned by all agents in the economy, so that urban considerations inter alia influence rural tax rates.

Finally, it should be noted that the tax rule described in the preceding paragraph will generally imply non-zero taxes for land and labour despite the fact that neither can be taxed directly. This apparent paradox can be resolved by realizing that the taxes used in the tax rule refer to divergences between consumer prices and shadow prices, while the tax restriction refers to divergences between consumer prices and producer prices. Thus, the taxes reported below for land and labour in agriculture are wedges between consumer prices and shadow prices that are caused by distortions elsewhere in the economy, not by taxation applied directly to them.

Migration

A marginal rural to urban migrant has no direct effect on social welfare. This is because the extra urban utility gained, $\frac{1}{\mu}[\theta V_M]^{\mu}$ exactly balances the rural utility lost $\frac{1}{\mu} V_A^{\mu}$, by the migration condition. In Model 2, where the entire population owns land, indirect net benefit of the move is the cost of the rural consumption bundle he give up, less the cost of the urban consumption bundle to which he lays claim, where both are evaluated at shadow prices. Since this must be zero at an optimum, the shadow cost of a consumption bundle in the two sectors must be equal. This is easily shown to imply an equal tax burden per capita on rural and urban consumers at an optimum. Thus, this condition states that the tax system should be such as to neither encourage nor discourage migration. It is similar to the production efficiency result in that it forbids government intervention in all allocative processes despite the existence of distortionary taxation.

The situation is different in Model 1 where rural and urban migrants lose all rights to land. This creates an externality in the form of an increase in land income for those left behind in the rural sector. The corresponding policy intervention takes the form of the tax burden per capita on rural consumers exceeding that on urban consumers by the net social value of the ensuing change in land incomes.

These results on optimal migration in the presence of distortionary taxation have not been derived before in the published literature.

4. Static Results

The results of solving the Models 1 and 2 with specific functional forms for single period optimal policies are reported in Tables 4.1 and 4.2

respectively. [1/] The case that is of central concern is characterized by untaxability of trades internal to agriculture and by the commodity-specific tax restriction. The figures appear in column 1. As mentioned before, the untaxed numeraire is food for Model 1 and land for Model 2. In interpreting the results, it should be noted that a positive tax on a good supplied by consumers constitutes a transfer from the government to those consumers.

In column 1 of Table 4.1,, urban consumers gain both from the negative taxes on clothing and services and from the positive tax on labour. The only good that is supplied by the rural sector is food, which is the untaxed numeraire. However, it benefits from the negative taxes on all its purchases: clothing, utilities and fertilizer. Land and labour in agriculture are assumed to be untaxed and so the taxes reported on them do not represent transfers of resources, simply differences between private prices and shadow prices that result from the other distortions in the economy. To remind the reader of this fact, these taxes are enclosed in parentheses in the table.

The reason for the government's ability to transfer resources to both urban consumers and the rural sector is that it receives all the profits from the urban sector, where it owns the capital stock. In distributing these profits to the two groups, and in deciding the tax rates on particular goods, it is guided by considerations of efficiency. Equity is not an issue because the migration mechanism completely determines the relative utilities of the

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1/ Demand functions are obtained from a linear expenditure system that applies to both sectors. The choice of technique is made from a nested constant elasticity of substitution production function. In the rural sector, land and fertilizer are combined to form a subaggregate, which is then combined with labour to produce output. The manufacturing industries are treated similarly but capital and energy form the subaggregate, instead of land and fertilizer.

Table 4.1

Optimal Taxes in the Static Model
With Land Owned by the Rural Population
(Food untaxed)

	(1)	(2)	(3)	(4)	(5)	(6)	([illegible])	(8)	(9)
	Base Case $s_E = 1$	Base Case $s_E = 5$	No C-S-T $s_E = 1$	No C-S-T $s_E = 5$, $\mu = 1$	Base Case No migration $s_E = 1$, $\mu = 1$	Base Case No migration $s_E = 1$, $\theta = 1$	Base Case No migration $s_E = 1$, $\mu = -5$	No Tax Restrictions $s_E = 1$	No Tax Restrictions $s_E = 5$
Urban									
Clothing	-22.2%	-19.1%	16.1%	15.7%	-36.1%	-33.9%	-24.1%	14.6%	14.7%
Services	-7.7%	-3.4%	21.3%	21.3%	-19.9%	-17.9%	-9.1%	19.7%	19.6%
Labour	77.9%	77.3%	124.7%	118.3%	37.3%	43.8%	72.7%	104.9%	102.4%
Rural									
Clothing	-22.2%	-19.1%	-30.8%	-27.9%	-36.1%	-33.9%	-24.1%	0%	0%
Services	-31.2%	-28.1%	-28.3%	-25.3%	-43.1%	-41.4%	-32.9%	0%	0%
Labour	(-6.2%)	(-5.3%)	(-6.5%)	(-5.6%)	(-10.6%)	(-9.8%)	(-6.7%)	0%	0%
Land	(9.5%)	(7.9%)	(9.7%)	(8.0%)	(18.1%)	(16.4%)	(10.4%)	87.2%	74.6%
Fertilizer	-6.0%	-5.2%	-1.9%	-1.4%	-8.2%	-4.8%	-6.2%	0%	0%
V_A	3.90	3.71	3.91	3.72	4.32	4.25	3.95	3.94	3.74
θV_M	3.90	3.71	3.91	3.72	3.51	3.68	3.86	3.94	3.74
L_A	0.50	0.49	0.52	0.51	0.50	0.50	0.50	0.59	0.57

Notes: (1) s_E stands for the world price of energy

(2) C-S-T is an abbreviation for Commodity Specific Tax Restriction.

two groups. However, in pursuing its goal of efficiency, the government must take account of three factors: (a) the tax rate on clothing is restricted to be the same for both groups; (b) it is assumed that no trades within agriculture can be taxed; and (c) the tax rates will have an effect on migration.

The significance of point (a) can be evaluated by looking at column 3 of Table 4.1, which reports the optimal taxes when the commodity-specific tax restriction is dropped. A comparison of columns 1 and 3 shows that, as expected, the tax rates on clothing change when the restriction is eliminated. However, the effects are not limited to that. In the urban area there is a shift away from commodity subsidies and an increase in the (utility raising) tax on labour. Also, in the rural sector, the increased subsidy on clothing allows the government to reduce the subsidies to services and fertilizer. Thus, the impact of the commodity-specific tax restriction on the pattern of taxes is considerable. However, the reported values for V_A and ΘV_M show that its impact on utility levels is small.

The significance of point (b), in addition to point (a), can be evaluated by looking at column 9 of Table 4.1, which reports the optimal taxes in the (unlikely) event of the government's being able to tax trades internal to the agricultural sector. The most dramatic effect can be seen in agriculture where the possibility of taxing land means that the government can transfer resources to agriculture in a non-distortionary manner. Thus, no other taxes are levied on that sector. The pattern of taxation in the urban sector is not affected significantly in comparison to column 3, but all the tax rates are reduced, so that each urban consumer receives less from the government. This is consistent with the fact that rural employment (L_A)

increases, in response to the less distortionary form of rural subsidy, thus leaving fewer urban workers and a smaller urban profit for the government to distribute. The figures for V_A and θV_M indicate a more substantial utility gain to the relaxation of the untaxability of agriculture than the commodity-specific tax restriction.

