The Microeconomics of a Corrupt Tax Bureaucracy

by

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Abstract

This paper is the first of a series of papers exploring the problems of evasion and corruption, and how these influence the effectiveness of government policies. This paper develops a simple model of the corrupt tax bureaucracy based on institutional structure which characterizes many developing countries. It is shown that higher penalties as recommended by earlier models may not be the best of increasing tax compliance. A reward scheme for tax bureaucrats is derived, which could result in less corruption and higher tax collections. Finally it is shown that the bureaucratic wage bill which maximizes net government revenues may be the same as that which minimize the resources wastage in work seeking and tax evasion.
1. Introduction

The issue of tax evasion has been of long standing concern to many developing countries where non-trade taxes constitute a substantial share of tax revenues. This is particularly so in countries in which government efforts to raise either government savings or consumption expenditures has led to progressively higher tax rates. This concern has, however, often been publicly muted because of the intimate links between evasion by private individuals and the bribery and corruption of public officials, and the delicate political nature of the latter problem. Even though the socio-political aspects of the problem will not be directly addressed, the link between the two cannot be entirely ignored as it has been in the existing literature.

The literature on income tax evasion (Allingham and Sandmo (1972), Srinivasan (1973), Kolm (1973)) uses what may be termed the IRS model of evasion. This assumes a remote, honest and impartial tax bureaucracy which is, however, limited in its ability to check evasion. It therefore randomly selects, investigates and imposes penalties on discovered evaders. There is a complete asymmetry between the honesty of the bureaucracy and the evader. In most developing countries the relationship between the tax bureaucrat (both income and commodity taxes) and the tax payee is much more intimate. Each bureaucrat has the responsibility of dealing with a specific set of tax-payers whose returns he investigates and from who he has the responsibility of collecting taxes. More importantly, it seems arbitrary and unjustified to assume that all bureaucrats are totally honest while all tax payers are not.
In the somewhat different context of government contracting Rose-Ackerman (1975) has investigated the conditions under which bureaucratic corruption and bribery are likely. The problem addressed here is quite different, and one important difference is worth noting; An element of information asymmetry between the 'evader' and the bureaucrat is an essential element of the tax evasion problem. In contrast Rose-Ackerman does not explicitly consider this issue as it is not as essential to the contracting problem.

The present paper therefore attempts to develop a simple general model of tax evasion and bribery. At this stage, details specific to a given tax such as Income or Commodity tax are avoided. The objective is to understand the complex interaction between the tax evader, the bureaucrat dealing with him and receiving a bribe, and the government as an institution to which taxes and penalties accrue. In addition three specific results of interest for tax administration are derived: In the IRS model an increase in penalties is found to be a substitute for increasing the probability of detection. The latter involves higher administrative costs, however. It follows that higher penalties are the best means of improving net revenues or even eliminating evasion entirely [Singh (1973)]. In the presence of corruption, the present paper shows that higher penalties do not lead unambiguously to higher revenues. These results are more consistent with casual observation that governmental concern about increased evasion seldom leads to imposition of higher penalties on evaders. The second issue to be analyzed in the paper is a reward scheme for inducing bureaucrats to reduce bribe taking and increase government revenue collections. Conditions under which a scheme which shares
increased tax collection with the tax bureaucrats, is feasible are derived. The third issue to be analyzed is the connection between salaries of government bureaucrats and the rents arising from evasion. This is an issue which has not been considered in the growing literature on rents and resource wastage [Kruger (1974), Bhagwati & Srinivasan (1980, 1982), Buchanan, Tollison & Tullock (1980)]. It is shown that under certain conditions the wage rate that optimizes tax revenues net of the government wage bill is identical to the wage rate that minimizes total resource wastage.

At the next stage (in succeeding papers) this model will be embedded in analysis dealing with income and commodity taxes respectively. The approach adopted in the present paper is, however, influenced by this more general objective. It is therefore useful to state it briefly. The objective is to develop a model of steady state tax evasion and bribery which can be fully incorporated into the traditional static equilibrium model. Two sets of economic analysis can then be carried out: One is to determine the effect of evasion on various parameters of interest such as revenues and welfare and the other is to evaluate the effects of different policies, particularly tax policies in the presence of evasion.

The next section presents a view of the corrupt bureaucracy which underlies the present model. It can serve as a preview for those unfamiliar with this problem as well as a check on the model for others. Section 3 outlines the model and presents a solution of the bribery-evasion problem. Section 4 analyzes the effect of changes in exogenous parameters, and the introduction of an incentive scheme for bureaucrats. Section 5 examines the problem of rents and rent seeking, and the resource wastage arising from
evasion. Section 6 presents a simplified model which may be applicable to many countries. Section 7 concludes the paper.

2. A View of the Corrupt Bureaucracy

As tax evasion and corruption are areas on which it is inherently very difficult to collect data and do empirical work, an evaluation of any model must be based to a much greater degree on its general plausibility and the realism of its assumption. It is therefore useful to first summarize the perceptions on which the current model is based. We conceive of this problem in terms of a bureaucratic and information hierarchy. The taxpayers at the bottom of the hierarchy have the most information, assumed perfect, of their legal tax liability. As most income tax evasion in developing countries involves self-employed persons or entrepreneurs, for whom it is difficult to distinguish between labor and capital income, and commodity taxes are collected and paid to the government by firms, we will refer interchangeably to tax payers as firms.

The next level consists of the bottom rung of the tax bureaucracy, the tax inspectors, who interact directly with these firms. Each of these inspectors is assigned a specific set of taxpayers. It is the responsibility of each inspector to determine and collect all legally due taxes from the firms in his assigned set. By the very nature of evasion the tax related information available at this level is in general less than that to the firms. The inspector also has the direct responsibility for detecting and exposing evasion, and recommending to his supervisor the appropriate penalties to be imposed. Obviously the inspector also has the option of not exposing a
part or whole of the detected evasion in return for an appropriate bribe. This is done directly or more often indirectly through the tax accountants representing the firm. It should also be noted that even if the tax bureaucrat knew exactly the average level of evasion and/or the suspected level for each firm, there will be great variability in the confidence with which a legal case can be made for imposing penalties. As most such penalties can be appealed against to a quasi-legal or legal authority, the probability with which different proportions of the suspected tax evasion will be subject to penalties will in general vary from zero to one.

The next level of the information and bureaucratic hierarchy consists of the next-in-line supervisors under whom the tax inspectors are grouped. Though the supervisors have both the authority and the ability to sample the tax compliance of firms and the performance of the inspectors, this ability is clearly limited by their smaller numbers. They are more likely to keep a watch on the (actually) perceived or potentially large taxpayers. The larger the amount of evasion detected by their inspectors the more likely is the information likely to reach them either directly from the concerned inspectors or indirectly through the general information pool in the department. Sometimes when the type of evasion detected is particularly blatant, and the potential exposure from the lower level high, they may be approached by the evading firm in the hope of bypassing the lower level. In a different situation, the supervisor may be collaborating directly with lower levels in extracting bribes. In this case the more difficult or large cases are sent up as matter of course to be dealt with directly by them.
The higher level bureaucracy above the supervisory level does not in general have regular direct contact with any tax payer. Most information is received through the lower levels. This includes information on taxes paid, evasion detected and penalties imposed. Information about cases of evasion which are large by supervisor level standards is, however, likely to percolate up through this level. These higher levels are the ones at which performance standards for tax collection and evasion exposure are set. So are the departmental reward and punishment mechanisms for directing lower levels towards the institutional goal. The extent and diligence with which such incentive systems, are implemented, and to a lesser degree the formal and informal rules themselves, are influenced by the general socio-economic, moral and political environment. At the highest levels they will also, in a democratic framework, be subject to direct pressure from public opinion.

