Policy Research Working Paper 9017

Eurobonds

A Quantitative Analysis of Joint-Liability Debt

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WORLD BANK GROUP
Macroeconomics, Trade and Investment Global Practice
September 2019

Policy Research Working Paper 9017

Abstract

This paper assesses the consequences of implementing a joint liability debt system in a two-country small open economy model. With joint liability a default of one country makes the other participant liable for its debt. The results highlight a trade-off between the *contagion* risk, in the sense that this instrument may push some member states to default even though they are individually solvent, and *cheaper access* to credit on average, since lenders are at risk only if no participating sovereign is willing to service the debt. The findings suggest that the welfare consequences of this policy proposal hinge critically on the timing of its introduction: Introducing such instruments at the peak of the Eurozone crisis would have helped the Periphery and harm the Core member states, while its adoption during normal times has the potential to make all participants better-off.

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Eurobonds: A Quantitative Analysis of Joint-Liability Debt

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1 Introduction

This paper introduces bonds with joint liability in a model where two small open economies borrow from risk neutral international lenders. Under joint liability a default in one country makes the other country liable for its debt. This feature introduces the potential for contagion of default decisions, while introducing further repayment guarantees for lenders. Hence, the introduction of this instrument generates *contagion* risk, in the sense that this instrument may push some countries to default even though they are individually solvent. On the other hand, it may generate *cheaper credit*, thereby helping enhance financial stability. In this paper I quantify the effects and generate predictions about the welfare implications of introducing joint liability bonds under different underlying fundamental conditions.

The recent Eurozone crisis has highlighted the necessity for the development of financial instruments that mitigate the effects of the financial crisis and stabilize the yields of sovereign bonds. One of the mechanisms that was proposed by the European Commission (2011) as a potential shield of future financial crises is the implementation of bonds with joint liability (Eurobonds).[1] On the one hand, some member states may increase their debt accumulation with this mechanism, since they will have easier access to financial markets. This would be

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Acknowledgment: The views expressed in this article are mine and do not necessarily reflect the views of the World Bank. I am greatly indebted to Juan Carlos Conesa and Eva Carceles-Poveda for their encouragment and guidance. I would also like to thank Alexis Anagnostopoulos, Marina Azzimonti, Gabriel Mihalache, Pau Pujolas, Pavlos Karadeloglou, Doerte Doemeland, Tito Cordella, and participants of Stony Brook University workshop, The Economic Seminar of Central Bank of Ireland, Irish Economic Association Annual Conference, The 11th Annual Economics Graduate Students Conference, The 69th Annual New York State Economics Association, The 27th International Conference on Game Theory, and the seminar of University of Piraeus.

[1] Not to be confused with Eurobond, which are bonds denominated in a currency other than the local currency of the issuing country.

problematic since a failure of a country to repay may trigger a contagion effect if the other countries do not have enough resources to absorb the troubled debt.[2] On the other hand, the introduction of joint liability bonds like Eurobonds could provide better access to financial markets especially to those countries under stress. Moreover, it could decrease the incentives for some member countries to abandon the Euro or default, by promoting stability and setting the basis for a prospective fiscal integration, European Commission (2014).

I consider two economies with exogenous incomplete markets. In the benchmark case countries can issue only individual sovereign bonds, following Arellano (2008). Then, I study the interactions between two countries that can issue bonds with full joint liability. Following the literature, I study an endowment economy and I abstract from production and input decisions. The endowments follow a stochastic process taken from data on the performance of Core and Periphery countries in the Eurozone.[3] For my measurement I use two groups of countries, the first group is wealthier with less income volatility than the other group and represents the Core member states of the Eurozone (Germany, France, Netherlands), while the other group represent the Periphery member states of Eurozone (Portugal, Italy, Greece, Ireland, Spain). In the analysis, the prices of the bonds are endogenously determined and depend on both countries choices, generating a strategic interaction between the two countries. In particular, there exist a two-stage Nash equilibrium. In the first stage countries make their repayment decisions and, conditional on this, they make their borrowing decisions on the second stage. I do not allow for partial default, and the penalty of default is a permanent output loss and exclusion from financial markets.

This paper is related to the novel literature that studies the effects of Eurobonds. Delpla & von Weizsacker (2010) discusses the 'blue and red bond' proposal, in which they propose pooling debt up to 60% of GDP (blue bonds) and using individual bonds issued by each country separately (red bonds) beyond that threshold. Hellwig & Philippon (2011) foresees a mutualization of 10% of GDP for the short term debt. Claessens *et al.* (2012) discusses in depth various proposals of Eurobonds and analyze potential effects in the Eurozone, and Beetsma & Mavromatis (2014) and Tirole (2015) analyze stylized finite-period models of the strategic interactions between two countries that can issue joint liability bonds. They find that Eurobonds might be beneficial under some circumstances. This paper complements that literature by providing

[2] The European Commission (2011) has tried to asses the feasibility of common issuance of sovereign bonds among Member States of the Eurozone, while it highlights the "moral hazard" as a potential problem. As a solution to this problem, all the proposals suggest borrowing limits in order to mitigate this potential problem.

[3] The distinction between Core and Periphery countries in Eurozone follows the description of the related literature.

quantitative predictions in an infinite horizon general equilibrium model of debt and default.

This paper also builds on the literature on the quantitative implications of debt dynamics and default in incomplete asset markets models: Eaton & Gersovitz (1981), Aguiar & Amador (2013), Aguiar & Gopinath (2006), Cuadra *et al.* (2010), Pouzo & Presno (2014), and Yue (2010).[4] In fact, the benchmark for comparison is Arellano (2008), which accounts for the empirical regularities in emerging markets as an equilibrium outcome of the interaction between risk-neutral creditors and a risk averse borrower that has the option to default.[5] Hatchondo *et al.* (2017) studies the effects of introducing a limited non-defaultable financing option in a small-open economy. However, they abstract from the strategic interactions that might be generated among the participating member states. Their results suggest that access to such an asset for a given country could produce substantial welfare gains and lead to significant reductions in sovereign debt and spreads. Arellano & Bai (2014a), Arellano & Bai (2014b) and Lizarazo (2009) examine the contagion across sovereign defaults through the existence of common lenders. In this paper, I extend this idea and I develop a model that nests common lenders and borrowers. Àbrahàm *et al.* (2015) develop a model of the Financial Stability Fund (FSF) across sovereigns as a long-term partnership with limited ex-post transfers. To the best of my knowledge, none of the papers in the quantitative default literature addresses the impact of the strategic interactions that joint liability bonds might generate.

The benchmark model is calibrated for the case of a single country issuing individual bonds. Then, I compare this benchmark to a world where two countries can issue joint liability bonds under two different scenarios: (i) the two countries are subject to different processes of idiosyncratic income risk (asymmetric case), and (ii) the two countries are subject to the same process of idiosyncratic income risk (symmetric case). The findings show that countries have cheaper access to financial markets in both scenarios, even though the welfare implications differ drastically between both scenarios. In the symmetric case (two core countries issuing joint liability bonds), the model predicts welfare gains for both countries since the cheaper credit effect dominates to the contagion effect. In contrast, in the asymmetric case Periphery countries experiment welfare gains, while Core countries face welfare losses when both countries start with large debt-to-output ratios. If the Periphery countries start with low debt-to-output ratios, then the Core countries could also benefit from the introduction of Eurobonds.

The paper is structured as follows: Section 2 presents the theoretical models for the bench-

[4]See Aguiar & Amador (2014) or Tomz & Wright (2012) who explore more key issues in this literature.

[5]Alternative models of default focus on rollover risk, such as Cole & Kehoe (2000) and Conesa & Kehoe (2015), but I do not consider this issue in my analysis.

mark economy and the Eurobonds, Section 3 calibrates the model and assesses the quantitative implications of the model, and Section 4 concludes.

2 The Model

I consider two cases of sovereign bonds markets: first the benchmark economy, in which countries issue only individual bonds to the international markets, i.e. no joint liability. Second, both countries are allowed to issue only bonds with joint liability.