Finally, the significance of migration can be considered by looking at columns 5, 6 and 7 of Table 4.1, which report the optimal tax rates with all the tax restrictions in place, but with no migration. For ease of comparison, the distribution of the labour force between urban and rural areas has been fixed at the optimal level in column 1. Two sets of results are reported because the absence of migration means that the urban-rural utility differential is no longer fixed. Thus, the government will take distributional considerations into account in setting taxes. Column 5 gives the results for $\mu = 1$, where the government is not concerned about inequality. Column 7 gives the results for $\mu = -5$, where the government is greatly concerned to reduce inequality. 1/

A comparison of columns 1 and 5 shows that the elimination of migration unambiguously increases the subsidies to rural areas. There is a significant reduction in the (utility-raising) tax on urban labour, and some increase in the subsidy to urban services. The increase in the subsidy to urban clothing is required by the commodity-specific tax restriction when the subsidy to rural clothing is increased. In fact, rural utility increases while urban utility falls. The reason for this shift in government support is the structure of rights to land income in Model 1. Recall that migration

1/ The results for column 6 are discussed in the footnote on page 29.

creates an externality in the form of an increase in land income for those left in the rural sector. This tends to lead to too little migration, which the government attempts to correct by giving larger subsidies to urban consumers. However, when migration is prohibited and the distribution of labour fixed at its optimal base case level, as in column 5, this reason for differential subsidy disappears and resources are switched back into agriculture. This point will be reinforced in the discussion of Table 4.2 for Model 2 where no such externality is present.

This inequality in outcome in column 5 is clearly not what is required by a government that desired equality. Thus, column 7 shows a pattern of taxation much closer to that in column 1, which produced the same value of L_A when equality was imposed through the migration mechanism.

The next set of experiments consider the impact of increases in energy prices on optimal government policy. Clearly, much of that impact will relate to such issues as the rate of investment. However, there will be some impact on tax rates and patterns of production in the short run. Accordingly, columns 2, 4 and 9 report the optimal taxes that correspond to the models of column 1, 3 and 8, but, in each case, it is assumed that the world price of imported energy has increased by a factor of five.

A comparison of columns 1 and 2 shows the effect on optimal taxes in the base case of an increase in the energy price. This shows that the government reduces (the absolute size of) all taxes, thus reducing transfers to both sectors of the economy. This is partly the result of the reduced profitability of urban production and partly the result of a slight shift of the labour force out of the relatively lightly subsidized rural sector into the urban sector, thus increasing the cost of any subsidy programme. This shift in the labour force is mainly accounted for by import substitution in clothing in response to increased energy prices. The reduction in energy imports is accompanied by a fall in production of services -- the most energy-intensive sector in the economy.

Another variable of interest, apart from the taxes, is the terms of trade between agricultural products and manufactured goods. In this model the effect of the energy price rise is to increase the prices of both clothing and services relative to the price of food in both the urban and the rural sectors. Thus, the relative price of clothing rose from 0.78 to 0.81 for both urban and rural residents. The relative price of services rose from 0.99 to 1.12 for urban residents and from 0.73 to 0.85 for rural residents.

As well as looking at the impact of energy price increases in the base case, it is of interest to see whether the extent of tax restrictions on the government affects its ability to deal with those increases. Thus, a comparison of columns 3 and 4 shows the effect of energy price increases when there is no commodity-specific tax restriction, while a comparison of columns 8 and 9 shows the effect when there are no tax restrictions at all. It turns out that the effects are very similar to those in the main case: all tax rates are reduced, with the exception of a slight increase in the tax on urban clothing in column 9. Furthermore, the magnitude of the utility reductions are all very similar (either 0.20 or 0.19). Thus, there appears to be little connection between the extent of tax restrictions and the ability of the government to deal with energy price changes, at least over the range of restrictions analyzed here.

Table 4.2 reports the corresponding figures for Model 2 where the entire population owns land. In looking at these results, it is worth

Table 4.2

Optimal Taxes in the Static Model With Land Owned by the Entire Population (Land Untaxed)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Base Case $s_K = 1$	Base Case $s_K = 5$	No C-S-T $s_K = 1$	No C-S-T $s_K = 5$	Base Case No Migration $s_K = 1$ $\mu = 1$	Base Case No Migration $s_K = 1, \theta = 1$	Base Case No Migration $s_K = 1, \mu = -5$	No Tax Restrictions $s_K = 1$	No Tax Restrictions $s_K = 5$
Urban									
Food	-67.8%	-66.2%	-74.6%	-73.1%	-78.4%	-79.0%	-69.7%	-72.1%	-70.8%
Clothing	-79.0%	-77.7%	-74.6%	-73.1%	-79.0%	-79.0%	-79.0%	-72.1%	-70.8%
Services	-76.7%	-75.3%	-74.6%	-73.1%	-78.8%	-79.0%	-77.0%	-72.1%	-70.8%
Labour	-76.7%	-75.3%	-74.6%	-73.1%	-78.8%	-79.0%	-77.0%	-72.1%	-70.8%
Rural									
Food	(3.6%)	(3.0%)	(-1.0%)	(-1.1%)	(0.3%)	(0%)	(3.2%)	-72.1%*	-70.8%*
Clothing	-32.4%	-32.0%	-45.5%	-44.2%	-2.4%	0%	-28.6%	-72.1%	-70.8%
Services	-42.9%	-41.9%	-42.7%	-41.4%	-4.4%	0%	-39.2%	-72.1%	-70.8%
Labour	(7.8%)	(6.5%)	(-1.4%)	(-1.7%)	(0.6%)	(0%)	(0.5%)	-72.1%*	-70.8%*
Fertilizer	-7.0%	-6.9%	-6.1%	-5.9%	-0.5%	0%	-6.1%	0%	
V_A	3.963	3.769	3.977	3.781	3.015	2.946	3.829	4.031	3.828
θV_H	3.963	3.769	3.977	3.781	4.340	4.444	4.024	4.031	3.828
L_A	0.247	0.258	0.269	0.277	0.247	0.247	0.247	0.401	0.394

* These tax rates apply to rural consumers. The corresponding tax rates for rural producers are zero.

recollecting two features. First, with the migration equilibrium condition equalizing utilities in the two sectors, it is optimal for the government to neither encourage nor discourage migration. The tax burden per capita is the same for rural and urban consumers. Second, the untaxed numeraire, land, is also inelastically supplied. Therefore, if there were no restrictions other than the unavailability of lump sum instruments in a particular sector, it would be optimal to have a uniform tax structure there. When the commodity specific tax restriction is removed, this situation obtains in the urban areas, as is shown by the uniform subsidy on all commodities in column 3. If internal trades could be taxed in agriculture, uniformity would be optimal there as well, as column 9 confirms.

The urban tax structure in the base case does not deviate very much from uniformity. Compared to model 1, there is now no argument for favored treatment of the urban sector. Clothing and services are subsidized in rural areas. Taxes on rural food and rural labour are enclosed in parentheses to remind the reader that these do not represent transfers of resources.

