ECONOMIC SYSTEM MODELS FOR TRANSPORT SECTOR STUDIES

A critique of the Harvard Transport Program's multiregional economic model and a proposal for a more practical one.

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Appendix I: Data on the Colombia Economy

Appendix II: Derivation of Some Price Relations
ECONOMIC SYSTEM MODELS FOR TRANSPORT SECTOR STUDIES

A critique of the Harvard Transport Program's multi-regional economic model and a proposal for a more practical one.

I. Background and Conclusions

The purpose of the Harvard Transport Research Program was to develop a more comprehensive method than had theretofore been used for evaluating transportation-system development plans. Initially supported by the Agency for International Development and the Brookings Institution, it was later continued as a project for the Planning Department of the Colombian Government, which borrowed funds for the purpose from the World Bank. The final report\(^1\) was a study of transportation development plans for Colombia, using, among other methods, a computer simulation of the transport system together with a computer simulation of the economy on a multiregional basis.

The present paper concerns only the economic system model.\(^2\) Its formulation has been studied in detail, and attempts have been made to gather the empirical information that would be needed for a meaningful application of it to Colombia. On the basis of these efforts, conclusions are presented about (1) the


\(^2\) An evaluation of the transport system model will be forthcoming in a separate paper by Messrs. de Weille, Harral and Miller. Adaptations of submodels from the transport model are described in following Working Papers Nos. 62, 63, and 64.
theoretical formulation, (2) the empirical aspects of its application in
general and (3) the specific data problems of the Colombian application.
It is hoped that the discussion will bring out the interrelatedness of these
three aspects of the simulation, which are too often thought of as independent.

The simulation model, as presented in the report and as embodied in
the computer program, is in abstract (non-numerical) form, designed to be
applied to different countries by specifying different numerical data,
without making major changes to the basic theoretical structure or computer
program. Harvard's use of the model for the Colombian study, of course,
required specifying voluminous numerical inputs, but since the project did
not include a major empirical study, which would be required for quantifying
such a model, many of the numbers used were rough estimates or simply
arbitrary choices.

The two main purposes that the Harvard economic system model was in-
tended to serve in studies of transportation system development are stated
in the final report of the project as follows:

"For purposes of transport planning, there are at least
two specific requirements which the macroeconomic model must
fulfill. First, it must provide the transport model with
estimates of the demands which will be placed upon the trans-
port system and of the general economic conditions within which
the system will operate. Second, the macroeconomic model must
recognize and respond to changes in the operating characteristics
of the transport system."[1] /

The latter of those objectives is the more difficult to attain,
involving, as it does, the responses of multisectoral investment and location
decisions to changes in transportation system characteristics. Our first
conclusion is that the state of knowledge about such responses is presently
inadequate for a reliable model of the influences of transport-system changes

on the evolution of the economy of Colombia or probably of any other country. The major problem is not a matter of how the model is formulated; neither is it a problem that would be solved by more data of conventional sorts; it is a lack of fundamental knowledge about the real phenomena that are to be simulated. A corollary of this conclusion, unfortunately, is that within the existing limitations of empirical knowledge, there is no reliable way to compare the benefits of significantly different transport system development plans or to evaluate the benefits of most major projects in terms of all of their effects—direct and indirect—on the development of an economy. 1/

This conclusion, however, does not rule out fulfillment of the first objective in the quotation above. The second conclusion is that it would be feasible and worthwhile to use this type of model to project the growth of supplies and demands for various goods in different regions as a basis for projecting traffic, while using more conventional criteria to compare alternative transportation system plans. Such a model would be useful in order to keep the projections consistent in terms of income and expenditure, foreign accounts, and the supplies of and demands for different kinds of goods for both final and interindustry uses. The application would be feasible

1/ Although it is not feasible to project all of the repercussions of transport system changes on the rest of the economy, there are ways of approximating the effects in terms of partial analysis. The methods, and the conditions under which such partial analysis may be expected to yield a good approximation, are set forth in World Bank Staff Occasional Paper No. 13, The Economic Benefits of Road Transport Projects by Herman G. van der Tak and Anandarup Ray, "BRD, 1971. See also Clell G. Harral, Preparation and Appraisal of Transport Projects, U. S. Department of Transportation, 1968.
if an adequate collection of the right kinds of data were assembled. In most countries, including Colombia, this would require a large amount of preparatory statistical research.