Where a vigilance department exists, it is nominally considered to be a part of this higher level. The job of the vigilance department is to detect corruption and bribery at lower levels. The members of the vigilance department generally belong to the same socio-economic levels as the inspectors and supervisor they are supposed to check on. Their pay scales and status are often virtually the same. Though in the initial stages they may act as higher level organ checking on the lower levels they are subject to similar temptations and often gravitate to the same level of honesty.

In any real situation we expect to find honest and dishonest individuals at every level, and in varying proportions. They are subject to different degrees of pressure to do their work honestly, both from within and from outside the organization. For our present purpose it is not very fruitful to
attempt a representation of all features of such a complex organization. One simplified representation would be to think of the organization as consisting of a lower level which is dishonest and a higher level which is perfectly honest. In a model of the corrupt bureaucracy we find it more useful to think of two forces: One of which we label as the tax bureaucrat and the other the tax institution. We think of the former as a representative of all bureaucrats in the organization who view and evaluate a potential bribe largely in terms of an economic calculus. We think of the institution as an embodiment of all the forces, rules, regulations and honest individuals pushing on the tax bureaucrats to perform their assigned duties honestly. The view underlying this representation, is that just as an individual wanting to be corrupt is constrained by institutional pressures, so too an individual wanting to be honest can be constrained by the ethos of corruption (more so if he is at lower levels) and by the need to utilize institutional mechanisms. If he is at a higher level he may also have to work through corrupt lower level officials.

One final point needs to be noted. When the problem of corruption becomes of any significance, the tax inspectors are not merely passive acceptors of bribes, but actively seek them. On the other hand even when the problem becomes quite extensive an effort to maintain a modicum of secrecy is never abandoned. Thus the inspectors may continue to communicate through trusted intermediaries even though the list of such intermediaries expands phenomenally. Similarly even though firms know that other firms are giving bribes, and inspectors know that other inspectors are taking bribes, the precise amounts and levels of activity will not be communicated to those not
directly involved in a particular transaction. Thus secrecy is an important element of the "market" for bribes.

3. A Model of Tax Evasion and Bribery

Before setting out the model it is useful to provide for reference a list of symbols used in the paper.

\[ z \] = Legal tax liability
\[ x \] = Tax evaded
\[ \bar{x} = z - x \] = Taxes paid
\[ y = zh(\frac{x}{z}) \] = Tax evasion detected by the tax bureaucracy.
\[ w \] = Part of detected tax evasion which is exposed to the institution
\[ K \] = Equilibrium level of \( w \)
\[ P(y) \] = Penalty function
\[ G(y) \] = Function summarizing the institutional rewards and penalties faced by bureaucrats.
\[ C(x) \] = A function specifying resource cost of evasion to the firm.
\[ b \] = Bribe per firm.
\[ S_G \] = Government Share
\[ S_B \] = Bureaucrats Share of total gain from evasion detected by
\[ T \] = Tax collections of government including exposure and penalties.

In the present paper, we will assume that the amount of tax which is legally due \( (z) \) is fixed and known perfectly to the firm. The firm expends
certain resources, or incurs certain costs (C) in an effort to evade a certain amount (x) of these taxes. This involves concealment of a part of the base on which such taxes are levied. This may involve keeping a double set of accounts, hiring a trusted accountant, obtaining spurious receipts, or even physically concealing part of the output, inventories or sales of commodities. The tax bureaucrat's aim is to detect this evasion, and the amount he can detect (y) will in general be less than or equal to the amount evaded.

We can think of this as a problem of asymmetric information: The firm interposes an information filter between the actual tax information, and the tax bureaucrat. In a conventional information framework we could think of the bureaucrat as making an estimate of the total evasion based on all the information he obtains. That is he would have probabilistic estimate of evasion conditional on his information. There are two reasons why the present problem differs significantly from this one. Firstly, in general the bureaucrat never finds out with certainty what the actual evasion was at any time in the past. There is therefore no way to rationally check expectations and forecasts against actuality. Consequently there is no reason for the expected evasion, as perceived by the bureaucrat, to converge to the value of the actual evasion, even if the latter is constant. Secondly, given the institutional-legal framework for imposing taxes and penalties, "expected evasion" (or suspected evasion) has no operational significance. This is true to a large extent, even for bribes, which must be based on the amount of tax and penalties avoided in return for the bribe. Another, more useful, way of putting it is that for any given amount of evasion and the consequent
suspected evasion there are different degrees of confidence with which evasion can be asserted and proved. One can conceive of the quality of evasion deteriorating with the proportion of the tax evaded, so that the marginal evasion is much easier for the bureaucrat to detect and prove, than the initial amounts. Similarly we expect the average quality of evasion to be lower the higher the proportion of payable taxes evaded. As we are interested in a steady-state equilibrium we will summarize these facts by assuming that on average, the amount of tax evasion formally detected by the tax bureaucrat \( y \), is related to the actual tax evasion \( x \) as follows;

\[
\frac{y}{z} = h\left(\frac{x}{z}\right) \quad 0 < h(.) < 1, \quad h' > 0, \quad h'' > 0
\]

\( y < x < z \)

where \( z \) is the real legal tax liability of the firm.

In contrast, the firm has information about the legal tax liability, the amount of evasion and the amount detected by the firm. It can in principle therefore estimate the function defined in (1). We will assume that the firm knows this function, that is, it knows exactly how the average level of detection is related to the average level of evasion.

Once evasion has been detected, the honesty or dishonesty of the tax bureaucrat must be explicitly introduced. If the bureaucracy was perfectly honest the entire amount of detected evasion would be subject to penalties. There would of course be no bribes and we would have a model very similar to the IRS one. If the entire bureaucracy was totally dishonest no penalties would ever be imposed and we would have an anomalous situation in which the whole system is corrupt but nobody is ever penalized for it (the Degenerate...
Both specialized models are equally unrealistic for the developing country situations we have in mind. As already indicated we will assume that the tax bureaucrat is corrupt, but must act within the limits imposed by the institution and be subject to the pressures towards honesty which arise within and outside the institution. This will be summarized by an institutional constraint to be made more precise subsequently.

In the assumed institutional setting, the difference between 'detected' and 'exposed' evasion is important. The former refers to the evasion of which the tax bureaucrat has knowledge, and the later to that part of detected evasion which is on the books of the institution and on which penalties are levied. 'Exposure' here refers to a formal written report made by the tax bureaucrat (inspector and/or supervisor) detailing evasion. It is observed that once a precise written determination of evasion is made it becomes institutionalized, and much more difficult for the tax bureaucrat to modify.\(^5\) In our simplified framework, the part of detected evasion which is exposed or institutionalized in this way is then assumed to result in payment of extra taxes and penalties with perfect certainty.