Columns 5, 6 and 7 report the case of no migration with the distribution of the labour force between urban and rural areas fixed at the optimal base case level of column 1. With $\mu = 1$, which corresponds to no concern for equality, urban utility is higher than the rural; with $\mu = -5$, which corresponds to a substantial concern for inequality, the differential is considerably narrowed. When $\theta = 1$ (column 6), urban agents' utility (θV_M) is socially no less valuable than those of rural agents (V_M) and efficiency requires that the government intervene solely in the sector with fewer tax restrictions, viz., the urban sector which then has a uniform tax structure. [1/]

The remaining columns report the effects of a five-fold rise in energy prices on the tax rates. The results are similar to those of Model 1.

Government Revenue

Model 1 is next used to explore the effects of introducing an exogenously specified government revenue requirement on shadow tax rates. This may be set in terms of any of the goods and, to fix ideas, it is set in terms of fertilizer. Specifically, it is assumed in the experiments that the shadow value of fertilizer demanded by the government as a proportion of national income evaluated at shadow prices varies from 1% to 5%.

Column 1 of Table 4.3 reproduces the base case of Model 1. The last row calculates the government's shadow "nontax" budget surplus, i.e., the excess of shadow rents from public ownership of capital over the exogenously specified shadow revenue as a proportion of shadow value added, the latter being national income calculated at shadow prices. This figure is 37.4% in the base case, indicating, as argued before, that the government's problem here is the efficient disposal of subsidies through the indirect tax system. A comparison of columns 1 and 2 shows that the introduction of a revenue requirement cuts the shadow nontax budget surplus and hence urban subsidies on clothing, services and labour. Similarly, subsidies on rural purchases of clothing, services and fertilizer are also reduced. As expected, these trends are intensified in column 3, where the revenue requirement as defined above is 5%.

1/ For completeness, the $\theta = 1$ case is also reported in column 6 of Table 4.1. The unavailability of an inelastically supplied factor in Model 1 implies that uniformity is not optimal there.

Table 4.3

Optimal Taxes in the Static Model with Land Owned by the Rural Population Varying Revenue Requirement (Food Untaxed)

	(1) Base Case (Zero Revenue)	(2) 1% Revenue Requirement	(3) 5% Revenue Requirement	(4) 36.6% Revenue Requirement
URBAN				
Clothing	-22.2%	-21.2%	-16.8%	42.9%
Services	-7.7%	-6.7%	-3.1%	33.5%
Labour	77.9%	77.7%	76.1%	56.8%
RURAL				
Clothing	-22.2%	-21.2%	-16.8%	42.9%
Services	-31.2%	-30.1%	-25.3%	49.7%
Labour	-6.2%	-5.9%	-4.6%	5.2%
Land	9.5%	8.9%	6.7%	-5.9%
Fertilizer	-6.0%	-5.7%	-4.7%	3.0%
V_A	3.90	3.87	3.76	2.73
L_A	0.503	0.501	0.493	0.427
Shadow Nontax Budget Surplus / Shadow Value Added 1/	37.4%	36.4%	32.9%	0%

1/ $\dfrac{s_K K - s_F G_F}{SVA}$

In the final column, the revenue requirement is increased to the point where the shadow nontax budget surplus is driven to zero. In this situation, with no subsidy to dispose of, the sole reason for intervention is the externality associated with rural and urban migration. The subsidy to urban labour required to offset that distortion is financed through taxation of all goods both in the urban and rural areas. It is interesting that the revenue requirement needed to generate a zero shadow nontax budget surplus (column 4) is close to the shadow nontax budget surplus when the revenue requirement is zero (column 1).

5. Steady-State Results

The results in the previous section showed that an increase in world energy prices had some effect on optimal government policy. However, that static effect ignored the possibility that energy price rises might alter the optimal rate of investment. It is the aim of this section to investigate that possibility.

There are two different ways in which energy prices might affect the optimal rate of investment. The first, which may be regarded as the "demand side" effect, is the effect that energy prices might have on the quantity of investment that yields any particular rate of return. This could take the form of a reduction in the marginal product of capital consequent upon a rise in the price of energy, which in the model is a complement to capital 1/. Alternatively, the demand for investment could be affected by a change in industrial structure that resulted from the rise in the energy price. The

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1/ This reduction in the marginal product of capital was confirmed in the experiments that lie behind the results discussed in Section 4.

second, or "supply side," effect would be on the quantity of resources that the government would be prepared to devote to investment at any particular rate of return. In contrast to the demand side effect, which would be expected to cause a reduction in investment when energy prices rise, the supply side effect could either increase or reduce investment. It would reduce investment if the government felt that the reduction in the real income of the country meant that it could no longer afford as much. On the other hand, it would increase investment if the government saw a higher capital stock as a way of compensating for the reduction in real income.

In order to investigate these different effects, it would be desirable to find the infinite horizon optimal investment path and then analyze how it is affected by changes in energy prices. In practice, of course, it is only possible to compute finite horizon paths which approximate the early stages of the optimal infinite horizon path. However, the necessary complexity of the model makes the computation of even short finite horizon paths a very difficult exercise. Some progress has been made in this direction, but it is better at this stage to concentrate on the results of a somewhat different approach: computation of steady-states. As is explained below, this technique is not as satisfactory for answering some questions as the computation of intertemporal investment paths. However, it is worth noting that preliminary results with intertemporal investment paths tend to confirm the results obtained from steady-state analysis.

The basic idea behind the steady-state analysis is that, in this type of model, there is some size of the capital stock at which it is not worthwhile to undertake further investment. Thus, if the country somehow attained this capital stock, the government would not invest and the economy would remain unchanged from then on. One reason for being interested in such steady-states is that for some types of model all infinite horizon optimal investment paths make the economy's capital stock approach the steady-state capital stock closely. 1/ Thus, knowledge of the steady-state implies some knowledge of the optimal investment path. For example, if economy A was known to have a steady state capit al stock higher than that of economy B, and if both economies start from the same capital stock, it must be true that over at least some time period economy A's optimal investment rate is higher than economy B's.

Column 1 of Table 5.1 reports the steady state optimal tax policy for Model 1 in the base case, with migration and all tax restrictions. The last row reports the capital stock in the steady-state (K*) divided by the initial capital stock (K^0). Thus, the economy in column 1 has the same population as the economy in column 1 of Table 4.1. This enormous difference in capital-intensity has produced some changes in the tax structure. For example, the urban areas have experienced a shift from consumption subsidies to income subsidies and, via the tax restriction, this has altered the relative subsidies to rural areas, where very few people now work.

The effect of the energy price increase can be seen by comparing column 2 of Table 5.1, where the energy price has been increased by a factor of five, with column 1. A comparison of the taxes shows that the net

1/ It is possible for the optimal investment path in models with more than one sector to go into a continual cycle rather than seek the steady state. Thus, it is not possible to be sure of the inferences about the optimal investment path that are drawn from the steady-state results. It is therefore reassuring that they are confirmed by preliminary results on intertemporal investment paths. It is hoped to report these results on another occasion.