I assume that the countries are risk-averse and they cannot affect the world risk free interest rate. The period utility function $u(.) : \mathbb{R}_+ \to \mathbb{R}$ and is assumed to be strictly increasing, strictly concave and satisfies Inada conditions. The lifetime payoff of each borrowing country i is $E_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t})$, where $\beta \in (0,1)$ is the discount factor, $i \in \{1,2\}$ is the index for each set of countries, $c_{i,t}$ denotes each country's level of consumption at period t. Moreover, in each period the countries receive a stochastic endowment of a single perishable consumption good $y_{i,t}$, which is drawn from a compact set $Y = [\underline{y}, \overline{y}]$. These shocks follow a Markov process with transition matrix $\pi_i(y_i', y_i)$.

In both models, the risk averse countries trade one-period asset with the risk-neutral competitive foreign lenders. The lenders have access to an international credit market where they can trade as much as they need at a constant risk free interest rate r. I assume that the lenders always commit to repay their debt. However, countries have no commitment and each period decide whether to repay their debt or to default.

The lenders have perfect information about the history of endowments and they can observe the demand for next period's assets. Given these two variables they estimate the probability that the countries will be insolvent and they offer an interest rate that compensates for the risk of default. Considering the risk-neutrality and the zero expected profits, the equilibrium prices q are given by,

$$q = \frac{1-\phi}{1+r} \tag{1}$$

where ϕ is the endogenous derived default probability. The bond price q lies in $[0, \frac{1}{1+r}]$, since, $0 \leq \phi \leq 1$. The probability of default is zero for any positive savings and the sovereign bond price indicates the price of a risk free bond $\frac{1}{1+r}$. When countries have negative savings there might be some positive probability ϕ for the government to default which has a negative effect on the price of the sovereign bond to compensate the international creditors.[6] The sovereign's interest rate is defined as the inverse of the bond price, $r^s = \frac{1}{q} - 1$ and the country's spread is the difference between the interest rate and the risk free interest rate, $s = r^s - r$.

Influenced by the default episodes in various emerging economies, the cost originated by default episodes is two fold: (i) de facto prohibited access to the financial markets because of high interest rates and (ii) a direct output loss due to liquidity problems, outflow of capital, banking problems. If a country chooses to default, I assume that it will remain in permanent financial autarky since the incidence of the insolvency has created bad reputation for the country from the international creditors. The output cost is a function $g(y_i) \leq y_i$ that country has when defaults and is an increasing function respect to y_i, as in Arellano (2008).

2.1 Benchmark

This section is the benchmark economy and follows Eaton & Gersovitz (1981) for the theoretical part and Arellano (2008) and Aguiar & Gopinath (2006) for the quantitative part. Define $V(b, y)$ to be the life-time value function for a country that starts the current period with assets b and endowment y. The country chooses to maximize the present value of its welfare by choosing to repay its debt or to default. Therefore, $V(b, y)$ satisfies

$$V(b, y) = \max \ \{W^{def}(y), W^r(b, y)\} \tag{2}$$

where $W^{def}(y)$ is the value that is associated with the default, while $W^r(b, y)$ is the pay-off function associated with repaying:

$$\begin{aligned} W^r(b, y) &= \max_{c,b'} u(c) + \beta E_{y'/y} V(b', y) \\ \text{s.t.} \quad c + &q(b', y) \cdot b' = y + b \\ &b > \underline{b} \end{aligned} \tag{3}$$

If the country defaults, it faces permanent financial autarky and its consumption equals the endowment, which entails some direct output costs. The value of default, $W^{def}(y)$ is given by the following:

$$W^{def}(y) = u(g(y)) + \beta E_{y'|y} W^{def}(y') \tag{4}$$

[6] Arellano (2008) models the price function in similar method.

Let $A(b)$ be the set of y's for which it is optimal for the country to default. The default set of the country, given that it has good credit history is:

$$A(b) = \{y \in Y : W^r(b, y) \leq W^{def}(y)\} \tag{5}$$

The country may have incentives to default because it had a bad shock in the output combined with a massive debt that is unsustainable. However, the country loses its ability to have an intertemporal consumption smoothing since it has no access to the financial markets. If the country has a bad credit history then the default set is $A(b) = Y$.

The default probability for the country is defined by:

$$\phi(b', y) = \int_{A(b')} d\pi(y'|y) \tag{6}$$

When the default set is empty, $A(b) = \emptyset$, then the equilibrium default probability is zero, since it is not optimal for the country to default. When $A(b) = Y$ then the probability to default is equal to one. In general, the probability changes in a positive manner as the assets shift (i.e. if the government debt is high then the probability is higher).

To derive the equilibrium prices I use Eq. 1, and we get:

$$q(b', y) = \frac{1 - \phi(b', y)}{1 + r} \tag{7}$$

The level of the asset's price depends on the probability that the country will default next period. In the extreme case that the probability is equal to one then the price is equal to zero and the country can not borrow. As the probability decreases, the price gets closer to the price of a risk free bond.

Definition 1: A Recursive Equilibrium for a single country consist of: *(i)* policy functions for borrowing and consumption $\{b'(b, y), c(b, y)\}$ and a value function $\{V(b, y)\}$ *(ii)* the price function for individual bonds $q(b', y)$ st:

1. Given the prices, the policy functions and the value functions of the country solve its maximization problem 2 - 4.

2. Taking as given country's policy functions and value function, the bond price function satisfies the maximization problem of the foreign lenders 7.

2.2 Eurobonds

In this part, I lay out the economy in which both countries can issue bonds with joint liability. The vector of endogenous aggregate states consists of the vector of countries' debt holdings, $\{b_i\}_{\forall i}$. Therefore, the economy's state space consists of the endogenous and exogenous states and is denoted by $S = \{b_1, b_2, y_1, y_2\}$. The countries' repayment strategy is denoted by, $\{h'_i\}_{\forall i}$. The repayment strategy is a binary variable, where $h'_i = 0$ stands for good credit, while $h'_i = 1$ stands for bad credit.

In this economy countries interact strategically about their borrowing and repayment decisions simultaneously in two stages as shown in figure 1. In the first stage they chose their repayment decision. Conditional on the decision of the first stage, they issue assets on the second stage. Hence, there are three possible scenarios.

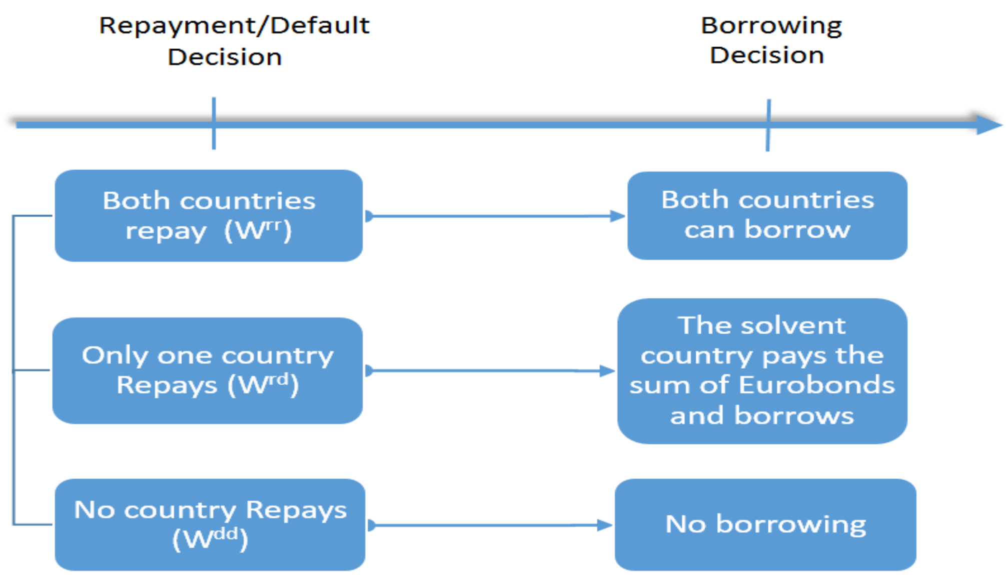

Figure 1: **Timing of Decision**

Scenario I - If both countries choose to repay, the payoff function $W_i^{rr}(S; b'_{i^-})$ of country i, given the arbitrary asset strategy b'_{i^-} for country i^-, solves:

$$W_i^{rr}(S;b'_{i^-}) = \max_{c_i, b_i{}'} \quad u(c_i) + \beta E_{y'_i, y'_{i^-}|S} V_i^E(S') \tag{8}$$

$$\text{s.t.} \quad c_i + q_E(b_i^{'}, b_{i^-}^{'}, y_i, y_{i^-}) \cdot b_i^{'} = y_i + b_i$$

$$b_i > \underline{b}$$

Let $V_i^E(S)$ be the associated value function for Eurobonds for each country i, given that both countries have good credit history. It is vital to know the level of debt for country i^-, since it influences the Eurobonds' price $q_E(b_i^{'}, b_{i^-}^{'}, y_i, y_{i^-})$. Next period, since both countries choose to repay, they will be able to borrow again with Eurobonds.