The third and last major conclusion is that it would be inappropriate to use the Harvard economic system model (as presently constituted) for the purpose just described, partly because of theoretical flaws, and partly because of the confusing, error-encouraging, way that some parts of it are formulated. The final chapter of this report presents a preliminary formulation for a new model, which is intended to overcome the flaws of the old one and which (taking advantage of the less ambitious objective for which it is designed, as well as of the experience of reviewing the earlier one) is simpler and clearer for the user.

Neither the criticisms of the particular model as formulated by the Harvard group nor the conclusion that present knowledge of responses to transport-system changes is inadequate should be taken to imply that attempts to model these incompletely known effects are useless. As a research technique, experimenting with a model that includes hypothetical or conjectural elements sometimes helps to screen out inappropriate hypotheses and sharpen the definition of others, and gives useful guidance to the design of empirical research. Such model experimentation and further empirical studies may eventually add enough to the store of knowledge to permit making this sort of model useful as an operational tool, but the research must be done before that stage can be reached.
II. The Harvard Transport Program's Economic System Model

IIia. Sector Analysis and the Rest of the Economy

Analysis of a transportation system is a particular variety of sectoral analysis. Like any other sector, a transportation system operates within conditions and constraints some of which are determined by the rest of the economy. In turn, the rest of the economy is influenced by the performance of the transportation system. These interactions, of course, take place dynamically as both the transportation system and the rest of the economy grow and change through time.

The conditions that the rest of the economy imposes on the analysis and planning of a transportation system may be called the context for the sector study. They include the evolving geographical patterns and volumes of production and use of various kinds of goods, which help to define what will need to be transported between various points. They also include prices of fuels and other inputs used in different modes of transportation, possible bottlenecks in supplies of inputs, and the prices and availabilities of the capital goods that would go into any expansion or improvements to the system.

The effects of the transportation system itself on the overall economy have two kinds of significance. First, the pattern of transport costs and capacities may affect the geographical distributions and magnitudes of production and new investment in the other sectors, which, in turn would have a bearing on the
context variables in the future. Secondly, there will presumably be effects on the growth of GNP, price stability, income distribution, and so on, which ideally would provide measures of relative benefits of alternative transportation development plans.

The Harvard Transport Program's economic system model was intended to simulate each of these kinds of interaction. As a multi-regional model of a developing economy it was to provide consistent projections as the context for the transportation system simulation with which it was associated. It was also designed to respond to differences in transportation cost patterns and to yield comparative measures of benefits for alternative plans.

The model generates consistent projections of production and use of various categories of goods in various regions of the country. Imports and exports are also accounted for, with constraints on imports to avoid a balance-of-payments crisis. A further refinement is that the growth of production in various localities is not independent of transportation-system characteristics but is influenced by the pattern of transportation costs. Thus, a transport plan that lowers the costs of shipping goods out of a particular region tends to stimulate production and investment in that region. These effects not only change the geographical pattern of activities but even affect the rate of growth of gross domestic product as a whole.
Making the geographical patterns of production and capacity expansion respond to differences in transport costs introduced many complexities and a number of unverified, hypothetical relations that could otherwise have been left out of the model. It required the inclusion of decision functions that would determine the magnitudes and geographical distributions of production and capital formation for each sector, and the postulated formulations of these functions necessitated other functions for the endogenous determination of prices, costs, and profits for each activity in each geographical region. For some of the functions involved, even the qualitative forms are subject to serious uncertainty, and the relevant quantitative data are scarce or totally lacking in most countries. These difficulties raise doubts about the desirability of including these responses in the model. However, the model designers believed that the added complexity and the associated empirical problems were offset by major advantages, not merely for projecting the demands for transportation, but for evaluating the relative benefits from different plans.

If it had indeed been feasible to make the economic system model work as the Harvard group intended and yield meaningful comparative results, then a profound and long-standing problem would have been solved. It would have been possible to compare alternatives in terms of their full effects on the growth of national income, on its regional and wage-profit distribution, on price stability, external and internal solvency, and other measures of economic objectives. The measurement of benefits in these ultimate terms was the grand ideal toward which the Harvard model was aimed, but which, as will be shown, is too ambitious to be attainable.