Table 5.1

Optimal Taxes in the Steady-State
With Land Owned by the Rural Population
(Food Untaxed)

	(1) Base Case $s_E = 1$	(2) Base Case $s_E = 5$	(3) Base Case with Revenue Required
Urban			
Clothing	0.8%	1.2%	0.8%
Services	1.0%	1.5%	1.1%
Labour	130.6%	130.2%	111.1%
Rural			
Clothing	0.8%	1.2%	0.8%
Services	-37.3%	-37.3%	-29.8%
Labour	-12.0%	-11.5%	-9.0%
Land	10.3%	10.6%	7.3%
Fertilizer	-12.3%	-12.2%	-9.5%
V_A	19.53	15.66	18.65
K^*/K^0	140.8	91.5	146.2
L_A	0.001	0.003	0.001

transfers from the government are slightly reduced, but that the effect is very small: less than 1% for both goods in both sectors. However, the steady state capital stock has been reduced very significantly. Thus, in steady-state, the effect of an energy price rise is almost entirely felt through a reduction in the capital stock.

It is of some interest to see whether the reduction in the steady-state capital stock is caused mainly by the effect that energy prices have on the balance of payments or by the effect they have on relative prices. In order to investigate this, an experiment was performed which involved a reduction in real income caused by the requirement to make a payment (such as loan interest) to a foreign country equal to approximately 10% of export earnings. Column 4 of Table 5.1 shows that this increased the steady-state capital stock. Thus, a worsening of the balance of payments situation increased the steady-state capital stock and the effect of energy price rises in reducing the steady-state capital stock can be attributed entirely to its relative price effects.

In using the steady-state results to predict investment behaviour, it must be realized that steady-states do not reflect all of the forces that affect the rate of investment in the period before the steady-state is reached. This is because the condition that a steady-state must satisfy, in order that no further investment be desirable, is that the rate of return on diverting one unit of current consumption to investment (evaluated at shadow prices) should equal the rate of pure time preference, δ . This clearly embodies all the demand side forces, which affect the rate of return on investment. However, it does not embody those supply side forces that affect the willingness to give up current consumption and which depend on current and future consumption at those different levels. The reason for this omission is that consumption levels, and thus the evaluation of increases in consumption, are constant in steady-state. This means that, although the steady-states represent the ultimate goal of investment policy, they do not reflect all the forces that influence how that goal should be achieved. An example from the results presented above is that although the requirement to make payments abroad increases the steady-state capital stock, it may not increase investment immediately. The loss in real income may be so severe that the government is not willing to compound the reduction in consumption by an increase in investment until the economy has become somewhat richer. Thus, the investment increase required to reach a higher steady-state capital stock may not materialize for some time after the imposition of the payments. As the rate of pure time preference was held constant, this means that the variations in capital stocks shown in Table 5.1 only reflect changes on the demand side for investment and ignore supply side changes. In this light, it is worth mentioning that preliminary results on intertemporal optimization do show that an increase in energy prices does have an immediate impact in reducing the level of investment.

6. Conclusions

This paper has explored the determinants of market prices and shadow prices in a developing economy both analytically as well as numerically. The framework used to demonstrate these results was characterized by (1) an inability to tax any agricultural transactions other than the sector's net trades with the rest of the economy and the outside world, (2) restrictions on the government's ability to implement discriminatory pricing policies in rural and urban areas, (3) labour market distortions arising from assumptions about the distribution of property rights in land. These features, which are characteristic of many developing countries, were shown to affect optimal government policy in important ways.

The qualitative features of optimum policies may be summarized in the following rules

(1) Production in the manufacturing sector should be characterized by overall efficiency. This implies that there should be no interventions in manufacturing producers' transactions with the rest of the world (pages 17 and 47).

(2) It is desirable to have taxes and subsidies on the agricultural sector's net transactions. This is an argument for optimal tariffs and subsidies even when the country cannot affect the international prices of commodities in which it trades (page 17).

(3) The government should not interfere with rural to urban migration, unless migration gives rise to externalities. This implies equality of the tax burden per capita on rural and urban consumers. When the migration confers an externality on those left in rural areas, the rural tax burden should exceed the urban tax burden by the social valueof the externality (pages 20, 56 and 58).

(4) Optimum consumer taxation in the manufacturing sector must be such that a small equiproportionate intensification in it, at constant shadow prices, would produce equal proportionate reductions in

compensated demand for all urban consumer goods. This rule needs modification for goods where dual pricing is not feasible. (pages 18 and 49).

(5) If land ownership is confined to the agricultural sector, optimum agricultural taxation must be such that a small equiproportionate intensification in it, at constant shadow prices, would produce equal proportionate reductions in ~~compensated~~ marketed surplus for all goods. This rule need modification for goods where dual pricing is not feasible. (pages 19 and 53).

Within those overall guidelines, the numerical examples of the paper clearly illustrate the sensitivity of optimal taxes and shadow prices to underlying assumptions.

Certain specific features of the model used in the paper deserve comment. First, questions of within-period income distribution have been virtually eliminated by supposing that a person's earning abilities depend only on whether he works in agriculture or in non-agriculture. With no intra-rural or intra-urban inequality, therefore, distributional questions can only arise in the relative treatment of the two sectors. However, with migration equalizing utilities in the two sectors, the equity issue disappears entirely in the base case of the static model. This is done to allow a more detailed modelling of the role of imported energy without producing excessive complexity. Notwithstanding this, however, the model allows an adequate treatment of equity across periods in the intertemporal case, which is one of the objectives of the wider program of work discussed in the introduction.

Second, it is assumed that the government owns the entire capital stock. With no government claims on national product, the optimization problem requires the rent from publicly-owned capital to be disposed of in the least distortionary way. Indeed, this is the only rationale for intervention in Model 2, an additional reason in Model 1 being offsetting the externality resulting from rural-to-urban migration. The assumption of complete public ownership is not very realistic. It was made for three reasons. *First*, there is the difficulty involved in modelling how individuals, particularly migrants, come to own parts of the capital stock. *Second*, data on capital ownership are generally quite poor. *Third*, the intertemporal analysis is simplified by allowing the government to take investment decisions directly rather than exercising indirect influence over private investment. While it would obviously be desirable to work on alternative assumptions about capital ownership at a later stage, it is possible to see what tax rates and shadow prices look like when the government does not have a surplus to distribute to the private sector. This was done in the experiments by varying levels of exogenous government requirements, including one where those requirements were high enough to absorb all the rents from public capital.

With the discussion of the static and steady state cases complete, it remains to look at the intertemporal development of the economy. It is hoped to report this elsewhere and thus to arrive at a more complete judgment on the lessons to be learnt from this approach to the problems of taxation,

Mathematical Appendix

This appendix gives a formal statement of the static model described in Section 2 and derives the results discussed in Section 3.

Although, in general, there are four sets of market prices the sector specific tax restriction requires rural consumer prices to equal rural producer prices. Thus, this appendix will deal with three sets of market prices: rural prices, p, urban consumer prices, q, and urban producer prices, m.