Given the level of detected evasion, \(y\), the problem is one of determining the proportion exposed, \(w\), the amount of bribe, \(b\), and the residual return to the firm. If we consider the determination of bribe and exposure as a market process several problems arise. Unlike the case of corruption in government contracting, there is no good or service being sold by the firm. One might stretch the definition of service and say that it is the tax bureaucrat who is selling a service—non-exposure. Clearly the price of the service in this case is the bribe. In abstract, we can imagine a
market consisting of firm-buyers on one side and bureaucrat-sellers on the other. In reality, given the necessity for confidentiality, and the secret nature of the proceedings it seems highly unlikely except in a completely degenerate system. An additional factor is that each bureaucrat usually deals with a specific set of firms and it is not possible for firms to switch from one "bureaucrat seller" to another.

This suggests a monopoly solution, or somewhat more appropriately a differentiated monopoly solution in which the bureaucrat can "sell" separately to each firm. Such a solution would imply an asymmetry in power which is in contradiction to the information asymmetry noted earlier. Casual observation on the extraction of bribes does not support this conclusion either: Given the variation in quality of evasion noted earlier, in the first stage the bureaucrat attempts to convince the firm of his knowledge of the extent and seriousness of the firm's evasion. This may involve some kind of written notice requesting explanation, or it may be carried on verbally. This is followed by direct or indirect offers of help in minimizing the seriousness of the offense in return for compensation. Finally a bribe price is negotiated along perhaps with some proportion of exposure and penalties. A bargaining or bilateral monopoly solution seems the most appropriate description of this procedure, and the one we will use in this paper.

The complete model can now be outlined: From the firm's perspective it involves a two stage procedure. The second stage involves determining the levels of bribe, exposure and penalties for a given level of detection. This yields the firm's residual share \( S_f \) as a function of detected evasion. Given this function the firm solves the first stage problem of how much taxes \( (x) \) to evade.
We will represent the second stage problem as a co-operative game between the firm and the bureaucrat. The bureaucrat has control over the level of exposure \( w \), while the amount of bribe \( b \) to be given is under the control of the firm. For a given level of detection we can write the receipts of the government \( S_G \), the bureaucrat \( S_B \) and the firm \( S_F \) as follows:

\[
S_G = w + P(w) \\
S_B = b + G(w) \\
S_F = y - w - P(w) - b
\]

where \( P(w) \) is the penalty as a function of the evasion exposed to the institution. \( G(w) \) is the function representing in monetary equivalents, the institutional rewards, penalties and pressures on the bureaucrat to do their job honestly.\(^6\) As indicated earlier, the taxes paid, the evasion exposed and penalties imposed are the only information received on a regular and sustained basis at the higher levels (or by the "institution"). In a steady state, i.e., when the initial zeal of the vigilance department has subsided, the primary factor in judging bureaucratic performance will be the level of exposure.\(^7\)

The net rewards from incremental exposure are likely to be high at low levels and low at high levels and the function \( G \) is therefore assumed to be concave. In some institutions rewards \( (G > 0) \), and in others punishments \( (G < 0) \), may predominate, but the exact level is not important for most of the
analysis. In general we would expect the $G$ function to be as in figure 1, with $G$ non-negative at the full exposure point ($w = y$).

![Figure 1](image)

An interesting observation follows from equations (2) to (5). The non-cooperative solution yields the same result as would occur if the bureaucracy was perfectly honest. That is if there was no way for the firm and the tax bureaucrat to communicate with each other and reach a joint agreement on bribe and exposure levels, the level of bribe would be zero, and the bureaucrat would expose all evasion detected. From (4) it can be seen that the best the firm can do given any fixed level of $w$ is $b = 0$. Similarly for the bureaucrat, given any fixed level of $b$ the best that he can do, (from equation (3) and the shape of $G$) is to set $w = y$. One policy implication follows directly from this observation: The more impersonal the system of checking or auditing returns, the less likely is (or more slowly will) corruption to develop. Thus, for example, a computerized system of auditing,
run by computer personnel who do not interact directly with the firms and the tax inspectors may retard the development of corruption.

The Nash solution for a co-operative game, which captures very well the bargaining process outlined earlier, will be used here. The two versions of this solution, which use either the threat point or the non-cooperative solution as a starting point for the bargaining process, are identical for our particular model, as the threat point is the same as the non-cooperative solutions presented above (see Appendix). The bargaining process consists of three parts: First the bureaucrat convinces the firm of the losses (through penalties) the firm would suffer from full exposure, and the advantage to the bureaucrat from exposure. The next step involves determination of the level of exposure that would result in the greatest total gain to the two parties. This involves a joint maximization of the value received by the two parties, i.e.,

$$\max H = S_B + S_F = y - w - P(w) + G(w)$$

which occurs at the point at which the marginal loss to the firm from exposure is equal to the marginal gain to the bureaucrat, that is (if an interior solution exists), 8/

$$1 + P^*(w^*) = G^*(w^*) \text{ or } w^* = k \text{ (say)}$$

The final step involves sharing of this total gain (as measured from the non-cooperative solution or threat point) between the two parties by means of an appropriate bribe. The Nash solution of this problem is simply to
Max \( U = \frac{\left| S_B - S_B |_{w=y} \right|}{S_F - S_F |_{w=y}} = \left[ \frac{S_B - G(y)}{S_F - (-P(y))} \right] \)

\[ \frac{|b=0}{|b=0} \]

\[ = [b + G(k) - G(y)] [y + P(y) - k - P(k) - b] \]

The solution to which is \( b = b^* \) where,

\[ b^* = \frac{1}{2} (y - k + P(y) - P(k) + G(y) - G(k)) \]

\[ \frac{1}{2} [y + P(y) + G(y) - (k + P(k) + G(k))] \]

(7)

**Figure 2:** Shares of Bureaucrat and firm as \( b \) varies

In figure (2) AB (CD) traces out the locus of shares of the bureaucrat and the firm as the bribe is varied between zero and its maximum.
feasible value \((S_F \geq -P(y))\). AB is however the maximum that they can jointly achieve, i.e. it is the value obtained at the optimal level of exposure determined in (6). Any other level of exposure will leave a smaller value to share between them and result in locii such as CD. The line EF perpendicular to AB and passing through the threat point E then determines the appropriate bribe at its point of intersection (F) with AB. We also observe from figure 2 that a necessary condition for a bribe taking Nash equilibrium is that the absolute gain to the tax bureaucrat from full exposure be less than the maximum bribe that the firm can given without being worse off than in the non-cooperative solution, i.e. \(G(y) - G(k) < y + P(y) - (k + P(k))\) or \(G(y) < y - k + P(y) - P(k) + G(k)\). This condition is satisfied if an interior solution of (6) exists.  

Substitution of (6) in (2) shows that the government's share from collection of exposed taxes and penalties on them, is independent of the level of detected evasion, and consequently of actual evasion. In principle this can provide a strong and clear test of the present simple model. From equation (7), it is also clear that the bribe is positively related to the amount of evasion so that the incentive for tax bureaucrats to detect evasion is maintained.