Scenario II/III - If country i chooses to repay while country i^- chooses to default, the payoff function $W_i^{rd}(S)$ solves:

$$W_i^{rd}(S) = \max_{c_i, b_i^{'}} \quad u(c_i) + \beta E_{y'_i|y_i} V_i(b_i^{'}, y'_i) \tag{9}$$

$$\text{s.t.} \quad c_i + q_i(b_i{}', y_i) \cdot b_i{}' = y_i + (b_i + b_{i^-})$$

$$b_i > \underline{b}$$

where, $W_i^{rd}(S)$ is the payoff function when country i chooses to repay while country i^- chooses to default. In this scenario, country i has to pay the sum of all the Eurobonds, while next period country i will land in the benchmark case from section 2.1. The next period Value function $V_i(b_i, y_i)$ is the same as in the benchmark economy, since next period the country will issue debt without any joint liability. The price $q_i(b'_i, y_i)$ that country i receives today is also derived by the benchmark economy, since it reflects the probability that the country has to default next period.

Scenario IV - if country i chooses to default, its payoff is:

$$W_i^{dd}(y_i) = W_i^{def}(y_i) \tag{10}$$

which is identical to the one in section 2.1.

I develop an intra-period game to derive the optimal strategy of repayment and borrowing for each country i, since it internalizes the effects of its strategies and the other's country strategies. The structure of the subgame depends on the aggregate state space S, as well as the repayment and borrowing decisions of both countries. The equilibrium strategies of repayment and borrowing $\{b_i'(S) = b_i^{BR'}(S, b_{i^-}^{BR'}, h_{i^-}^{BR'})\ ,\ h_i'(S) = h_i^{BR'}(S, h_{i^-}^{BR'}, b_{i^-}^{BR'})\}_{\forall i}$ are computed by solving a Nash Equilibrium, thus they reflect the best response of country i given the best response of country i^-.

The best response for the repayment strategy of country i, given the arbitrary current strategies $\{h_{i^-}^{'}, b_{i^-}^{'}\}$ is defined:

$$h_i^{BR'}(S; h_{i^-}^{'}, b_{i^-}^{'}) = \underset{h_i' \in \{0,1\}}{\text{argmax}} \begin{cases} (1-h_i') \cdot W_i^{rr}(S; b_{i^-}^{'}) + h_i' \cdot W_i^{dd}(y_i) & \text{,if } h_{i^-}' = 0 \\ \\ (1-h_i') \cdot W_i^{rd}(S) + h_i' \cdot W_i^{dd}(y_i) & \text{,if } h_{i^-}' = 1 \end{cases} \tag{11}$$

The best response for the debt strategy of country i, given the arbitrary current strategies $\{h_{i^-}^{'}, b_{i^-}^{'}\}$ is defined:

$$b_i^{BR'}(S; b_{i^-}^{'}, h_{i^-}^{'}) = \underset{b_i' \in B}{\text{argmax}} \begin{cases} W_i^{rr}(S; b_{i^-}^{'}) & \text{,if } h_i^{BR'} = 0 \ \&\ h_{i^-}' = 0 \\ \\ W_i^{rd}(S) & \text{,if } h_i^{BR'} = 0 \ \&\ h_{i^-}' = 1 \\ \\ 0 & \text{,if } h_i^{BR'} = 1 \end{cases} \tag{12}$$

Moreover, $\tilde{V}_i^E(S; \{b_i^{'}, h_i^{'}\}_{\forall i})$ is the payoff function of country i, given the arbitrary current strategies $\{b_i^{'}, h_i^{'}\}_{\forall i}$

$$\tilde{V}_i^E(S; \{b_i^{'}, h_i^{'}\}_{\forall i}) = \begin{cases} W_i^{rr}(S; b_{i^-}^{'}) & \text{,if } h_i' = 0 \ \&\ h_{i^-}' = 0 \\ \\ W_i^{rd}(S) & \text{,if } h_i' = 0 \ \&\ h_{i^-}' = 1 \\ \\ W_i^{dd}(y_i) & \text{,if } h_i' = 1 \end{cases} \tag{13}$$

Definition 2: Given the future value functions $\{V_i^E(S'), V_i(b_i', y_i')\ , V_i^{def}(y_i')\}$ and the prices $\{q_i(b_i', y_i), q_E(b_1', b_2', y_1, y_2)\}$, the intra-period Nash Equilibrium consists of the best response strategies for borrowing and repayment $\{b_i^{BR'}(S; b_{i^-}^{BR'}, h_{i^-}^{BR'}), h_i^{BR'}(S; h_{i^-}^{BR'}, b_{i^-}^{BR'})\}_{\forall i}$ s.t.:

1. The best response strategies for repayment and borrowing are the solutions to maximization problem 12 and 11

2. The equilibrium pay-off value function $V_i^E(S)$ is derived by the equilibrium strategies $\{b_i^{BR'}(S, b_{i^-}^{BR'}, h_{i^-}^{BR'})$, $h_i^{BR'}(S, h_{i^-}^{BR'}, b_{i^-}^{BR'})\}_{\forall i}$ and equation 13 s.t.:

$$V_i^E(S) = \tilde{V}_i^E(S; \{b_i^{BR'}, h_i^{BR'}\}_{\forall i})$$

Given the outcome of the intra-period Nash Equilibrium, let $D(b_1, b_2)$ be the set, for which both countries choose to default simultaneously:

$$D(b_1, b_2) = \{y_1 \in Y \ \& \ y_2 \in Y : h_1'(S) \cdot h_2'(S) = 1\} \tag{14}$$

To derive the equilibrium prices I use Eq. 1 as in the benchmark economy, which gives:

$$q_E(b_1', b_2', y_1, y_2) = \frac{1 - \iint_{D(b_1', b_2')} d\mu_1(y_1'|y_1) d\mu_2(y_2'|y_2)}{1 + r} \tag{15}$$

Note that this price reflects the probability that both countries will default simultaneously, and the analysis is similar to the benchmark case.

Definition 3: Given the price function $\{q_i(b_i', y_i)\}_{\forall i}$ and the value function $\{V_i(b_i, y_i), W_i^{def}(y_i)\}_{\forall i}$ from definition 1, a Markov Perfect Equilibrium for this economy consists of: *(i)* policy functions for repayment, borrowing, consumption $\{h_i'(S), b_i'(S)\ , c_i(S)\}_{\forall i}$, value functions $\{V_i^E(S)\}_{\forall i}$ and *(ii)* a price function for bonds $\{q_E(b_i', b_{i-}', y_i, y_{i-})\}$ st:

1. Given the prices $\{q_i(b_i', y_i), q_E(b_i', b_{i-}', y_i, y_{i-})\}$ and the equilibrium value functions from definition 1 and 2, the policy functions and the value functions are the solution to the maximization problem 8 - 13 and satisfy definition 3.

2. Taking as given both countries' policies functions and values functions, the bond price function $\{q_E(b_i', b_{i-}', y_i, y_{i-})\}$ satisfies the maximization problem of the foreign lenders 15.