The model did not have to stand or fall on its capability for measuring the increase in regional and national income, however. The transportation model
itself produced all the information necessary for computing more conventional criteria, such as savings in total transportation costs. Some such measure could be used for comparing different transport plans in a growth context generated by the economic system model. At the same time the transport model could give valuable information on the flows in various parts of the network, indicate where congestion and bottlenecks would occur, and so on. The economic system simulation would have made a significant contribution to the state of the art just by providing the consistent supply and demand projections needed for the functioning of the transport model.

\[1/\] The model did, in fact, compute a cost-saving criterion, but one that did not deal properly with "generated traffic." Nevertheless, a better criterion could have been formulated and included in the transport model. See papers by van der Tak and Ray, and by Harral, cited on page 3.
IIb. Structure of the Harvard Model

This section explains the main outlines and principles of the model. The reader who wants a more complete description is referred to the Brookings book\(^1\) or to the final report of the Harvard Transport Program\(^2\) or to a World Bank paper which is a corrected and somewhat expanded version of the pertinent chapter from the report.\(^3\)

The economic-system model is a dynamic simulation model, which progresses from one year to the next, changing its state on the basis of exogenous information, feedbacks from the transportation-system simulation, and internal dynamic relations.\(^4\) It describes the state of the economy at a given time in terms of a number of sectors, each distributed over a number of geographic regions. (In the Colombia application, ten sectors and ten regions were defined.) Each regional subdivision of each sector has, at a given time, a nominal capacity (based on past net capital accumulation), a level of production, a price for its product, various production costs, a profit rate, a rate of investment (gross and net) and other characteristics. There are taxes, tariffs, exports, imports, and import quotas. Much of this information has to be specified for the initial year from which the simulation is to proceed. After that, many of the variables in successive years are determined endogenously, partly from conditions of the year just past.

\(^1\) Kresge and Roberts, op. cit.

\(^2\) An Analysis of Investment Alternatives.

\(^3\) Regional Macroeconomic Model, IBRD Economics Department Working Paper No. 60, adapted by E.P. Holland and I. Isaac, 1970.

\(^4\) This type of simulation is fully explained, and differentiated from other procedures known as simulation, in Economics Department Working Paper No. 90, Simulating the Dynamics of Economic Development. The overall approach is recursive, moving through time, but each years cycle includes a matrix inversion and some iterative adjustments.
The steps in the cycle of computations, which is repeated for one year after another, are summarized in Table 1. In all except the initial year, the first step is to adjust previous values of unit labor costs, other costs, and prices for all products in each region on the basis of conditions determined in the previous year's calculations. Next come calculations of final demands for each sector's product. Inventory investments are proportional to outputs (the previous year's outputs, since those of the current year are not yet known). Gross fixed investment in each sector and region may be determined in any of several ways; the investment function includes terms, which may be used alone or in combination, proportional to cash flow, to the utilization of capacity, and to the current level of output, in addition to the options of a specified trend or a purely exogenous time series. For each sector, regional components are summed to get nationwide sectoral investment, which is then multiplied by a vector of coefficients (different vectors for different investing sectors, forming the "B" matrix) to generate investment demands for different products. Consumption (in each region) is based on the previous year's income, with an adjustment for effects of inflation, and allocated among the products of different sectors by a set of fixed coefficients. Exports and government demands are exogenous.

The total final demands for the products of all sectors, having now been established, are used with a matrix of input-output coefficients to determine intermediate-goods demands and total production of each sector on a nationwide basis. (Table 1, Step 6) Associated with each matrix or vector of domestic demand coefficients referred to thus far is a corresponding set of coefficients for imports - API (I) for consumers' imports, AIMP (I,J) for intermediate goods imports, and BIMP (I,J) for investment goods imports. Applying these
1. Compute new wages, unit costs, and prices by sector and region.

2. Inventory increase proportional to output.

3. Gross Fixed Investment by sector and region:
   - optional combination of cash flow, capacity adjustment, ratio to output, trend, exogenous.