The slope of the \(G(w)\) function in its interior is a useful measure of the general degree of honesty of the tax bureaucracy. Thus a steeper function would indicate higher marginal rewards from honesty and consequently a more honest bureaucracy (or vice versa). Thus a perfectly honest bureaucracy can be seen as a limiting case in which \(G\) becomes steeper and steeper until it collapses into the y axis. Similarly a perfectly dishonest bureaucracy can be
seen as a limiting case in which G becomes flatter and flatter until it collapses into the x axis. It is easily shown that in the first case the solution of (6') is the corner solution \( w = y \) so that all evasion detected is exposed. 10/ Similarly, in the second case the solution of (6') is the corner one, \( w = 0 \), so that no evasion is exposed. 11/ Both the perfectly honest (IRS type) and the perfectly dishonest (degenerate type) models can therefore be derived as special case of the present model. A possible policy for making use of this observation to expand net revenue collection will be analyzed subsequently.

Equations (6) and (7), on substitution in (4) give

\[
S_F = S_F(y) = \frac{y - P(y) - G(y)}{2} - \frac{k + P(k) - G(k)}{2} \\
S_B = \frac{y + P(y) + G(y)}{2} - \frac{k + P(k) - G(k)}{2}
\]

(8)

The first stage problem for the firm is then to choose the level of evasion, \( x \) which maximizes profits.

\[
\max \pi = x - C(x) - y : S_F = x - C(x) - \frac{y + P(y) + G(y)}{2} - f(k),
\]

\[
y = zh(x), f(k) = \frac{k + P(k) - G(k)}{2}, c > 0, c'' > 0
\]

(9)

The solution to this is given by,
which states that the marginal benefit from evasion must be equal to marginal resource cost of evasion plus the marginal cost of detected evasion; The last being a composite cost consisting of exposure penalties and bribe costs.

4. Analysis: Exogenous Changes and Incentive Schemes

To carry out the analysis for the general functions used it is useful to replace $G(g)$ by $\alpha G(.)$, $P(.)$ by $\beta P(.)$, and $h(.)$ by $\gamma h(.)$. Thus $\alpha$ is a measure of the degree of honesty of the system, and results in a rotation of the $G(.)$ function at its zero point. A change in $\beta(y)$ results in a change in the absolute value and the slope of the penalty (information transfer) function. Thus a fall in $\gamma$ means that it has become more difficult for the bureaucrat to detect evasion. If we differentiate (6) totally and solve we find that the equilibrium exposure level $k$ can be written as\(^{12/}\)

\[
1 = C'(x) + \frac{1 + P'(y) + G'(y) h'(\frac{x}{2})}{2} 
\]

(10)

Similarly differentiating and solving for the evasion amount from (10) we have (see Appendix);

\[
x = x(\alpha, \beta, \gamma, z) \tag{12}
\]
As we would expect a decline in the level of honesty of the bureaucracy results in a decline in evasion exposure and an increase in evasion; the latter resulting from the decline in the combined bribe-penalty costs. An increase in absolute and marginal penalties leads to a decline in exposure and evasion. The latter result is similar to that obtained in the IRS model. The former is somewhat more unusual, but follows from the fact that the joint (to firm and bureaucrat) costs of exposure, at any level of detection, rise with the penalty. It therefore becomes jointly profitable to reduce exposure and to compensate the tax bureaucrat for his institutional losses by a rise in bribe share.

The effect of changes in tax liability on taxes evaded and declared is more complicated. Firstly from the first equation of (12') the amount of taxes declared and voluntarily paid always increase with tax liability. Allingham and Sandmo obtain the same result in their IRS type model with risk aversion; the taxes declared would be independent of taxes due, however, if the tax payer were risk neutral. This can only happen in the present model if the marginal resource cost of evasion, to the taxpayers, is constant, and all taxes are being evaded. From the first equation of (12') we also find that the proportion of taxes evaded declines with tax liability (or equivalently the proportion of taxes declared rises with tax liability). This result is similar to that obtained by Srinivasan but the conditions required reflect the difference in the modelling approaches used. In the present case

\[
\frac{\partial (x/z)}{\partial z} < 0 \text{ as } c''(x) > 0, \quad \frac{\partial (z-x)}{\partial z} > 0
\]
marginal resource costs of evasion must be increasing with evasion, while in Srinivasan's case (marginal tax rate constant) the probability of detection must be increasing with tax liability. The results are identical if we think of detection probability as equivalent to marginal (resource) costs of evasion. In Allingham and Sandmo the sign depends critically on whether risk aversion is increasing or decreasing. For the risk neutral case their results are again the same as the present ones if the marginal cost of evasion is constant. Finally, it is useful to explore in the context of the present model, why absolute evasion may increase or decrease with tax liability. If we think of the marginal change in the total bribe with a change in amount detected \((y)\), as a bribe-price, it can be shown that a negative sign is possible only if the bribe price decreases with detected evasion (a necessary but not a sufficient condition). This in turn is more likely if the marginal benefit of exposure, to the bureaucrat, is declining sharply, relative to the increase in marginal penalties. This result appears more likely for linear than for convex penalty functions. The total effect of changes in \(z\) on \(x\) will be negative, if the decline in bribe price with detected evasion overwhelms the increase in marginal detected evasion \((y)\) with increasing evasion \((x)\). Intuitively the effect is quite similar to the effect of increased demand on prices in cases where a firm's marginal total costs are declining.

In most developing countries there is a corporate sector which is required by law to keep proper books of account, and an unincorporated sector which is not. We would expect information transfer about evasion to be more in the former than in the latter. The predictions of the present model that the latter would have a higher level of evasion for given tax liability is
consistent with the accepted beliefs on this topic. An alternative explanation of difference in costs of evasion (C) yields the same prediction. Another example of the differential information transfer problem is found in the case of technically very similar chemicals taxed at very different rates. The model would predict a much higher level of evasion in a firm which produces both chemicals than in one which is legally bound (through license) to produce only the one subject to the high tax rate. This also suggests that an increase in the complexity of tax laws with no corresponding increase in the sophistication of the tax bureaucracy will reduce the information flows to them. This will tend to increase tax evasion. To the extent, that firms have to employ more qualified tax accountants, however, resource costs of evasion will increase. This will mitigate the effect on evasion.

Finally consider the tax collections of the government

\[ T = z - x + k + \alpha P(k) \]

we have

\[ \frac{\partial T}{\partial \alpha} = - \frac{\partial x}{\partial \alpha} + (1 + \beta P'(k)) \frac{\partial k}{\partial \alpha} > 0 \]

A deterioration in the honesty of the bureaucracy will lead to a decline in revenues. Similarly,

\[ \frac{\partial T}{\partial \beta} = - \frac{\partial x}{\partial \beta} + (1 + \beta P'(x)) \frac{\partial k}{\partial \beta} + P(k) > 0 \]
as the middle term is negative. Thus a rise in penalties does not have an unambiguous effect on tax revenues. This results contradicts the conclusions of the IRS model (see e.g. B. Singh (1973)) that penalties can be raised to point at which there is no evasion and consequently revenues are maximized (in that model). The present model is consistent with the observation that few governments raise penalties to counter a perceived deterioration in tax compliance. On the contrary they often come out with formal or informal schemes which allow tax payers to declare previously evaded taxes with reduced penalties.