3 Quantitative Analysis

3.1 Calibration

Most parameter values of the benchmark economy are set following the literature or exogenously estimated from the data. First, I cluster the Core countries (Germany, France, Netherlands) and the Periphery countries (Portugal, Greece, Italy, Spain, Ireland) of Eurozone. Then I estimate the stochastic processes for the outputs of these groups from their time series. I assume that the stochastic processes of these two groups are independent and follow a log-normal AR(1) process $\log y_t = \rho \log y_{t-1} + \epsilon_t$, where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$.[7] The stochastic process is discretized into

[7] In future work, I will examine the spill-over effects that may be generated by introducing correlation in the endowment processes of the two countries.

an independent Markov Chain by using Tauchen & Hussey (1991). Furthermore, the differences between Core and Periphery are not only in their income process, but also the Core is richer than the Periphery by 20% on average according to Eurostat data. A period in the model refers to a quarter, and the risk free interest rate is set equal to 1.7% as in Arellano (2008). The utility function displays a constant coefficient of relative risk aversion form,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \text{ with } \sigma \neq 1$$

The risk aversion coefficient σ is set to 2, which is a common value used in real business cycle studies. All the Eurobonds proposal had some form of borrowing limit for the member states in order to mitigate moral hazard concerns. For this reason, I set an exogenous borrowing limit, of 66% debt-to-income ratio for the Core and of 83% for the Periphery.[8] As in Arellano (2008), I assume that default entails some direct output cost of the following form:

$$g(y_i) = \begin{cases} \gamma E(y_i) & , \text{ if } y_i > E(y_i) \\ y_i & , \text{ if } y_i \leq E(y_i) \end{cases} \tag{16}$$

where γ is the exogenous output cost that I set equal to 0.96, as in Arellano (2008). Finally, I calibrate the discount factors of the benchmark model to match the sovereign spreads of Core and Periphery and I set them equal to 0.89 and 0.88, respectively.

Table 2 presents some results on the performance of the benchmark models in comparison with the data. To derive the business cycle statistics, I run many simulations of the model over time until a default occurs and I evaluate the mean statistics of these simulations.

The model matches relatively well the spread for both countries. It predicts that the mean interest rate spread for the Core is 0.5%, while in the data is 0.6%. The model is less successful for the Periphery, since it generates a mean interest rate spread of 1.9%, while in the data is 2.4%. Moreover, the model has an exogenous debt-to-output ratio to match the data.

The model predicts lower volatility than the data. The volatility of the interest rates for the Core is 0.9 % and the Periphery is 2.16% in the data; the model under-predicts the volatility for both countries, since for the Core is 0.019% and the Periphery is 0.042%.

[8] I am in the process of relaxing this assumption. I am solving for the economy that has no exogenous borrowing constraints.

Table 1: **Calibration**

	Values	Target
Risk aversion	$\sigma = 2$	Arellano (2008)
Output cost after default	$\gamma = 0.96$	Arellano (2008)
Risk free interest rate	1.7 %	Arellano (2008)
Core's income process	$\rho = 0.96$, $\sigma_\epsilon = 0.003$	Data
Periphery's income process	$\rho = 0.92$, $\sigma_\epsilon = 0.004$	Data
Output difference	$\bar{y}_c/\bar{y}_p = 1.2$	Data
Core's borrowing limit	66%	Treaty
Periphery's borrowing limit	83%	Treaty
Calibrated parameters		
Core's discount factor	$\beta_c = 0.89$	0.6% spread
Periphery's discount factor	$\beta_p = 0.88$	2.4% spread

Table 2: **Business Cycle Statistics: The Benchmark Model and the Data**

	Data		**Benchmark**	
mean(%)	Core	Periphery	Core	Periphery
Debt/Y	66	83	66	79
Spread	0.6	2.4	0.5	1.9
C/Y	77	80	98.8	98.6
std(%)				
Debt/Y	8.10	19	0.16	0.36
Interest rate	0.9	2.16	0.019	0.042
C/Y	1.15	1.9	0.14	0.15

3.2 Results

This section first analyzes the policy functions of the benchmark and the Eurobonds models and then examines the quantitative performance of the Eurobonds model in comparison with the benchmark model.

The introduction of joint-liability bonds generates two opposing forces. On the one hand, this instrument generates a contagion effect, in the sense that it may push some countries to default even though they are individually solvent. On the other hand, joint-liability bonds may create cheaper access to credit, since insuring other countries allows for lower rates.

Figure 2 shows the effects of introducing Eurobonds and having cheaper access to the financial markets for the Core countries. It compares the spread that is generated by the benchmark and the Eurobonds model. The Eurobonds' price depends also on the Periphery's debt, which is fixed to 55% debt-to-output ratio. When the Core has below 53% debt-to-output ratio, there is no positive externality from the introduction of Eurobonds, since if the Core defaults the Periphery would also be dragged to default with high probability. It would have been very expensive for the Periphery to cover the Core's inherited debt, thus there is no significant effect on the spreads. However, in the region 53%-47%, the Periphery would be willing to cover the inherited debt if the Core defaults, since the Periphery inherits a lower amount of debt. Hence, the Periphery would not default and for this reason the spread decreases and the Core receives a positive externality. In the region above 47%, the Core would not default neither in the benchmark nor in the Eurobonds model, for this reason there is no difference between the two models.

Figure 3 shows the effect of introducing Eurobonds and having cheaper access to the financial markets for the Periphery countries. As in figure 2, the Eurobonds' price depends also on the Core's debt, which is fixed to 50%, and it compares the spread that is generated by the benchmark and the Eurobonds model. When the Periphery has above 53% debt-to-output ratio, there is no positive externality from the introduction of Eurobonds, since the Periphery would not default neither in the benchmark nor in the Eurobonds model. However, below 53% the Periphery receives lower spreads in the Eurobonds model, because of the insurance mechanism of the Eurobonds. If the Periphery defaults, the Core will cover the inherited debt with high probability, for this reason international lenders are willing to buy bonds at a relatively lower interest rate. As the level of the Core's inherited debt increases the probability, that Core has to payoff the debt, decreases and for this reason the spread increases. It is clear from figure 2 and 3 that the price effects for the Periphery are bigger than the Core, nonetheless Core also receives some positive externalities by issuing debt with joint-liability.

Figure 4 and 5 compare the changes on the repayment policy functions for both the Benchmark and the Eurobonds model, given a certain combination of income level of Core and Periphery. The x-axis and y-axis measure the level of asset holding over the average income level for Core and Periphery, respectively. The red region represents the combination of the inherited asset levels for which both countries decide to default simultaneously, as in Arellano & Bai (2014a). In the dark green area both choose to repay, while in the light green area I come across with multiple pure strategy Nash equilibrium on the repayment decision and countries choose either to repay or default. In case of multiple equilibrium, I choose by assumption the outcome that yields the highest aggregate welfare, which is the scheme that both economies

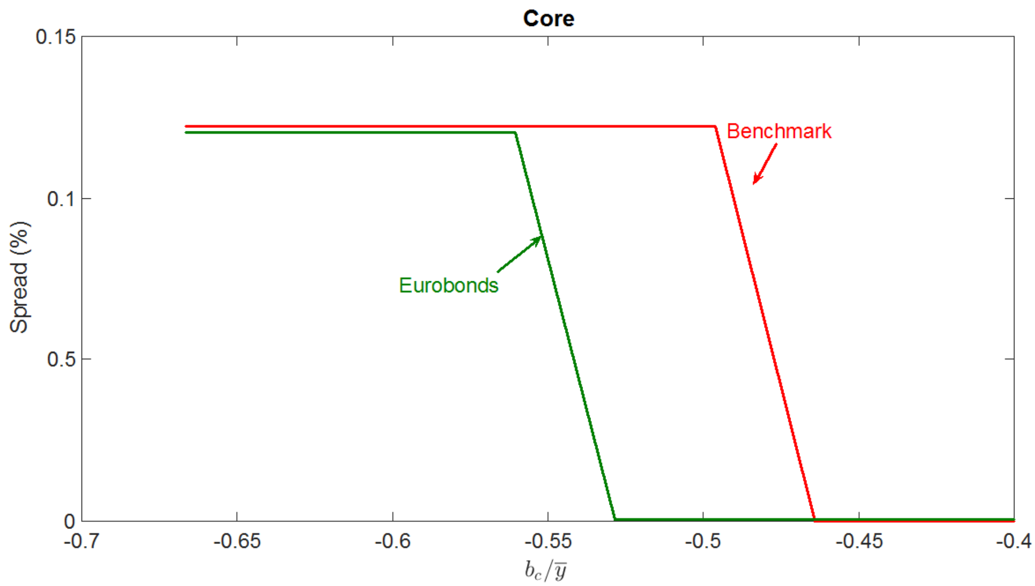

Figure 2: Core's spread in the Benchmark and the Eurobonds model, for the same level of debt-to-output ratio and income realization. Periphery's debt-to-output ratio is fixed to 55%.