The text deals with two models. In Model 1, migrants to urban areas lose all rights to land. In that case, it is necessary to indicate explicitly the dependence of the rural sector's demand and indirect utility functions on rural prices, p, as well as the proportion of people in the sector, $1-\ell$ (ℓ being the proportion of people in manufacturing). This is because land income is given by $\frac{N}{1-\ell}$; variations in ℓ therefore affect income and hence demands and indirect utilities. In Model 2, land is owned by the entire population and the above complications do not arise. However, urban demands and indirect utilities then depend on rural prices, in particular the price of land. In what follows, the derivations for Model 1 are presented, with necessary changes for Model 2 being indicated by the corresponding equations with a prime. Thus the government objective function can be written as:

$$\ell \frac{1}{\mu} [\theta V_M(q)]^\mu + (1-\ell) \frac{1}{\mu} [V_A(p,\ell)]^\mu \tag{A.1}$$

$$\ell \frac{1}{\mu} [\theta V_M(q,p)]^\mu + (1-\ell) \frac{1}{\mu} [V_A(p)]^\mu \tag{A.1'}$$

where

V_M is the urban indirect utility function

V_A is the rural indirect utility function

Demands for goods and supplies of factors can be represented by vectors where positive entries represent demands and negative entries represent factor supplies. Thus, total consumer demand and supply can be represented as:

$$\ell\, D_M(q) + (1-\ell)\, D_A(p,\ell) \tag{A.2}$$

$$\ell\, D_M(q,p) + (1-\ell)\, D_A(p) \tag{A.2'}$$

where D_M is the urban demand function

D_A is the rural demand function.

The producer prices will determine the choice of technique within each industry. This choice of technique can be summarized as a vector which represents the inputs and outputs of the chosen technique when operated at unit level. Outputs are represented by positive numbers and inputs by negative numbers. Thus, the agricultural inputs and outputs are given by:

$$A_A(p)\, Y_A$$

where $A_A(p)$ is the unit supply vector

Y_A is the level of operation.

Similarly, for each manufacturing industry, inputs and outputs are given by:

$$A_M^f(m)\, Y_M^f$$

where $A_M^f(m)$ is the unit supply vector

Y_M^f is the level of operation

f is an index that runs over the different manufacturing industries.

In addition to domestic production, there are trade activities that can be represented by a matrix, G, which when multiplied by a vector of activity levels, X, gives a vector of imports (positive numbers) and exports (negative numbers).

Finally, the government has an exogenously specified vector of net demands, R. One of its components (with a negative sign) is the government's endowment of capital. Thus, the market clearance condition is: 1/

$$\sum_f A_M^f(m)\, Y_M^f + A_A(p)\, Y_A + GX - \ell D_M(q) - (1-\ell)D_A(p,\ell) - R \geq 0 \qquad (A.3)$$

$$\sum_f A_M^f(m)\, Y_M^f + A_A(p)\, Y_A + GX - \ell D_M(q,p) - (1-\ell)D_A(p) - R \geq 0 \qquad (A.3')$$

In addition to satisfying condition (A.3) the producer prices must be such that no activity can make a positive profit and that those activities that are in operation should make zero profit. Formally:

$$m^T A_M^f(m) \leq 0 \quad \text{for all} \quad f \qquad (A.4)$$

$$p^T A_A(p) \leq 0 \qquad (A.5)$$

1/ Market clearing holds as an equality for rural labour and land where trades are internal to rural households.

$$\sum_f m^T A_M^f(m) Y_M^f = 0 \tag{A.6}$$

$$p^T A_A(p)\, Y_A = 0 \tag{A.7}$$

The consumer prices must also be chosen to satisfy the commodity specific tax restriction:

$$q = Qp \tag{A.8}$$

where Q is a diagonal matrix with elements $\frac{1}{1+\psi}$ and dimensionality equal to the number of commodities subject to the restriction.

Also, if there are to be agents in both sectors, the migration equilibrium condition must be satisfied:

$$V_A = \Theta V_M \tag{A.9}$$

It can be shown that the government's budget constraint is automatically satisfied in this economy, but this will be done later when taxes have been defined.

The government's problem is to choose p, q, m, Y_M^f, Y_A, X, ℓ to maximize (A.1) subject to (A.3) - (A.9). The Lagrangean for this problem is:

$$\begin{aligned}\Lambda = {} & \ell\frac{1}{\mu}[\Theta V_M(q)]^\mu + (1-\ell)\frac{1}{\mu}[V_A(p,\ell)]^\mu + \lambda s^T[\sum_f A_M^f(m)Y_M^f + A_A(p)Y_A \\ & + GX - \ell D_M(q) - (1-\ell)D_A(\ell) - R] - \sum_f \phi_M^f m^T A_M^f(m) - \phi_A p^T A_A(p) \\ & + \nu[\Theta V_M(q) - V_A(p,\ell)] + \omega^T[q-Qp] + r_M \sum_f m^T A_M^f(m) Y_M^f + r_A p^T A_A(p) Y_A\end{aligned} \tag{A.10}$$

$$\Lambda = \ell \frac{1}{\mu} [\Theta V_M(q,p)]^\mu + (1-\ell) \frac{1}{\mu} [V_A(p)]^\mu + \lambda s^T [\sum_f A_M^f(m) Y_M^f + A_A(p) Y_A$$

$$+ GX - \ell D_M(q,p) - (1-\ell) D_A(p) - R] - \sum_f \phi_M^f m^T A_M^f(m) - \phi_A p^T A_A(p)$$

$$+ \nu[\Theta V_M(q,p) - V_A(p)] + \omega^T(q-Qp) + r_M \sum_f m^T A_M^f(m) Y_M^f + r_A p^T A_A(p) Y_A \qquad \text{(A.10')}$$

where s is to be interpreted as a vector of shadow prices, the ϕ_M^f are scalar multipliers corresponding to (A.4), and λ is a scalar which enables s to be normalized.

The first order conditions are:

$$\frac{\partial \Lambda}{\partial q} = \ell \Theta^\mu V_M^{\mu-1} V_{Mq} - \lambda \ell (D_{Mq})^T s + \nu \Theta V_{Mq} + \omega \le 0 \qquad (q \ge 0) \qquad \text{(A.11)}$$

$$\frac{\partial \Lambda}{\partial m} = \lambda \Big[\sum_f A_{Mm}^f(m) Y_M^f\Big]^T s - \sum_f \phi_M^f A_M^f(m) + r_M \sum_f A_M^f(m) Y_M^f \le 0 \qquad (m \ge 0) \qquad \text{(A.12)}$$

$$\frac{\partial \Lambda}{\partial p} = (1-\ell) V_A^{\mu-1} V_{Ap} + \lambda \big[A_{Ap}(p) Y_A\big]^T s - \lambda (1-\ell) D_{Ap}^T s$$

$$- \phi_A A_A(p) + r_A A_A(p) Y_A - \nu V_{Ap} - Q^T \omega \le 0 \qquad (p \ge 0) \qquad \text{(A.13)}$$

$$\frac{\partial \Lambda}{\partial p} = \ell \Theta^{\mu-1} V_m^{\mu-1} V_{mp} + (1-\ell)\, V_A^{\mu-1} V_{Ap} + \lambda [A_{Ap}(p) Y_A]^T s$$

$$- \lambda \ell (D_{Mp})^T s - \lambda (1-\ell) D_{A_p}^T s - \phi_A A_A(p)$$

$$+ r_A A_A(p) Y_A - \nu V_{A_p} - Q^T \omega \le 0 \qquad (p \ge 0) \qquad \text{(A.13')}$$