The analysis of corner solutions suggests the possibility of introducing a reward scheme for bureaucrats. Schemes which give the bureaucrats a share of increased revenues from detection have often been mentioned and occasionally tried. In the context of the current model such a reward scheme can be described as follows: Given a certain level of detected evasion, there is an equilibrium level of sharing out between the firm, the bureaucrat and the government. By directly rewarding the bureaucrat for increased exposure, and in effect providing a substitute for the bribe, it should be possible for the government to obtain most of the firm's share in terms of higher revenues. The model shows, however, that the solution is not so simple. A higher reward to the bureaucrat not only increases his incentive for exposure, but also increases his bargaining power (or threat point) with the firm. This suggests that it might still pay him to continue dealing with the firm to extract most of the firm's share for himself. To disentangle the effects it is useful to look at a specific reward scheme.
Consider a reward function $N(w)$ which relates exposure levels $w$ to rewards $N$ to be paid by the government to the bureaucrat. The popular discussion of this reward scheme as well as the logic of the present model suggests that this function must satisfy the following conditions.

$$N(w) > 0, \quad N'(w) > 0, \quad 0 < w < y$$

That is the scheme does not involve absolute penalties or disincentives for increased exposure. The latter assumption is intuitively appealing, while the former is consistent with the view underlying the model, that there is an asymmetry between bureaucratic rewards and penalties. The effectiveness of penalties is much more strongly dependent on the general degree of honesty of the bureaucracy. Penalties would tend to modify the institutional constraint, but not in a simple linear fashion. The reward function modifies the shares given in equations (2) to (4) to

$$S_G = w + P(w) - N(w) \quad (2')$$

$$S_B = b + G(w) + N(w) \quad (3')$$

$$S_F = y - w - P(w) - b \quad (4')$$

The second stage problem of joint firm-bureaucrat maximization, for given detection, is modified from $(G')$ to
Max $\overline{H} = y - Q(w) + N(w)$, \hspace{1cm} \begin{align*}
Q(w) &= w + P(w) - G(w) \\
N(w) &= \begin{cases} 
0 & \text{if } w < k \\
\hat{Q}(w) - Q(k) & \text{if } k < w < \overline{w} \\
Q(\overline{w}) - Q(k) & \text{if } \overline{w} < w < y 
\end{cases}
\end{align*} \hspace{1cm} (13)

To simplify the problem of determining the best reward scheme, consider the following one ($\overline{w} < y$ is a constant):

\[
N(w) = \begin{cases} 
0 & \text{if } w < k \\
\hat{Q}(w) - Q(k) & \text{if } k < w < \overline{w} \\
Q(\overline{w}) - Q(k) & \text{if } \overline{w} < w < y 
\end{cases}
\hspace{1cm} (14)
\]

The bribe exposure problem can now be solved using the procedure of the previous section to obtain (on simplification):

\[
\overline{b} = y + P(y) - w^* - P(w^*) 
\hspace{1cm} (7')
\]

\[
\overline{S}_B = y + P(y) - k - P(k) + G(k) 
\hspace{1cm} (15)
\]

\[
\overline{S}_G = k + P(k) + G(w^*) - G(k) 
\hspace{1cm} (16)
\]

\[
\overline{S}_F = \overline{S}_B - P(y) 
\hspace{1cm} (17)
\]

\[
\begin{align*}
& \text{for } k < w^* < \overline{w}
\end{align*}
\]

The reason for the multiple solutions and the choice of the functional form for $N$ is illustrated in the following figure:
Thus the inclusion of a reward function, of the form in equation (14), modifies the joint bureaucrat-firm share function from $H$ to $\bar{H}$. Assuming for expositional simplicity that for two solutions yielding equal returns to the bureaucrat, he will pick the more honest one (with higher exposure), this shifts the exposure level from $k$ to $\bar{w}$ [i.e. $w^* = \bar{w}$ in equations (7'), (15), (16), and (17)]. Any reward function which left $H(w^*) < H(k)$ for all $w^*$, $\bar{w} > w^* > k$, would have no effect on exposure levels and consequently on revenues. It would merely (or at most) increase the bargaining power of the bureaucrat, and consequently his bribe. A reward function which made $H(w) > H(k)$ would result in a higher reward to the bureaucrat and smaller revenues to the government, without changing the bribe solution. This is because the firm is already paying out (in bribes) the maximum penalty that could be levied given the detected exposure. The chosen function is therefore a plausible candidate for the optimal reward function.
Government revenues are therefore maximized by setting \( w = y \) from equation (16), so that the solution of equations (7') and (15) to (17) is given by \( w^* = y \). The effect of the reward scheme is to shift the bribe-evasion solution from E to the zero bribe point at A [figure 2 or 4 (below)], the position of the joint maxima, AB being unchanged; The outward shift in AB due to the addition of extra joint revenues in the form of the reward, is exactly equal to the inward shift in AB due to higher exposure. In the process, the governments revenue collection from exposed evasion is increased by \( G(y) - G(k) \) [Shift from I-I to II-II in figure 4].

The firm's 1st stage problem is completely transformed: From obtaining positive gains (even) form detected evasion \( S_F > 0 \) in (3) to paying full penalties. Equation (9) is therefore modified to

\[
\text{Max } \Pi = x - C(x) - y - P(y), \quad y = zh \frac{x}{z}^{2}
\]  

\[(9')\]
whose solution \((x = \bar{x})\) is

\[
1 = C'(\bar{x}) + h'(\bar{z}) + P'(y) \cdot h'(\bar{z}), \quad y = zh(\bar{z}) \tag{10^-}
\]

which can be compared with "no-reward" solution \((x = x^*)\):

\[
1 = -\frac{1 - P'(y^*) - C'(y^*)}{2} \cdot h'(\bar{x}) + C'(x^*) + h'(\bar{x}), \quad y^* = zh(\bar{x}) \tag{10}
\]

We can prove the following result:

**Proposition 1:**

The reward scheme outlined above will result in a reduction of evasion (i.e. \(\bar{x} < x^*\)) and an increase in government revenues.

The first part of this proposition is proved in the appendix, assuming feasibility (see below). For the 2nd part we have, the total change in the government's revenues is therefore

\[
\Delta T = -(\bar{x} - x^*) + C(y) - C(k) > 0, \tag{18}
\]

so that a reward scheme for raising government revenues and eliminating corruption appears feasible.