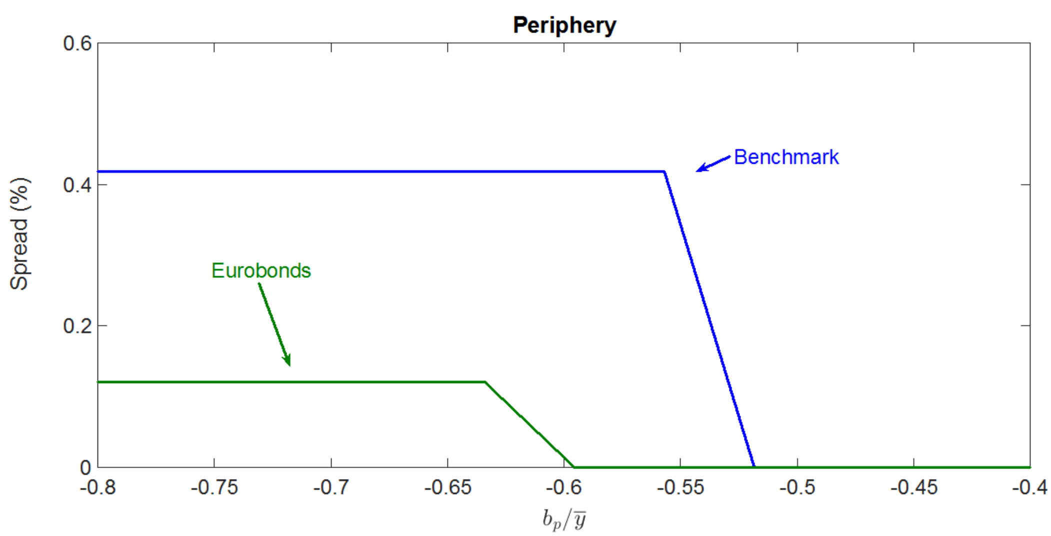

Figure 3: Periphery's spread in the Benchmark and the Eurobonds model, for the same level of debt-to-output ratio and income realization. Core's debt-to-output ratio is fixed to 50%.

repay simultaneously. The dark blue region shows the synthesis of asset level for which Core defaults while Periphery repays the sum of Eurobonds and then issues individual bonds. Finally, in the white area no pure strategy Nash Equilibrium exists. For this reason, I solve for the unique mixed strategy Nash Equilibrium of the repayment strategy. The solid yellow lines exhibit the threshold at which countries would default below that level of asset for a specific income realization in the benchmark model.

The analysis of repayment policies explains which of these two opposing effects dominates in the Eurobonds model in comparison with the benchmark model. Figure 4 shows the repayment policy functions when the Core and the Periphery face the lowest possible income realization, 4% below the trend of each country. In this figure, the contagion effect dominates, since after the introduction of Eurobonds the region that both countries default simultaneously is growing. On the left panel is the repayment policy function for the benchmark economy, in which there is no form of joint liability or any strategic interaction among the countries. Below the horizontal yellow line the Core countries default while below the vertical yellow line the Periphery countries default. Below the vertical and the horizontal yellow line both countries choose to default simultaneously while the vertical and the horizontal yellow line above both countries are solvent. On the right panel is the repayment functions for the Eurobonds economy, where countries issue assets with joint liability and they interact strategically on their repayment and borrowing decisions. In this particular case, the contagion effect dominates, since there are regions in which countries choose to default even though they would not in the benchmark economy. The area above the vertical yellow line and below the horizontal yellow line turns from blue in the benchmark to red in the Eurobonds. Here the result is driven by the fact that in this region the Periphery prefers to be insolvent while the Core inherits the sum of Eurobonds. Hence, the Core does not have the means to pay the whole sum of Eurobonds and it is dragged to the default region.

Figure 5 presents the repayment policy functions for which the cheaper access to credit effect dominates. In this figure, the cheaper credit effect dominates, since after the introduction of Eurobonds the region that both countries repay simultaneously is growing. The Core faces an income realization of 2% below its trend, while the Periphery has 4% below its trend (same as figure 4). On the left panel is the repayment policy function for the benchmark economy and the threshold for the Periphery is the same as in the previous figure although the Core now does not default. Here, the lower interest rate effect dominates since there is a region below the horizontal yellow line that turns from blue in the Benchmark model to dark green in the

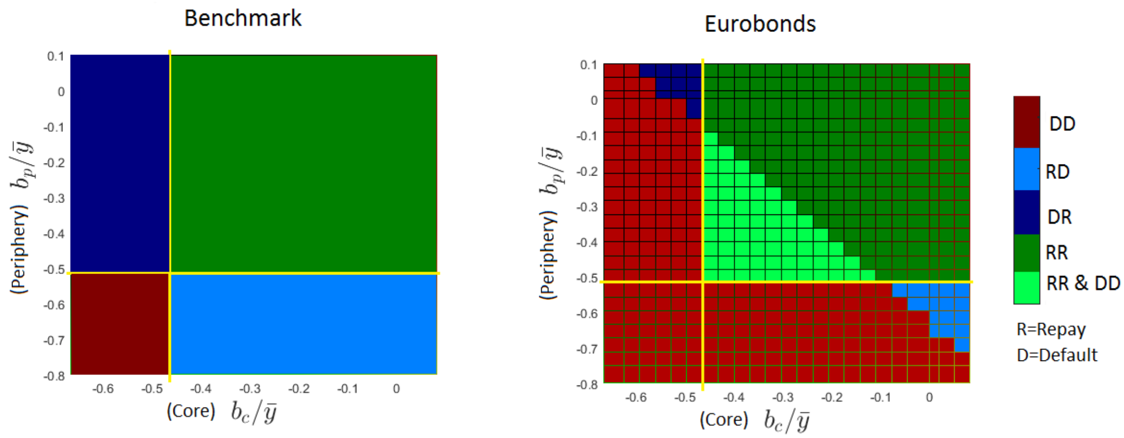

Figure 4: Policy function for repayment. Both countries have a deep recession, 4% below s steady state. DD is when both countries default, RR both countries repay, RD is when Core repays and Periphery defaults, DR is when Periphery repays and Core defaults.

Eurobonds model. This happens because the Periphery takes advantage of the relatively better income realization of the Core economy and receives a better interest rate. Hence, the Periphery has less incentives to default in this environment with joint liability.

Table 3 shows some quantitative predictions of the Eurobonds model. To derive the business cycle statistics, I run many simulations over time and report the mean, until at least one of the countries defaults in the Eurobonds. I use the same parameters as in the benchmark economy, to examine the effects after the introduction of joint liability bonds.