$$\frac{\partial \Lambda}{\partial Y_M^f} = \lambda s^T A_M^f(m) + r_M m^T A_M^f(m) \leq 0 \qquad (Y_M^f \geq 0) \qquad \text{(A.14)}$$

$$\frac{\partial \Lambda}{\partial Y_A} = \lambda s^T A_A(p) + r_A p^T A(p) \leq 0 \qquad (Y_A \geq 0) \qquad \text{(A.15)}$$

$$\frac{\partial \Lambda}{\partial \ell} = \frac{1}{\mu}\,[(\theta V_m)^\mu - V_A^\mu] + [(1-\ell)V_A^{\mu-1} - \nu]\,\frac{\partial V_A}{\partial \ell}$$

$$- \lambda(1-\ell)s^T \frac{\partial D_A}{\partial \ell} - \lambda s^T D_M + \lambda s^T D_A \leq 0 \qquad (\ell \geq 0) \qquad \text{(A.16)}$$

$$\frac{\partial \Lambda}{\partial \ell} = \frac{1}{\mu}\,\{(\theta V_M)^\mu - V_A^\mu\} - \lambda s^T D_M + \lambda s^T D_A \leq 0 \qquad (\ell \geq 0) \qquad \text{(A.16')}$$

$$\frac{\partial \Lambda}{\partial X} = G^T s = 0 \qquad \text{(A.17)}$$

where subscripts denote derivatives and each inequality bears the relation of complementary slackness with the corresponding variable appearing in brackets on the right.

Equations (A.11) to (A.17) are necessary conditions for a restricted tax optimum, provided certain regularity conditions are satisfied. Equation (A.17) is the standard condition that government sector projects (in this case, trade) should not make profits at shadow prices. Equations (A.14) and (A.15) imply that any chosen private activity that breaks even at producer prices must do so at shadow prices as well (cf. Diamond and Mirrlees (1976)). A widely known special case of this result is the desirability of shadow pricing a small country's tradeables at their world prices [Little and Mirrlees (1974)].

Production rules: manufacturing sector

From (A.14),

$$s^T \sum_f A_M^f Y_M^f = 0$$

Premultiplying (A.12) by s^T and using the above, one has

$$\lambda s^T \left(\sum_f A_{Mm}^f Y_M^f\right)^T s - \sum_f \phi_M^f s^T A_M^f \leq 0 \qquad \text{(A.18)}$$

By complementary slackness, $\phi_m^f > 0$ only if $m^T A_M^f = 0$. From (A.14), this implies that $\phi_m^f > 0$ only if $s^T A_M^f \leq 0$. Thus, from (A.18)

$$s^T \left(\sum_f A_{Mm}^f Y_M^f\right)^T s \leq 0 \quad .$$

Since A_{Mm}^f is positive semidefinite (by homogeneity), $s^T \left(\sum_f A_{Mm}^f y_m^L\right)^T s = 0$. By homogeneity,

$$m^T \left(\sum_f A_{Mm}^f Y_M^f\right)^T m = 0$$

Hence, it is possible to choose

$$m = ks \qquad \text{(A.19)}$$

where k is a constant. Furthermore, if A_{Mm}^f is of maximal possible rank, i.e., one fewer than the number of commodities in the manufacturing sector, (A.19) is the only possible solution.

This is stated as

Proposition 1: The manufacturing sector of the dual economy with tax restrictions in the agricultural sector should be characterized by production efficiency.

Setting Consumer Prices: Manufacturing Sector

Consider the ith equation in (A.11). Using Roy's identity,

$$-\left[1 + \frac{\nu\Theta}{\ell\Theta^{\mu}V_M^{\mu-1}}\right]\beta_M D_{Mi} - \lambda\ell \sum_j s_j D_{Mji} + \delta_i\omega_i = 0 \qquad \text{(A.20)}$$

where $\beta_M = \ell\Theta^{\mu}V_M^{\mu-1}\alpha_M$

α_M = private marginal utility of urban income (so that β_M is the gross social marginal utility of urban income).

D_{Mi} is the ith element of $D_M(q)$

D_{Mji} is the derivative of D_{Mj} with respect to the ith element of q.

δ_i = 1 for goods subject to (A.7)

= 0 , otherwise.

Define consumer taxes as deviations of consumer prices from shadow prices, i.e., $t_{Mj} = q_j - s_j$ (A.21)

The budget constraint facing a consumer in the manufacturing sector is

$$\sum_j q_j D_{Mj} = 0$$

Differentiating with respect to q_i

$$\sum_j q_j D_{Mji} = -D_{Mi} \qquad \text{(A.22)}$$

Substitution of (A.20) and (A.21) in (A.19) yields

$$- \left[1 + \frac{\nu\Theta}{\ell\Theta^{\mu}V_M^{\mu-1}}\right]\beta_M D_{Mi} + \lambda\ell \sum_j t_{Mj} D_{Mji} + \lambda\ell D_{Mi} + \delta_i\omega_i = 0 .$$

The Slutsky equation gives

$$D_{Mji} = D^c_{Mji} - D_{Mi} \frac{\partial D_{Mj}}{\partial I_M}$$

where the superscript c denotes compensated demands and

I_M: lump sum income of an urban consumer.

Substitution into (A.22) and symmetry of compensated demand derivatives $D^c_{Mji} = D^c_{Mij}$ leads to

$$- \sum_j t_{Mj}\ell D^c_{Mij} = - \left[1 + \frac{\nu\Theta}{\ell\Theta^{\mu}V_M^{\mu-1}}\right]\frac{\beta_M}{\lambda} D_{Mi} + \ell D_{Mi}$$

$$- \ell D_{Mi} \sum_j t_{Mj} \frac{\partial D_{Mj}}{\partial I_M} + \frac{\delta_i\omega_i}{\lambda} .$$

Dividing by ℓD_{Mi} ,

$$- \frac{\sum_j t_{Mj}\ell D^c_{Mi}}{\ell D_{Mi}} = \left\{1 - \sum_j t_{Mj} \frac{\partial D_{Mj}}{\partial I_M} - \frac{\beta_M}{\ell\lambda}\left[1 + \frac{\nu\Theta}{\ell\Theta_M^{\mu}V^{\mu-1}}\right]\right\}$$

$$+ \frac{\delta_i\omega_i}{\lambda\ell D_{Mi}} \qquad \text{(A.23)}$$

Define $b_M = \frac{\beta_m}{\ell\lambda}\left[1 + \frac{\nu\theta}{\ell\theta^{\mu}V_M^{\mu-1}}\right] + \sum_j t_{Mj}\frac{\partial D_{Mj}}{\partial I_M}$.

b_M is the net social value of an extra unit of income accruing to an urban consumer. It has three components. The first, $\frac{\beta_M}{\ell\lambda}$, is the direct social valuation, divided by λ , the shadow price of government revenue, and thus expressed in terms of revenue. The second term, $\frac{\beta_M\nu\theta}{\lambda\ell^2\theta^{\mu}V_M^{\mu-1}}$ equals $\frac{\alpha_M\nu\theta}{\ell\lambda}$, which is the effect on social welfare (again expressed in terms of revenue) of the change in average urban utility compared to rural utility. The third, $\sum_j t_{Mj}\frac{\partial D_{Mj}}{\partial I_M}$, is the extra tax revenue accruing to the government as a result of increased urban consumption.