The change in the level of evasion by the firms and consequently in the evasion detected by the bureaucrat also modifies his final returns from the new system. The change in his returns \(\Delta_B\), using equations \(8^-\) and \(15\) and simplifying is
Thus a necessary condition for the particular scheme to produce a stable, rational expectation equilibrium is that \( \bar{y} + P(\bar{y}) \geq M(\bar{y}, \bar{y}, k) \). If this condition does not hold, a simple (but inelegant) modification can produce the desired stability. This is to replace \( N(w) \) by \( N(w) + M(\bar{y}, \bar{y}^*, k) - \bar{y} - P(\bar{y}) \). As the second and third terms are constants they do not affect the marginal condition derived in (13). Similarly the bribe-evasion solution is not affected because the bureaucrats gets this amount whether he exposes fully or not \( \{S_B + M - \bar{y} - P(\bar{y}) - [G(y) - N(y) + M - \bar{y} - P(\bar{y})] = \bar{s}_B - G(y) - N(y) \} \) as before. Another, condition must be satisfied, however, as the net revenue gain to the government is reduced. This is merely that the total gain to the government and the bureaucrat (the sum of (19) and (20)) must be positive.

5. Analysis: Bribe Rents and Bureaucratic Wages

In this section we return to the model of section 3, and modify the analysis so as to analyze the problem of rents created by tax evasion. Rents arising from quantitative controls and taxes on trade have been dealt with by Krueger (1974), Bhagwati and Hanson (1973), Bhagwati and Srinivasan (1980) and others. Most of this literature traces the effect of such rents on welfare or
on prices, through the resource wastage that competition for such rents gives rise to. This section will look very briefly at this rent problem from a completely different perspective: The basic objective of raising revenues (through taxation) is assumed to be fixed and given, so the comparison of social welfare with and without taxation is not very useful at this stage. What is of much greater interest is the analysis of policies for reducing or eliminating rent. In particular, this section focuses on the possibility of reducing bureaucratic rents through appropriate government wage policy.

The basic idea is very simple. The net rent obtained by the tax bureaucrat is given by

\[ R = b + r_g - r \]

where \( R \) is the net rent, \( b \) is the bribe (rent), \( r_g \) the bureaucratic wage, and \( r \) the appropriate market wage (or opportunity cost to the bureaucrat of not working in the private economy). For expository simplicity, we will continue to assume that there is only one bureaucrat, so that \( r_g \) and \( r \) are the average wages for the bureaucracy, over the time period of tax collection (usually a year). It is clear from (21), that in principle rents could be eliminated by reducing the government wage rate; as long as \( b < r - r_g \), where \( r_g \geq r_g \) is an exogenously imposed constraint on government wage rates. This idea is not a new one: Tullock (1980) gives an example from the British colonial period, where officer level positions in the British army were sold by the government for a price. In terms of equation (21) this can be seen as an attempt to reduce the real government wage rate in the presence of a
constraint on reduction of nominal wages. The bribe rent in this case was the expected value of the "loot" that the officers obtained in the colonies.

The problem is, however, no longer as simple as it might have been for a colonial army. Popular discussions of the problem of increasing bureaucratic corruption in developing countries have sometimes suggested that it may be due to the decline in the real wage rate of bureaucrats compared to the private sector. In other words there is a contrary effect operating: A decline in wage \( r_g \) may itself result in a decline in the level of honesty of the tax bureaucracy. In terms of our earlier analysis, this would imply a decline in the slope of the \( G(.) \) function, and a consequent decrease in evasion exposure and an increase in tax evasion. Though the formal model is a static equilibrium one, we expect this shift in the \( G(.) \) function to be a gradual one. As rents \( R \) and net government revenues \( (T - T - r_g) \) will respond almost instantaneously, a government with a high discount rate on revenues may be tempted to allow bureaucratic wages to fall even if the new equilibrium involves a higher level of rent and/or lower level of revenues.

More formally, we assume that the institutional constraint \( G(.) \) now has the form

\[
G(w, r_g) = \alpha(r_g/r) G(w)
\]

where \( \alpha \) defined in section 4 is now a function of the relative wage rate. The form of the solution is unchanged and still given by equation (6) and (10) and the bribe given by (7). We expect the function \( \alpha \) to satisfy the following conditions: \( \alpha' > 0 \), \( \alpha'' < 0 \) for \( r < r_g \) we find that equations (11) and (12) can now be rewritten as
Net government revenues $t$ are the difference between gross revenues and the bureaucratic wage bill, i.e.

$$t = T - r_g = z - x + k + P(k) - r$$  \hspace{1cm} (22)

and total resource cost $RC$ from competitive rent seeking is the sum of rents from resources employed in seeking jobs in the bureaucracy $R$ and resource cost of evasion by firms $C$, i.e.

$$RC = R + C(x) = b + r_g - r + C(x)$$  \hspace{1cm} (23)

Differentiating and simplifying equations (22) and (23) and using the previous analysis, we can prove the following proposition (assuming that an interior optimum exists).

Proposition:

The value of the government wage rate ($r_g$) that maximize the net revenues of the government ($t$) is the same as the wage rate that minimizes the total competitive resource wastage ($RC$).
The proof of this proposition simply involves showing that

\[
\frac{\partial (RC)}{\partial r_g} = - \frac{\partial r}{\partial r_g},
\]

so that it is immediately apparent that the proposition is strictly valid only if an interior solution to the problem exists. The other limitation which must be kept in mind is the simplifications involved in developing the model. Keeping these limitations in proper perspective the result is very interesting. It means that in devising a wage policy for tax bureaucrats the government does not need direct information on resource wastage by firms or on rents obtained by bureaucrats. All it needs to do is to determine, by trial and error if necessary, the wage rates which will yield the maximum net revenues. The rent seeking problem is then taken care of automatically, to the extent possible with this policy tool.

The precise solution for the optimal bureaucratic wage rate is of course obtained (assuming that an interior solution exits), by setting the differential of the net revenue with respect to the wage rate equal to zero. This yields the following equation.

\[
G'(y) h'(x/z) \alpha'(r_g/r) \frac{\partial A}{\partial r} + \frac{r}{\alpha'(r_g/r)} [P''(K) - \alpha''(K)] = 1
\]

(24)

\[
A = C(x) + \frac{1}{2} h'(x/z) [P''(K) + \alpha''(K)] + \frac{1}{2z} \frac{P'(K) + \alpha'(K)}{2z} h''(x/z)
\]

where \( A > 0 \) from 2nd order conditions of the bribe-evasion problem. As this solution depends on 2nd differentials, further analysis becomes very difficult as we have virtually no information on the third differentials of the various functions. This suggests that the optimum value of \( r_g \) will be difficult to
determine using this model, and may have to be done by experimentation. This raises the difficult problem of short run effects masking the long run impact. As already indicated the net revenues of the government are likely to respond immediately while the effect on the degree of honesty of the bureaucracy is likely to be a gradual one. Thus a steady reduction in the relative real wage of the bureaucracy could result in increasing revenues over the medium term, and incorrectly suggest that this is the best long term policy.

Section 6: Simplification

Casual observation suggests two changes in the model, which may be applicable in several developing countries, and which considerably simplify the model for both empirical and further analytical purposes. Firstly the monetary penalty function in most countries appears to be an almost linear function of evaded taxes (or the tax base on which evasion takes place). Non-linear functions can be justified in such situations only if non-monetary costs like possible jail sentence are included. In a corrupt system it may be a reasonable approximation to ignore such costs, as jail sentences can often be avoided by reaching with bribes into higher levels of the tax bureaucracy or the political system. In this section we will therefore assume a linear penalty function.