The Eurobonds model predicts that interest rates will decrease significantly in the long run, not only for the Periphery but also for the Core, because of the cheaper credit effect. I conduct two experiments for the Eurobonds model, *(i)* two asymmetric countries (i.e. Core and Periphery) and *(ii)* two symmetric countries (i.e Core and Core). Both experiments predict lower mean spread than the benchmark model. In particular, in both experiments the mean interest rate spread drops to 0.1% for both countries, while in the benchmark economy it was 0.5% for the Core and 1.9% for the Periphery.[9] Moreover, the volatility of interest rates reduces significantly in the Eurobonds model. The volatility of interest rate drops to 0.002% and 0.001% for the asymmetric and symmetric case respectively, while in the benchmark economy it is 0.019% for the Core and 0.042% for the Periphery. It is important to mention that the debt-to-output ratio

[9]Both countries receive the same interest rate, since they issue debt with joint liability. The interest rate in the Eurobonds model reflects the probability that both countries will be insolvent simultaneously.

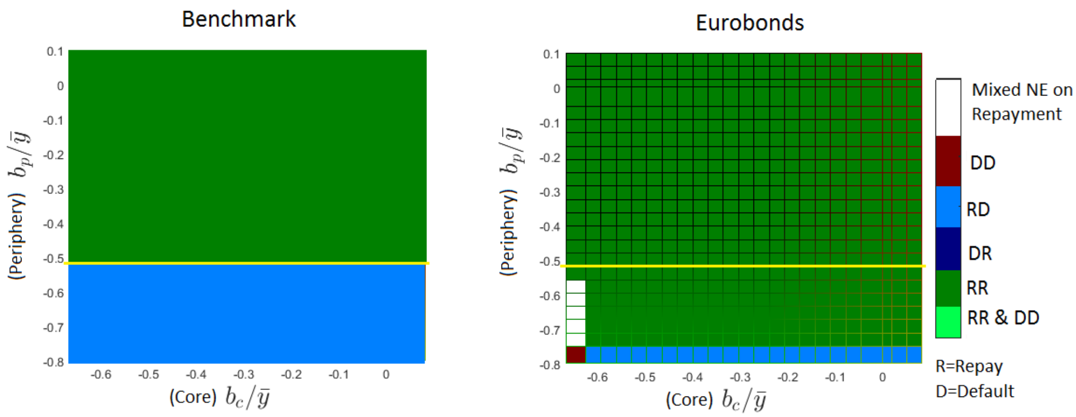

Figure 5: Policy function for repayment, Core has a mild recession and Periphery has a severe recession, 2% and 4% below steady state, respectively. DD is when both countries default, RR both countries repay, RD is when Core repays and Periphery defaults, DR is when Periphery repays and Core defaults.

is the same in the benchmark and Eurobonds models due to the exogenous borrowing limit, therefore there is no need for comparison. However, the goal of this paper is not only to forecast the effects on the spreads per se, but also the potential consequences of the Eurobonds on the countries' welfare, as it will be shown in the next section 3.3.

Table 3: **Business Cycle Statistics: The Eurobonds Model**

	Asymmetric		**Symmetric**
mean(%)	Core	Periphery	Core
Debt/Y	66	79	66
Spread	0.1	0.1	0.1
std(%)			
Debt/Y	0.035	0.034	0.029
Interest rate	0.002	0.002	0.001

3.3 Welfare Effects of introducing Eurobonds

I first solve for the benchmark economy in which there is no form of joint liability. Then, I measure the welfare effects of an unanticipated announcement explaining that from now on, Core and Periphery will be forced to issue debt with joint liability and interact strategically on their borrowing and repayment decisions. I measure the welfare effects as the proportional changes of consumption that would leave the consumer indifferent between living in the benchmark environment or in the Eurobonds environment, given the stationary ergodic distribution of income. This consumption change is given by

$$\lambda_i = \left(\frac{V_i^E(S)}{V_i(b_i, y_i)} \right)^{\frac{1}{1-\sigma}}$$

where V_i^E and V_i denote the value functions with and without joint liability, respectively.

Figure 6 shows the unconditional expected welfare effects for both countries in the asymmetric environment. The Core's debt-to-output is fixed to 50% and countries have welfare gains above the zero line, otherwise they have losses. The Core is getting better off as the Periphery's debt-to-output ratio decreases. This is happening for two reasons, first the Core receives a better price because the Periphery has a lower debt-to-output and therefore a lower probability to be insolvent. Second, the contagion effects decreases, thus the Core has smaller negative externalities if the Periphery defaults. On the other hand, the Periphery is overall better off after the introduction of Eurobonds. More specifically, when the Periphery's debt-to-output ratio is between 80% and 54%, the Periphery faces welfare improvements as the debt level decreases, since the country inherits lower level of debt. However, when the Periphery has a debt-to-output ratio below 55%, the expected welfare effects are getting stagnant, because the Periphery faces negative externalities from the fact that the Core has a relatively high debt-to-output ratio. When the Periphery has a debt-to-output ratio below 38% there is a Pareto improvement, since both countries have welfare gains. The model also predicts that Eurobonds should not have been implemented when they were suggested at the peak of the Eurozone debt crisis. At that time, most of the Eurozone member states, especially the Periphery members, had relatively high debt-to-output ratios. Moreover, as the model predicts, the Periphery member states were in favor of Eurobonds while the Core were not. Nonetheless, the model foresees that when the Periphery has relatively low debt-to-output ratio, all the member states are better off with the introduction of Eurobonds. Hence, if Eurobonds had been introduced before the financial crisis, when almost all the member states had low level of debt, then it would have been beneficial for all the member states.

Figure 7 presents the expected Pareto Effects in the asymmetric environment. In contrast with figure 6, where the Core has a fixed level of debt, this figure examines the Pareto Effects for all the possible asset combinations. The green region shows all the asset combinations for which there exist a Pareto Improvement. As it is explained in figure 6, in order to have Pareto

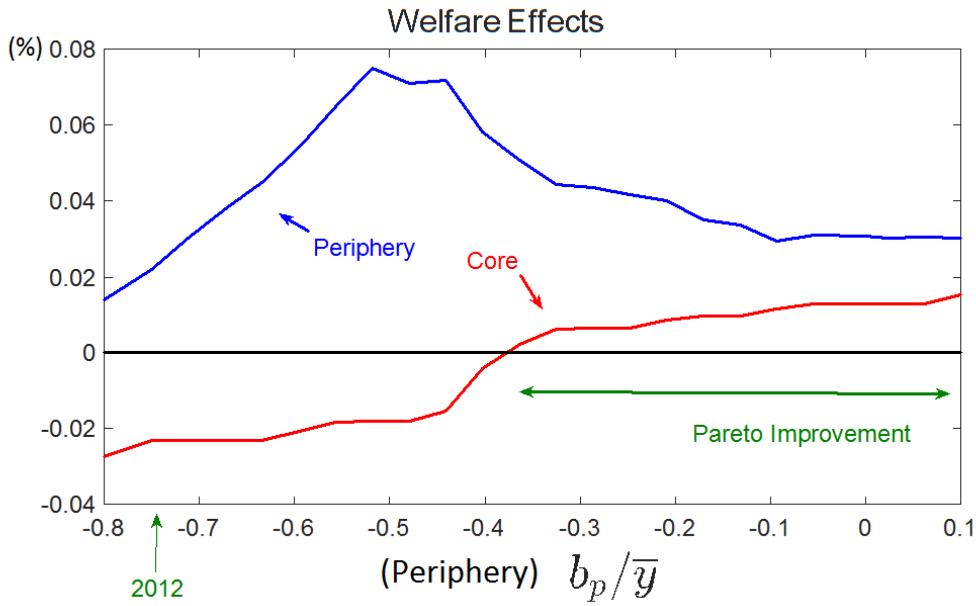

Figure 6: Above the zero line countries face welfare gains, while below welfare losses. Core's debt-to-output ratio is fixed to 50%.

Improvement the Periphery countries should have a relatively low level of debt. Otherwise, there is a Pareto loss, mostly because the Core countries have welfare losses.

Figure 8 shows the expected Pareto effects in the symmetric environment for all the possible asset combinations. As in Tirole (2015), as the countries get more symmetric the welfare improvements are bigger after the introduction of bonds with joint liability. In particular, the Core member states would be better off if they had a Eurobonds agreement with symmetric countries instead of the Periphery countries. For example, Germany would be better off if it had a Eurobonds agreement with France instead of Spain. The main force for this result is the fact that the contagion effect is smaller in comparison with the asymmetric case. Moreover, table 3 shows that there is no significant difference on the average spread between the asymmetric and symmetric environment, thus the cheaper credit effect is similar in both environments.