The tax rule then becomes:

$$-\frac{\sum_j t_{Mj}\ell D^c_{Mij}}{\ell D_{Mi}} = \left(1-b^M\right) + \frac{\delta_i\omega_i}{\lambda\ell D_{Mi}} \qquad (A.24)$$

where $\delta_i = 1$ for goods subject to (A.7), = 0 otherwise.

Equation (A.24) may be stated as:

<u>Proposition 2</u>: The proportionate reduction in the compensated demand for good i that would result from a small equiproportionate intensification of urban consumer taxation if shadow prices remained constant (the left hand side of A.24) is commodity-independent for those goods not subject to a commodity specific tax restriction. The rule needs modification in other cases to account for the fact that ratios of consumer prices must be equal in rural and urban sectors.

Taxation in the Agricultural Sector

Consider the ith equation in (A.13). Using Roy's identity.

$$-\left[1-\frac{\nu}{(1-\ell)V_A^{\mu-1}}\right]\beta_A D_{Ai}-\lambda(1-\ell)\sum_j s_j D_{Aji}+\lambda\sum_j s_j \frac{\partial a_{jA}}{\partial p_i} Y_A$$

$$-\phi_A a_{iA}+r_A a_{iA} Y_A-\frac{\delta_i\omega_i}{1+\psi} = 0 \qquad \text{(A.25)}$$

where $\delta_i = 1$ for goods subject to (A.7), = 0 otherwise, a_{kA} is the kth element of the vector $A_A(p)$, and β_A, D_{Ai} and D_{Aji} are defined as the rural equivalents of β_M, D_{Mi}, D_{Mji}.

Define taxes as deviations from shadow prices

$$t_{Aj} = p_j - s_j \qquad \text{(A.26)}$$

Define $S_j = a_{jA}Y_A$ as the net supply of good j in the agricultural sector,

whence $\frac{\partial S_j}{\partial p_i} = \frac{\partial S_i}{\partial p_j}$ by symmetry of the net supply function.

Also, by homogeneity:

$$\sum_j p_j \frac{\partial S_i}{\partial p_j} = 0 .$$

Thus (A.25) becomes:

$$-\left(1-\frac{\nu}{(1-\ell)V_A^{\mu-1}}\right)\beta_A D_{Ai}+\lambda\sum_j (t_{Aj}-p_j)\left[(1-\ell)D_{Aji}-\frac{\partial S_i}{\partial p_j}\right]$$

$$- \phi_A a_{iA} + r_A S_i - \frac{\delta_i \omega_i}{1-\psi} = 0 .$$

The budget constraint facing a rural consumer is

$$\sum_j p_j D_{Aj} = 0$$

Differentiating with respect to p_i ,

$$\sum_j p_j D_{Aji} = -D_{Ai} .$$

The Slutsky equation yields

$$D_{Aji} = D^c_{Aji} - D_{Ai} \frac{\partial D_{Aj}}{\partial I_A} = D^c_{Aij} - D_{Ai} \frac{\partial D_{Aj}}{\partial I_A} ,$$

where I_A: lump sum income of a rural consumer.

Substitute in the tax equation to get

$$-(1 - \frac{\nu}{(1-\ell)V_A^{\mu-1}})\beta_A D_{Ai} + \lambda \sum_j t_{Aj}[(1-\ell)D^c_{Aij} - S_{ij}]$$

$$-\lambda(1-\ell)D_{Ai} \sum_j \frac{\partial(t_{Aj}D_{Aj})}{\partial I_A} + \lambda(1-\ell)D_{Ai} - (\phi_A a_{iA} - r_A S_i + \frac{\delta_i \omega_i}{1+\psi}) = 0$$

which may be rewritten

$$-\frac{\sum_j t_{Aj}[(1-\ell)D^c_{Aij} - S_{ij}]}{(1-\ell)D_{Ai}} = (1-b_A) - \frac{1}{\lambda(1-\ell)D_{Ai}} (\phi_A a_{iA} - r_A S_i + \frac{\delta_i \omega_i}{1+\psi}) \qquad (A.27)$$

where

$$b_A = \frac{\beta_A}{\lambda(1-\ell)}\left[1 - \frac{\nu}{(1-\ell)V_A^{\mu-1}}\right] + \sum_j t_{Aj}\frac{\partial D_{Aj}}{\partial I_A}$$

the total effect of an extra unit of income accruing to a rural consumer.

Equation (A.27) may be simplified considerably when it is noted that land is in fixed supply. Thus if k is the index for land:

$$D^c_{Akj} = 0 \quad \text{for all} \quad j \;.$$

Also, the agricultural techniques can all be normalized so that they use the same quantity of land at unit level of operation. Thus:

$$S_{kj} = 0 \quad \text{for all} \quad j \;.$$

Finally, land is not affected by the commodity specific tax restriction ($\delta_k = 0$), and supply must equal demand for land as it is not traded:

$$S_k = (1-\ell)D_{Ak} \;. \qquad \text{(A.28)}$$

This enables (A.27) for $i = k$ to be written as:

$$0 = \lambda(1-b_A)(1-\ell)D_{Ak} - \phi_A a_{LA} + r_A S_k$$

$$= \lambda(1-b_A)\, S_k - \phi_A a_{kA} + r_A S_k$$

Therefore:

$$\frac{\phi_A}{Y_A} - r_A = \lambda(1-b_A) \qquad (A.29)$$

Substitution back into (A.27) gives:

$$-\lambda\sum_j t_{Aj}\left[(1-\ell)D^c_{Aij} - S_{ij}\right] = \lambda(1-b_A)\left[(1-\ell)D_{Ai} - S_i\right] - \frac{\delta_i\omega_i}{1+\psi} \qquad (A.28)$$

Now define marketed surplus as the excess of agricultural supply over agricultural demand for any good:

$$M_{Ai} = S_i - (1-\ell)D_{Ai}$$

$$M^c_{Aij} = S_{ij} - (1-\ell)D^c_{Aij}$$

This notation enables (A.28) to be written as

$$-\frac{\sum_j t_{Aj}M^c_{Aij}}{M_{Ai}} = (1-b_A) + \frac{\delta_i\omega_i}{\lambda(1+\psi)M_{Ai}} \qquad (A.29)$$

Equation (A.29) has an appealing form and is stated as:

Proposition 3: At a restricted tax optimum, for an economy with land ownership confined to the rural sector, an equiproportionate intensification of agricultural taxation at constant shadow prices will produce a commodity-independent proportionate reduction in the compensated marketed surplus of all goods not subject to the commodity specific tax restriction. There are divergences from this rule for those goods that are subject to the commodity specific tax restriction.

The results require some modification for Model 2 since the price of land enters the demand and indirect utility functions of urban consumers. Since land is owned by the entire population, its supply-demand equality reads

$$S_k = \ell D_{Mk} + (1-\ell)\, D_{Ak} \qquad \text{(A.28')}$$

Manipulation of the land tax equation in a way analogous to that following (A.27) leads to

$$\frac{\phi_A}{Y_A} - r_A = \lambda(1-\bar{b}) \qquad \text{(A.28')}$$

$$\text{where} \quad \bar{b} = \frac{\ell D_{Mk} b_M + (1-\ell)\, D_{Ak} b_A}{\ell D_{Mk} + (1-\ell)\, D_{Ak}}$$

Thus, $\bar{b}$ is a weighted average of net marginal social values of income in the urban and rural sectors, with the weights being the proportions of land owned by each sector. The examples worked out in the paper assumed that all land was divided equally, in which case $D_{Mk} = D_{Ak}$, and the above reduces to

$$\bar{b} = \ell b_M + (1-\ell)\, b_A$$

where the weights are the proportion of the population in each sector.