Secondly in a tax bureaucracy rules are often framed in terms of quotas for different bureaucrats. Thus each tax bureaucrat may be expected to expose a certain amount of evasion. If he fails to expose this amount, strong negative pressure (penalties) may be imposed. On the other hand there is
little positive incentive to expose more than the assigned amount. In a sense the corrupt bureaucracy informally decides on an appropriate level of total exposure needed to keep institutional and outside pressures in control. Given this level of exposure tax bureaucrats are tacitly allowed to collect as much bribe as they can. In the present paper such a situation can be represented by the kinked \( G(.) \) function given figure 5. Assuming that the kink occurs at

\[ w = k, \quad \text{with} \quad G_o < 1 + P_o < G_1, \]

the solution is identical to that obtained earlier. This modification however, allows us to simplify \( S_B, S_F \) and \( \Pi \) to

\[
S_B = \frac{y + P + G_o}{2} - \frac{1 + P_o - G_o}{2} k
\]

\[
S_F = \frac{1 - P_o - G_o}{2} y - \frac{1 + P_o - G_o}{2} k
\]

(25)
with optimal evasion given by

\[ l = C'(x) + \frac{1 + p_o + g_o}{2} h'(\frac{x}{z}) \]  

(27)

in this case the second order conditions are satisfied if either \( C'' > 0 \) or \( h'' > 0 \).

Analysis shows that \( x \) can now be written as

\[ x = x(G_o, P_o, \gamma, z) \]

so that the effect of \( x \) of changes in \( G_o, P_o, \gamma \) is identical in sign to that obtained in the more general model for changes in \( \alpha, \beta \) and \( \gamma \) respectively.

The effect of \( z \) on \( x \) is now unambiguously positive as is the effect of changes in penalty rate \( P_o \) on tax collections \( T \).

7. Conclusion

The paper developed a simple model of the corrupt bureaucracy, within which the degree of tax evasion, the bribe payments to the bureaucrat and the level of detected evasion which the bureaucrat reveals to the government can be determined. It was also shown how the two special cases of the perfectly honest bureaucracy (IRS type) and the perfectly dishonest bureaucracy (degenerate type) can be obtained as limiting institutional cases of the more general institutional structure portrayed in the model.
Two important issues which do not arise in an honest bureaucracy and cannot therefore be analyzed in an IRS type model were addressed. The first one involves a scheme by which part of any increased revelation/exposure of evasion detected by the bureaucrat is allowed by the government to be retained by the bureaucracy. Conditions under which an effective scheme for eliminating corruption exists were derived. It was shown that a scheme for increasing tax payment through reduced evasion can in principle be found. The second issue involved the problem of rents and rent seeking that bribery gives rise to. The model was modified to take account of the possible effects of bureaucratic wages on the honesty of bureaucrats. It was then shown that (if an interior solutions exists), the wage rate which maximizes government tax collections net of the bureaucratic wage bill is identical to the rate which minimize total resource wastage under competitive rent seeking.

In an area in which data collection and empirical verification is inherently very difficult more than the usual emphasis was placed on the plausibility of the model as a representation of reality. Further under these circumstances the trade-off between the generality of the model and its simplicity is more heavily weighted in favor of the latter. Keeping this in mind the model was then simplified further. This version of the model will be used in subsequent papers to model separately issues specific to income tax evasion, commodity (excise and sales) tax evasion, and tariff evasion.
Footnotes


2/ Tariff evasion through smuggling is much more clearly perceived as an illegal activity by citizens of all countries, than is tax evasion. The assumption of asymmetry in honesty between the evader and the bureaucrat can be a reasonable one for smuggling (Bhagwati and Hanson (1973), and Bhagwati and Srinivasan (1973) and Sheikh (1974)). Activities like over invoicing of imports and underinvoicing of exports do not come under the ambit of smuggling in this context. They essentially aim at circumventing either (quantity) controls on purchase of foreign currencies, or the implicit tax on sale of foreign exchange (controlled rate). The comments in the text would still apply to these activities.

3/ The non-linear nature of most income tax schedules, would complicate the exposition and analysis by an order of magnitude. Similarly commodity taxes cannot be explicitly treated without considering the changes resulting in commodity markets from the existence of tax evasion. Both issues are best left to a subsequent paper.

4/ Both these types of solutions will be derived as special cases of the present model.
Loosely worded notices, having sufficient loopholes for subsequent modification such as a “show cause notice”, may however be used as a pressure tactic for extracting a bribe.

Alternatively we can assume that each bureaucrat deals with \( n = n(i) \) identical firms. In this case the \( G(.) \) function can be written as \( G(nw) \) and the bureaucrats share per firm is \( S_B = b + \frac{G(nw)}{n} \) where \( b \) is the bribe from one firm as before. This formulation yields the same results as the simpler formulation used in the text.

In fact quotas for exposure are often assigned to the different inspectors. This yields a special \( G(w) \) function which will be considered in a subsequent section.

The second order condition \(-P''(w) + G''(w) < 0\) is satisfied if either \( P'' \) or \( G'' \) is non-zero. For an interior solution we must have \( G''(0) > 1 + P''(0) \) and \( G''(y) < 1 + P''(y) \). These are satisfied for example if

\[
\lim_{w \to 0} G''(w) = \infty \quad \text{and} \quad \lim_{w \to y} G''(y) = 0.
\]

The first inequality follows from \( P(y) - P(k) \geq (y - k)(1 + P''(k)) = (y - k)G''(k) \geq G(y) - G(k) \).

The second inequality follows from equation (6). The third one follows from the concavity (non-convex) of \( G(.) \).
10/ As \( G'(w) \to -\infty \) and \( P'(w) \to \infty \), \( \frac{\partial H}{\partial w} = -1 - P'(w) + G'(w) > 0 \) for all \( w \).

Therefore the solution is \( w = y \).

11/ \( \frac{\partial H}{\partial w} = -1 - P'(w) < 0 \) in this case so that the solution is \( w = 0 \).

12/ The second order condition is \( -A = -\left[ c''(x) + \frac{1}{2} h''(P'(y) + G'(y)) \right. \]

\[ + \frac{1 + P'(y) + G'(y)}{zz} h'' \]

\[ \left. \right] < 0 \]

is assumed to be satisfied. Note that only \( \frac{h''G'}{z} \) is positive.
Appendix:

1. The Threat Point

Let \( L = S_B - S_F = b + G(w) - [y - w - P(w) - b] \)
\[ = G(w) - y + w + P(w) + 2b \]

The Threat point is obtained by solving the following problem

\[
\min \max L = w + P(w) + G(w) - y + zb
\]
\[
\begin{align*}
\frac{\partial L}{\partial w} &= 1 + P'(w) + G'(w) > 0 \quad \Rightarrow \quad w = y \\
\frac{\partial L}{\partial b} &= 2 > 0 \quad \Rightarrow \quad b = 0
\end{align*}
\]

2. The Optimal Bribe:

The solution of the sharing problem (Max U) is obtained by differentiating U with respect to b, where U is given by

\[ U = [b + G(k) - G(y)] [y + P(y) - k - P(k) - b] \text{ or} \]

\[ \frac{\partial U}{\partial b} = [y + P(y) - k - P(k) - b] - [b + G(k) - G(y)] = 0 \]

which gives the solution \( b = b^* \) in equation (7). Note that \( \frac{\partial^2 U}{\partial b^2} = -2 < 0 \) so that the 2nd order condition is satisfied.
Analysis of Equilibrium Conditions