It is likely that this model may underestimate the welfare gains from lowering the sovereign spreads mainly for two reasons. Firstly, lower sovereign spreads lead to better allocation of factors of production and therefore could create significant positive effects as in Mendoza & Yue (2012). Secondly, they decrease the probability of a credit crunch and/or a banking crisis as in Sosa-Padilla (2015) and Bocola (2014). In light of these findings, gains from introducing

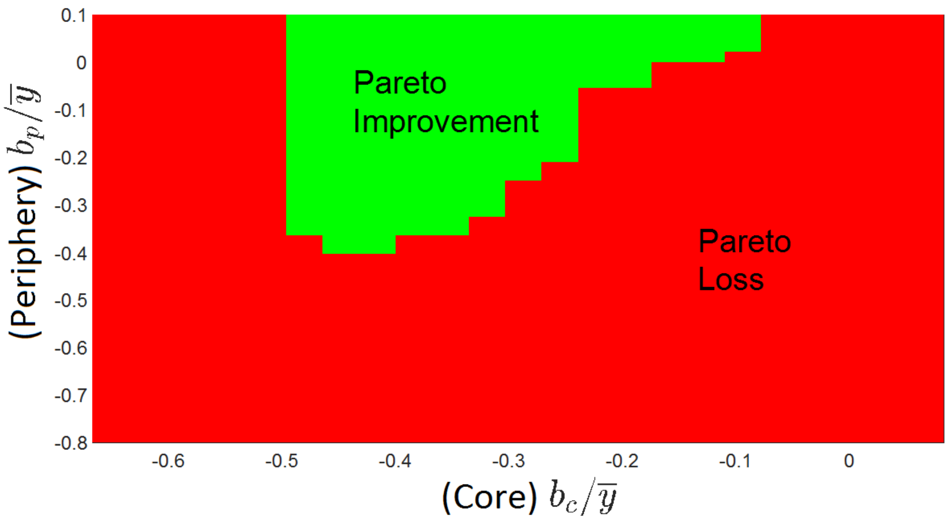

Figure 7: Expected Pareto Effects in the Asymmetric environment, for all the possible asset combinations.

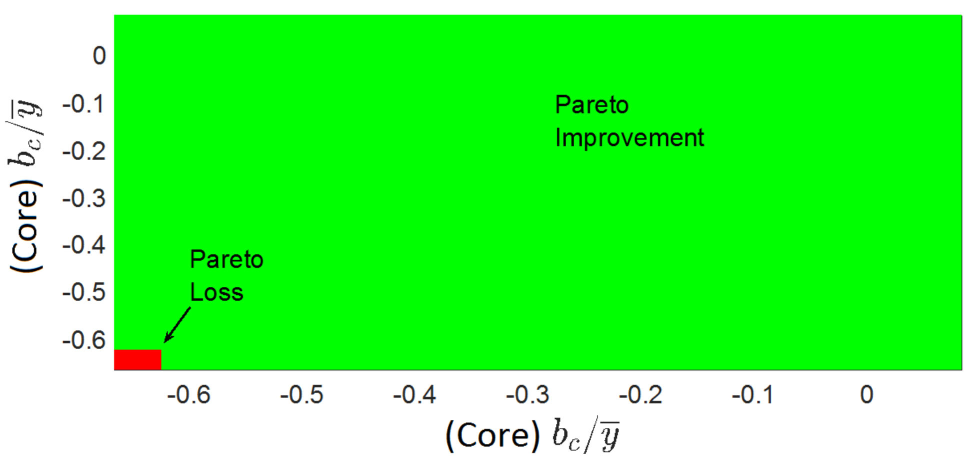

Figure 8: Expected Pareto Effects in the Symmetric environment, for all the possible asset combinations

joint-liability bonds may be larger than the ones I compute.

4 Conclusion

Europe faces the dilemma of whether to step forward to a higher degree of unification. This paper develops and analyzes a Eurobonds model where two small open economies issue bonds with full joint liability and interact strategically on their borrowing and repayment decisions. I compare this to the benchmark economy, which builds on a standard default model as in Arellano (2008), under two different scenarios. In the first scenario countries are asymmetric, one country is wealthier and less volatile than the other (i.e Core and Periphery member states of Eurozone), while in the second scenario there are two symmetric countries (i.e Core and Core). The findings show that in both scenarios Eurobonds decrease the yields of sovereign debt for all the member states in the long run. Nonetheless, the welfare consequences in the asymmetric scenario hinge critically on the timing of its introduction. More specifically, introducing such an instrument at the peak of the Eurozone crisis would have brought welfare gains for the Periphery member states and losses for the Core member states. However, adopting Eurobonds in "normal times", when member states have relatively lower debt-to-output ratios, has the potential to make all participants better-off. In the symmetric scenario, the implementation of Eurobonds produces welfare gains for all participants.

A natural extension of the model with the asymmetric scenario would be the analysis of whether or not member states would be willing to take austerity measures to reduce the current high debt-to-output ratios. This would allow the member states to reach the debt-to-output ratio levels at which all participants would be better off with the introduction of Eurobonds. Moreover, it would be newsworthy to explore the option of a joint liability mechanism that allows for bailing-out insolvent participants, as in Azzimonti & Quadrini (2016). In particular, in this environment countries will make transfers in order to decrease the default incentives. This mechanism has the potential to generate not only less default, but also reduce the contagion effect. Finally, it would be interesting to examine the case which countries are not forced to permanent financial autarky and they are permitted to issue debt after a few years of the default incident. I leave these for future research.

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A Appendix

A.1 Additional Results for the Effects of Introducing Eurobonds

Figure 9 presents next period's expected inherited debt for Core and Periphery in the asymmetric experiment, when Core's debt-to-output ratio is fixed to 50%. If the model does not allow for default, Core's expected inherited debt would have been an horizontal straight line and the 45 degree line for the Periphery. However, in this model countries are allowed to default and inherit zero debt. For this reason, when Periphery has low debt-to-output ratio the expected inherited debt for Periphery is the 45 degree line and for the Core the expected inherited debt is the horizontal line. Nonetheless, Periphery defaults more frequently as the debt increases, thus below -0.75 Periphery's expected inherited debt line is getting flatter and Core's expected inherited debt is getting larger. Hence, this figure shows the negative externalities that Core countries receive from the fact that inherit higher level of debt.

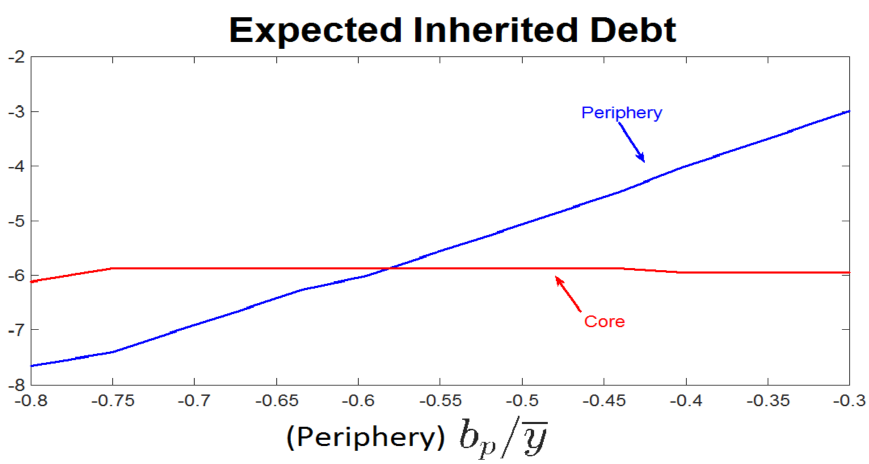

Figure 9: The Expected Inherited Debt for Core and Periphery, while Core's debt-to-output ratio is fixed to 50%.