Substitute back into (A.27) for commodities other than land to get

$$- \lambda \sum_j t_{Aj} \left[(1-\ell)\, D^C_{Aij} - S_{ij}\right] = \lambda\, (1-b_A) \left[(1-\ell)\, D_{Ai} - \left(\frac{1-\bar{b}}{1-b_A}\right) S_i\right]$$

The fact that $\bar{b} \neq b_A$, for income distributional reasons, makes clear that an equal proportionate reduction rule in compensated marketed surplus is not available in Model 2.

Migration

The allocation of labour across rural and urban areas is governed by (A.16) and (A.16'). It is easier to consider (A.16') first.

An extra urban worker improves social welfare both directly, $\frac{1}{\mu}[\theta V_M]^\mu$, and indirectly by giving up a rural consumption bundle which is worth $\lambda s^T D_A$ at shadow prices. The marginal cost of the move is the direct welfare loss, $\frac{1}{\mu} V_A^\mu$, and indirectly, the urban consumption bundle to which he lays claim, $\lambda s^T D_M$. (A.16') equates marginal costs and benefits at the optimum. Notice that the migration constraint (A.9) equates the direct marginal costs and benefits, so that the comparison is simply in terms of the indirect effects.

With the migration constraint holding as an equality, (A.16') becomes

$$s^T D_A \leq s^T D_M \qquad (\ell \geq 0)$$

Using the definitions

$$t_A = p - s$$

$$t_M = q - s$$

and the budget constraints

$$\sum_j p_j D_{Aj} = 0$$

$$\sum_j q_j D_{Mj} = 0$$

yields

$$t^T D_M \leq t^T D_A \qquad (\ell \geq 0) \qquad (A.30')$$

at a restricted tax optimum.

This establishes

Proposition 4: When income from land ownership is unaffected by migration, an optimum with migration is characterized by an equal tax burden per capita on both rural and urban consumers, where taxes are measured relative to shadow prices.

In Model 1, by contrast, land ownership is affected by migration which therefore confers an externality on rural agents. In the absence of intervention, this can be expected to lead to the extent of migration being suboptimal. It would therefore be optimal to tax the agricultural sector more heavily. The ensuing discussion establishes this formally as well as deriving the magnitude of the difference in the tax burden between the two sectors.

Equation (A.16) can be given an interpretation analogous to that given (A.16'). The extra terms arise from the fact that the movement of a worker to the urban area has three additional effects. First, it improves welfare in rural areas by $(1-\ell)\, V_A^{\mu-1} \frac{\partial V_A}{\partial \ell}$. Second, any increase in rural utility tightens the migration constraint, imposing a welfare cost equalling $\nu \frac{\partial V_A}{\partial \ell}$. Third, it raises net demands in the rural areas, the welfare cost of which is $\lambda s^T (1-\ell) \frac{\partial D_A}{\partial \ell}$.

Applying equation (A.9) to (A.16) and considering interior solutions,

$$[(1-\ell)\, V_A^{\mu-1} - \nu]\, \frac{\partial V_A}{\partial \ell} - \lambda(1-\ell)\, s^T \frac{\partial D_A}{\partial \ell}$$

$$- \lambda s^T D_M + \lambda s^T D_A = 0$$

Since $\dfrac{\partial V_A}{\partial \ell} = \alpha_A \dfrac{\partial I_A}{\partial \ell}$

where, as before,

α_A = private marginal utility of rural income

$I_A = r_N \dfrac{N}{1-\ell}$ (lump sum income of a rural consumer)

$$\{[(1-\ell)\, V_A^{\mu-1} - \nu]\, \alpha_A - \lambda\, (1-\ell) \sum_j s_j \frac{\partial D_{Aj}}{\partial i_a}\} \frac{\partial I_A}{\partial \ell}$$

$$- \lambda s^T D_M + \lambda s^T D_A = 0$$

Differentiating the rural consumer's budget constraint, $\sum_j p_j D_{Aj} = 0$

$$\frac{\partial}{\partial I_A} (\sum_j p_j D_{Aj}) = 0$$

Since $t_{Aj} = p_j - s_j$

the above may be written

$$\{[1 - \frac{\nu}{(1-\ell)\, V_A^{\mu-1}}] \frac{\beta_A}{\lambda\,(1-\ell)} + \frac{\partial}{\partial I_A} (\sum_j t_{Aj} D_{Aj})\} \frac{\partial I_A}{\partial \ell}$$

$$- \frac{s^T D_M}{1-\ell} + \frac{s^T D_A}{1-\ell} = 0$$

where $\beta_A = (1-\ell)\, V_A^{\mu-1} \alpha_A$

The term in curly brackets is the net marginal social value of rural income, b^A.

Thus,

$$s^T D_M - s^T D_A = b_A (1-\ell) \frac{\partial I_A}{\partial \ell}$$

which, on using the rural and urban budget constraints, becomes

$$t^T D_A - t^T D_M = b_A (1-\ell) \frac{\partial I_A}{\partial \ell} \qquad \text{(A.30)}$$

This result is intuitively appealing and is stated as

Proposition 5: When income from land ownership changes owing to migration, the tax burden per capita on rural consumers exceeds that on urban consumers by the net social value of the ensuing change in land income, where taxes are measured relative to shadow prices.

Finally, equations (A.11) and (A.13) show that, as stated in Section 3, the introduction of migration simply changes the weights on the derivatives of urban and rural utility functions, by introducing the terms $\upsilon\theta V_{Mq}$ and υV_{AP}.

The government budget constraint

It remains to show that the government's budget constraint may be derived from the equations of the model. Its revenue is

$$(p_F - s_F) F + (s_F - m_F) Y_F + (q_A - s_A) D_{AM} + (s_A - p_A)(Y_A - D_{AA})$$

$$+ (r_c - s_c)\ D_{cA} + (q_c - s_c)\ D_{cM} + (s_c - m_c)\ Y_c$$

$$+ (r_u - m_u)\ D_{uA} + (q_u - m_u)\ D_{uM} + (m_L - q_L)\ h_u L_M$$

$$+ (m_E - s_E)\ E + m_K K$$

The budget constraints for rural and urban consumers and the zero profit conditions in production allow this to be written as

$$- s_F F + s_F Y_F - s_A D_A + s_A Y_A - s_c D_c + s_c Y_c - s_E E + m_u R_u$$

where R's denote government demands.

This in turn may be written, using the market clearance conditions as

$$s_F\ (X_F + R_F) + s_A\ (X_A + R_A) + s_c\ (X_c + R_c) + m_u R_u - s_E E$$

This last, from the balance-of-payments constraint, equals

$$s_F R_F + s_A R_A + s_c R_c + m_u R_u$$

which is government expenditure.

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