Equation (6) can be rewritten as

\[ 1 + \beta P'(k) - \alpha G'(k) = 0 \]

Differentiating totally we have

\[ (\beta P'' - \alpha G'') \, dk + P \, d\beta - G \, d\alpha = 0 \]

\[ \frac{\partial k}{\partial \alpha} = \frac{G''}{\beta P'' - \alpha G''} > 0, \quad \frac{\partial k}{\partial \beta} = \frac{P''}{\alpha G'' - \beta P''} < 0 \]

Equation (10) can be rewritten as

\[ 1 - C'(x) - \frac{1 + \alpha G''(y) + \beta P''(y) \gamma h'(x/z)}{2} = 0 \]

Which on differentiating totally yields

\[- Adx - G' \frac{d\alpha}{2} - P' \frac{d\beta}{2} - \frac{1 + \alpha G'' + \beta P''}{2} \gamma h' d\gamma + Bdz = 0 \]

\[ A = C''(x) + \frac{1 + \alpha G'' + \beta P''}{2z} \gamma h''(x/z) + \frac{\alpha G''(y) + \beta P''(y)}{2} \gamma h > 0 \]

\[ B = \frac{1 + \alpha G'' + \beta P''}{2z^2} \gamma x h'' + \frac{\alpha G'' + \beta P''}{2z} \gamma h'^2 > 0 \]
\[ \frac{\partial x}{\partial a} = -\frac{G'}{2A} < 0; \quad \frac{\partial x}{\partial b} = \frac{-p'}{2A} < 0, \quad \frac{\partial x}{\partial y} - \frac{1 + 2G' + sp'}{2A} < 0 \]

\[ \frac{\partial x}{\partial z} = \frac{B}{A} > 0 \quad \text{as} \quad B > 0 \]

\[ \frac{\partial (x/z)}{\partial z} = 1 \cdot \frac{\partial x}{\partial z} - \frac{x}{z} \cdot \frac{1}{2} \cdot \left( \frac{B}{A} - \frac{x}{z} \right) = \frac{x}{2} \cdot \left( \frac{B}{x} - A \right) = -\frac{xc''}{2} < 0 \]

Note \[ A = c'' + B \cdot \frac{z}{x} > z \cdot \frac{B}{x} \quad \text{as} \quad c'' > 0 \]

i.e. \[ B > 0 \text{ insures } A > 0 \text{ but not vice versa.} \]

The only negative term in \( B \) and \( A \) is that arising from \( G'' \).

Note that \( B = \begin{cases} > 0 & \text{if } G'' = 0 \\ \geq 0 & \text{if } P'' = 0 \end{cases} \)

Further \[ G''(y) + P''(y) = \frac{a^2b}{3y^2} \]

4. The Optimal Bribe with Reward Scheme

Assuming exposure is at \( w = w^* \), we can rewrite \( U \) as

\[ \bar{U} = [S_B(w^*) - G(y) - N(y)] [S_F(w^*) + P(y)] \]

\[ \equiv [b + G(w^*) + N(w^*) - G(y) - N(y)] [y - w^* - P(w^*) + P(y) - b] \]

using equations (3^-) and (4^-)
\[
\frac{\partial U}{\partial b} = y - v^* + P(w^*) + P(y) - b - [b + G(w^*) + N(v^*) + G(y) - N(y)] = 0
\]

which on simplification gives

\[
\bar{b} = \frac{1}{2} [y + P(y) + G(y) + N(y)] - \frac{1}{2} [w^* + P(w^*) + G(w^*) + N(w^*)]
\]

Substituting \(N(y)\) and \(N(w^*)\) from (14) we obtain,

\[
\bar{b} = \frac{1}{2} [2(y + P(y)) - Q(k)] - \frac{1}{2} [2(w^* + P(w^*)) - Q(k)]
\]

\[
= y + P(y) - w^* - P(w^*)
\]

substituting in (2') to (4') we obtain

\[
\bar{S}_G = w^* + P(w^*) - N(w^*) = G(w^*) + k + P(k) - G(k)
\]

\[
\bar{S}_B = y + r(y) - w^* - P(w^*) + G(w^*) + N(w^*) = y + P(y) - [k + P(k) - G(k)]
\]

\[
\bar{S}_P = y - w^* - P(w^*) - b = - P(y)
\]

5. **Proof of Proposition 1** \((x < y^*)\)

Given the assumption that the reward scheme is feasible. The proof is by contradiction. Assume the contrary; that is let \(\bar{x} > x^*\). Using the second order conditions,
\[ 1 - C'(\bar{x}) - h'(\bar{z}) < 1 - C'(\bar{x}^*) - h'(\bar{z}^*) \]

which from (10) and (10') imply,

\[ -\frac{1 - P'(y^*) - G'(y^*)}{x} h'(\bar{z}) > P'(y) h'(\bar{z}) \quad \text{(Al)} \]

From the solution of \( y^* \) in the previous section, we must have (for \( y^* > k \)),

\[ 0 = -1 - P'(k) + G'(k) > 1 - P'(y^*) + G'(y^*) = -(1 + P'(y^*) - G'(y^*)) \]

or

\[ 0 > \frac{1 + P'(y^*) - G'(y^*)}{z} - 1 - \frac{P'(y^*) - G'(y^*)}{z} - P'(y^*) \]

or

\[ -\frac{1 - P'(y^*) - G'(y^*)}{z} h'(\bar{z}) < -P'(y^*) h'(\bar{z}) < P'(y) h'(\bar{z}) \]

as \( \bar{x} > x^* \), \( \bar{y} > y^* \). This is a contradiction of equation (Al), so that \( \bar{x} > x^* \) is not possible.

The second part of the proposition is proved in the text.

6. Proof of Proposition 2 \( \frac{\partial R}{\partial g} = -\frac{\partial t}{\partial g} \)

\[ \frac{\partial t}{\partial g} = -\frac{\partial x}{\partial g} + (1 + P(k)) \frac{\partial k}{\partial g} - 1 \]

\[ \frac{\partial R}{\partial g} = 1 + \frac{\partial b}{\partial g} = 1 + \frac{1 + P'(y) + G'(y)}{z} h'(\bar{z}) \frac{\partial x}{\partial g} - 1 + P'(k) + G'(k) \frac{\partial k}{\partial g} \]
where we have used optimality condition equations (10) and (6) respectively. Therefore

\[
\frac{\partial R}{\partial r} = - C'(x) \frac{\partial x}{\partial r} + \left[ 1 + \frac{\partial x}{\partial r} - (1 + P'(k)) \frac{\partial k}{\partial r} \right]
\]

\[
= - C'(x) \frac{\partial x}{\partial r} - \frac{\partial t}{\partial r}
\]

i.e. \[\frac{\partial RC}{\partial r} = \frac{\partial R}{\partial r} + C'(x) \frac{\partial x}{\partial r} = - \frac{\partial t}{\partial r}\]

QED
REFERENCES