Figure 10 shows the unconditional expected welfare effects of introducing joint liability bonds in the asymmetric environment. On the right graph is Periphery's welfare effects after the introduction of Eurobonds. As I discussed previously there are two opposing forces, the effect of cheaper credit and the contagion effect. The introduction of Eurobonds brings welfare gains for Periphery mainly because the cheaper credit effect dominates for all the combination of assets between Core and Periphery.

The left panel of figure 10 presents Core's welfare effects after the introduction of Eurobonds.

The welfare effects are mixed and they depend on the asset combination of the Core and Periphery. When Core and Periphery have relatively high debt-to-output ratio, the contagion effect dominates and Core is worse off. However, when Periphery has relatively low debt-to-output ratio then Core has welfare gains due to the cheaper credit effect. At this point I would like to mention that the combination of the left and right panel of figure 10 generates the Pareto Effects that are shown on figure 7.

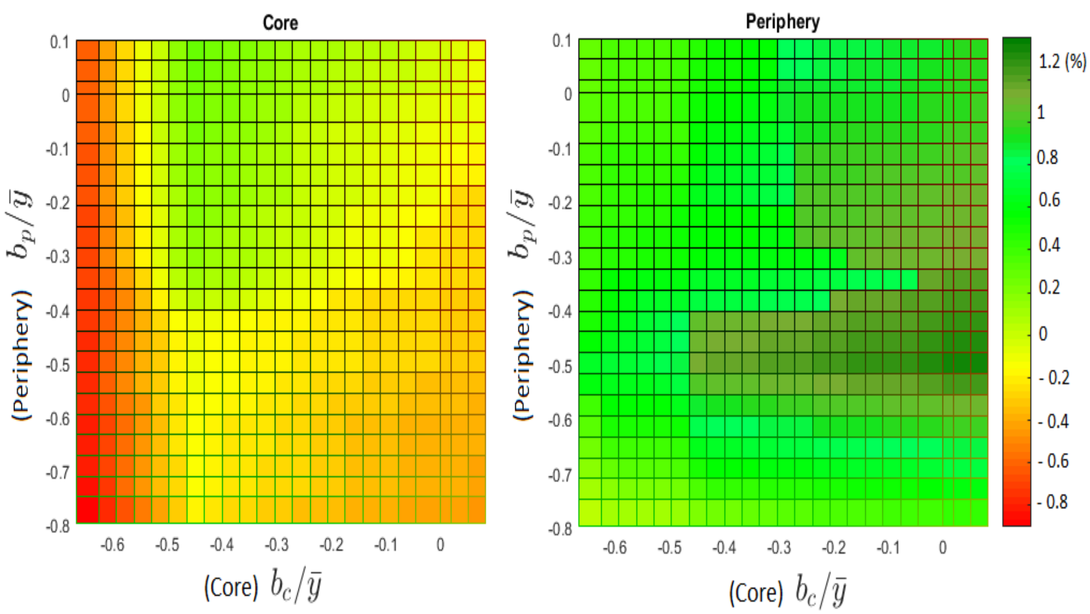

Figure 10: Welfare effects after the introduction of Eurobonds (Core & Periphery). Dark green and Red represent the welfare gains and losses, respectively.

Figure 11 performs the same experiment as in figure 10 for the symmetric environment. In this environment countries are better off because the contagion credit effect is smaller than the asymmetric case, thus countries will be more willing to issue debt with joint liability at any combination of assets. As I explained in the section 3.3 the cheaper credit effect is similar in the symmetric and asymmetric environment. The combination of this figure for both participating countries in the symmetric environment derives the Pareto Effects of figure 8.

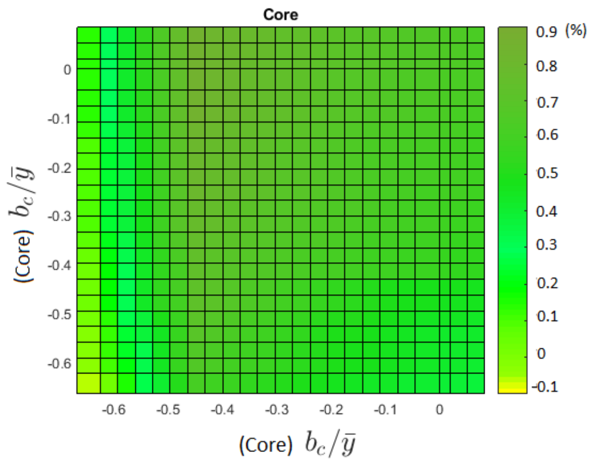

Figure 11: Welfare effects after the introduction of Eurobonds (Core & Core).

A.2 Computational Algorithm

The following algorithm is used to solve the Benchmark and Eurobonds models:[10]

1. Discretize the state space for assets $b = (b_1; b_2)$ consisting of a grid of 1600 points equally spaced and the endowment space $y = (y_1; y_2)$ into 25 pairs using Tauchen & Hussey (1991) method.

2. Solving the Benchmark model for the two countries separately (following Arellano (2008))

   (a) Start with some guess for the parameters to be calibrated: β_i and γ.

   (b) Start with a guess for the bond price schedule such that $q_i^0(b_i', y_i) = 1/(1+r)$ for all b_i' and y_i.

   (c) Given the bond price schedule, solve the optimal policy functions $c_i(b_i, y_i)$, asset holdings $b'(b_i, y_i)$, repayment sets and default sets $A_i(b_i)$ via value function iteration. I iterate on the value function until convergence for a given q_i^0.

   (d) Compute business cycles statistics from 3,000 simulations that each have 3,000 periods. If the model business cycles match the data we stop, otherwise we adjust parameters, and go to step 2.a.

3. Solving the Eurobonds model, given the parameters and the equilibrium outcomes of the Benchmark model:

[10]It is important to compute first the Benchmark model separately and then use the equilibrium parameters to compute the Eurobonds model.

(a) Given the price schedules $\{q_i(b_i, y_i)\}_{\forall i}$ and the value functions $\{W_i^{def}(y_i), V_i(b_i, y_i)\}_{\forall i}$ from the Benchmark models.

(b) Derive the pay-off functions $W_i^{rd}(S)$ $\forall i$, this is the scenario that country i repays while country i^- defaults.[11]

(c) Start with a guess for the eurobonds price schedule such that $q_E^0(S) = 1/(1+r)$ for all the possible combinations of b_i' and y_i.

(d) Given the price schedules $\{q_E^0(S), q_i(b_i, y_i)\}_{\forall i}$ and the pay-off functions $\{W_i^{def}(y_i), V_i(b_i, y_i), W_i^{rd}(S)\}_{\forall i}$. To solve for the value function and the intra-period Nash Equilibrium, for a given price schedule, the following algorithm is being used:

   i. Assuming that both countries choose to repay, I solve for the pay-off function $\{W_i^{rr}(S; b_{i^-}')\}_{\forall i}$, and the best response of debt policy function $\{b_i^{BR'}(S; 0, b_{i^-}')\}_{\forall i}$, for all the arbitrary next period asset decisions of country i^-, given that the country i^- is solvent.

   ii. Given the best response debt policy function for all the arbitrary debt decisions of the other country. I solve for the fixed point that yields the optimal best response of asset and repayment decisions $\{h_i^{BR'}(S; h_{i^-}^{BR'}, b_{i^-}^{BR'}), b_i^{BR'}(S; h_{i^-}^{BR'}, b_{i^-}^{BR'})\}_{\forall i}$ and update the value function for Eurobonds, $\{V_i^E(S) = \tilde{V}_i^E(S; \{h_i^{BR'}, b_i^{BR'}\}_{\forall i})\}_{\forall i}$. I iterate on the the value function for Eurobonds until convergence for a given $q_E^0(S)$.

(e) Given the optimal best response for repayment of both countries I update the price schedule of Eurobonds $\{q_E^0(S)\}$, and go to step 3.d until convergence.

[11] $W_i^{rd}(S)$ is one shot problem since in the first period country i has to pay the sum of Eurobonds and then continues as in the Benchmark model, without issuing assets with joint liability.