4. Consumption:
   - by sector and region; function of last year's income, inflation, and fixed APC's.


6. Intermediate goods:
   - Invert (I-A) matrix.
   - (National aggregates)

7. Imports (National and Regional)
   - Consumption: API's
   - Intermediate: AIMP's
   - Capital: BIMP's

8. Adjust for Import Restrictions
   - Shift from AIMP to A coefficient
   - " " BIMP to B coefficient
   - " " CIMP to C (flow)
   - Then: reduce investment.
   - Shift some more consumption.
   - Return to Step 6 and reiterate until restrictions are satisfied.

9. Compute "depreciation" and new level of nominal capacity by sector and region.

10. Allocate production to regions on basis of:
    - a. past shares of output.
    - b. shares of capacity.
    - c. shares of profits.

11. Reallocate production if, in any sector-region combination, it exceeds point of diminishing profits (PRDMX).

12. Determine intermediate demands by region and total demand and supply by region, for each product. Recompute wages and costs.

13. Pass Demands, Supplies, and other economic information to Transportation Model and stand by for feedback from that model.

14. First time around each year:
    - When information comes back on transportation costs, use it to revise input-output coefficients, etc., and repeat entire program from step 6 through step 13.

15. On second time around (each year) adjust flow discrepancies, and calculate transport cost per unit, taxes, profits, national aggregates of regional variables etc.

16. This completes the calculations for one year. Start again from Step 1 for the next year.
respectively to total consumption, production of each sector, and investment in each sector yields the flows of several categories of imported goods. (Step 7) Total imports thus determined must be checked against a "quota" or constraint. If they exceed the limit, a complicated series of adjustments is called into play, involving increases in domestic input-output and capital coefficients to offset reductions in their import counterparts, shifting consumer demand away from imports to equivalent domestic products, and, if these measures are not sufficient, reducing the level of investment to reduce the demand for capital goods and further restricting consumers' imports, diverting that expenditure to domestic consumption (Step 6). These adjustments require repetition of the input-output analysis and subsequent import calculations (Steps 6 and 7) and several iterations may be required.

Since the input-output and import analysis deal with nationwide totals of each variable concerned, it is necessary, after they are completed, to split up some of these quantities into regional components. First (Step 9), regional production capacities of each sector are computed from the previous year's capacities, reduced by depreciation and augmented by gross investments (which were determined regionally in Step 3 before being summed for the national totals). The capacities are nominal, rather than rigid constraints, and may be exceeded at the penalty of increasing production costs.

Each sector's production is distributed regionally (Step 10) according to a weighted combination of shares of output in the previous year, shares in sectoral capacity, and relative profits. A limit is imposed to avoid allocating so much production to one region that the rising cost for over-capacity operation begins to diminish profits (Step 11). Intermediate demands are then found for each region by applying the input-output coefficients
to production of each sector in that region (Step 12). Information is now complete on the production of, and demand for, each good in each region, as well as on import requirements and exports, and this information is transferred to the Transportation Model (Step 13).

The Transportation Model determines the pattern of flows of each category of good from every region to other regions, including the choices of routes and modes. It also determines the costs associated with transporting goods to the points at which they are to be used. When this information is returned to the economic-system model, the transport sector coefficients of the input-output table are adjusted accordingly, and most of the calculations already done (omitting import substitution) are repeated (Step 14). Then, after a series of adjustments to balance the flows of goods supplied and goods demanded, calculations are made to determine transport costs for various items, taxes, profits, national income concepts, and some other variables either for output information or for use in determining prices, etc., in the next cycle of calculations (Step 15).

This completes the cycle of calculations for one year. The process is then repeated for the next year, and so on for as many years as the projection is to run. For each alternative development plan and each set of alternative assumptions, a new projection is made.
III Basic Problems in Modeling Economic System Behavior

IIIa. Empirical Knowledge and the Validity of a Model

The first conclusion, stated at the beginning of this review, indicated that some of the objectives of the Harvard model could not be attained because of insufficient empirical knowledge about basic relationships. These deficiencies in knowledge will be examined more fully in this chapter.

Empirical knowledge of economic relations may be thought of in terms of two components -- the qualitative and the quantitative. Qualitative empirical knowledge consists in knowing what independent variables are significant in determining the behavior of another variable and in knowing how each exerts its influence: Does it have a linear effect, independent of the other variables? Does it act as a constraint, being sometimes dominant, other times out of action? Do two or more independent variables reinforce each other's effects? etc. Such knowledge -- or hypotheses and conjectures to take its place if it is absent -- is the basis for formulation of an abstract model.

An abstract model is a set of relations expressed in terms of mathematical symbols whose numerical values are not specified. An abstract model cannot be simulated on a computer and cannot be used for planning or comparison of alternatives until it has been transformed into a specific model (of a real or fictitious economy) by assigning numerical values to all of its parameters and some of its variables.

If the numerical inputs are to be based on empirical knowledge, data must be obtained and coefficients must be estimated. Both the determination of what data are needed and the process of estimating coefficients depend on the relations constituting the abstract model. However, it is likely that some of the relations are conjectural and need to be empirically verified or altered. Ideally,
the quantification of an abstract model and the verification or change of any of its abstract equations that might be in doubt would be simultaneously carried out in an empirical study of the economy to be modeled. In practice, unfortunately, this empirical side of model building seldom gets the emphasis it deserves, and abstract relations based on a combination of theory and conjecture are often quantified by means of "reasonable guesses," resulting in a model whose relation to reality is hard to ascertain.

It is not intended here to deny that experimentation with a partly conjectural model and "guesstimated" parameters can be useful. Indeed, it can be very fruitful for certain purposes provided it is not misunderstood. Sometimes there is a priori knowledge of the qualitative nature of some relationship but insufficient quantitative data to estimate the parameters. Then it may be possible to get some useful results from sensitivity testing, using different assumed values and determining how sensitive the results are to such differences. Such tests would indicate how worthwhile it might be to do further basic research to get some of the missing data. In addition, they may show which way to alter policies to overcome instability,1/ or in other ways may be useful for learning more about the system in question. However, the aim of the sort of model under discussion in this review is not to study the economic system but to evaluate alternatives in the transportation system. For that purpose, more reliable knowledge of the reactions of the economic system is essential.

The hypothetical nature of some abstract relations in the Harvard economic system model and the weakness of its quantitative foundations, in this reviewer's opinion, invalidate this model as a means of evaluating transport system plans.

1/ as in Samuelson's well-known exploration of the multiplier-accelerator model (Rev. Econ. & Stat. 1939) or Phillips's analog experiments (Econ. Journal 1957).
The designers of the model are not to be blamed if economic research has not yet defined all the relationships involved in the dynamic interactions between a transport system and a geographically distributed economy. Nor should they be chastized for using a combination of common sense and conjecture to fill some gaps in their extremely ambitious design, with the hope that they could later check their hypotheses or improve upon them. It is not a sin to aim for great objectives and find them only partially attainable. Their contribution would have been more valuable, however, had they frankly discussed the fundamental difficulties that they encountered instead of merely finding fault with the Colombian data and presenting the model as if it were something more useful than it really is.

Unfortunately, the report gives the impression that the model actually simulated the macroeconomic dynamics of Colombia sufficiently well to provide planning guidance.\(^1\) That impression is not warranted. When it comes to discussing the quantitative matching of the model to the Colombian economy, significant issues are simply ignored while it is pointed out that the rates of growth of GDP in constant and current prices were closely reproduced, as well as the variations in the savings rate (actually the investment-GDP ratio).\(^2\) No other calibration results are given. The reliability of the model for comparisons

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\(^1\) The first paragraph of the report says:
\[ "The Colombian transport plan, as reported in this document, is the product of a rather experimental approach to transport planning. Specifically, a very large scale planning model (Macro-Economic Transport Simulator or METS, for short) was developed which simulates the physical and economic processes of the transport system and how the transport system interacts and conditions the processes of economic growth in the general economy."
\]

(An Analysis of Investment Alternatives ..., op. cit., page 1.) Other statements throughout the report tend to bolster that impression.

\(^2\) ibid., pp. 181 - 183

\(^3\) Simulated patterns of production and growth by region and sector are also presented in the "Calibration" section, but there are no actual data with which to compare them.
of alternatives is not proven by these results. They merely demonstrate that it was possible to adjust various coefficients so that the model would reproduce the historical paths of three nationally and sectorally aggregated variables during a period of relatively steady growth.

A much more significant criterion of a model's validity for planning or policy evaluation is suggested by the statement just before the presentation of those results,

"The model must ... be calibrated so that major variables are reproduced quite accurately and, more importantly, so that growth rates, behavior patterns, and response characteristics are reasonable representations of real world mechanisms." 1/

But nothing further is said about behavior patterns or response characteristics, which, in fact, proved impossible to calibrate in the Colombian experiment. Behavior patterns and response characteristics are the keys to the capability of an economic system model to reflect the effects of transportation system changes. If their formulation and calibration in the model are to yield "reasonable representations of real world mechanisms" then it is important, first of all, to have empirical knowledge about those mechanisms.

Apart from the particular formulation arrived at by the Harvard group, and apart from their specific attempt to quantify it for use in Colombia, a larger question is the degree to which existing empirical knowledge is adequate for specifying any model of economic-system responses to transport-system changes. Below, there is a theoretical discussion of the variables likely to be involved in bringing about and shaping these responses. Then the adequacy of current knowledge about relations among these variables will be considered.

1/ ibid., p. 181, emphasis added
IIIb. Theory of Economic System Response

The primary direct impact of transport system characteristics on the rest of the economy is conveyed in terms of the unit costs (including imputed costs of delay, spoilage, inconvenience, etc., as well as actual charges) incurred in shipping various kinds of goods between various origins and destinations. The economic effects that are of concern for planning and policy purposes will generally include the projected evolution of GDP or national income, and may include its distribution among regions and socio-economic classes, as well as the projected paths of unemployment, prices, foreign payments, and the government's budgetary flows.

Few, if any, of these criteria are altered simply and directly by changes in unit transport costs. Generally, their responses are repercussions of intermediate effects. The principal direct effects of a change in the structure of transport costs (costs to the user of transport services) are:

1. Changes in the level of demand at any location for goods whose delivered prices change.
2. Changes in the relative quantities of goods supplied from different sources.
3. Changes in profitability of certain production activities in different localities.

The first of these effects combines adjustments in consumers' budgets, changes in derived demands for intermediate goods, adjustments in investment levels, and substitution between domestic goods and imports, all as responses to differences in delivered prices. These demand adjustments will alter the total quantities of various kinds of goods produced in the country as a whole (perhaps
with some lag, resulting in inventory changes).

The second kind of effect listed above simply represents the shift of orders for a product by some users to a different source that has become relatively cheaper for them because of differential changes in shipping costs.

The third effect—changes in profitability—follows from the other two, being the result of changes in delivered prices of the inputs to production and changes in demand for the outputs.

These primary reactions to changes in the transport cost pattern have further repercussions that lead to effects on the criterion variables in which we are ultimately interested.

The changes in the total demand for some goods (1) and in relative quantities supplied from different sources (2), affect the utilization of capacity in each sector and region.

The changes in capacity utilization and changes in profitability (3) both have effects on investment and location decisions in each sector.

Changes in investment decisions affect future investment disbursements, distributed through time, and the corresponding demand for various capital goods.

All of the effects mentioned tend to become cumulative, through processes of the income-expenditure-multiplier-accelerator nature, so that a single change in shipping costs affects the whole future path of national income, and cannot be evaluated in terms of a comparative static equilibrium solution.

It is important to recognize that the variables usually thought of as ultimate criteria for evaluating policies or plans—like changes in the growth rates of income and employment and in the rate of inflation—are affected indirectly through complex processes that involve behavioral decisions and time lags.
The use of such criteria depends on the ability to develop a dynamic model that adequately simulates the response mechanisms involved.

To reproduce the effects described in general terms above, a model would have to include, among other things, mechanisms corresponding to the following relations:

- the responses of delivered (and hence producers') prices to changes in shipping costs.

- the responses of consumers to changes in price patterns, in their disposable incomes, and in the general inflation rate.

- the time distributions and quantities of inputs to the construction of investment projects in each sector, from the time of decision until the availability of full output from the newly created capacity.

- the effects of capacity utilization, profitability, and perhaps other factors on the decisions of entrepreneurs in each sector as to (a) how much to invest, and (b) the geographical distribution of investment projects.
The State of Existing Knowledge

Creation of a model that adequately reproduces the response mechanisms described above would require reasonably correct specification of each relation in abstract terms on the basis of qualitative knowledge, plus quantitative estimation of all the coefficients and other numerical quantities. The degree of difficulty of carrying out these steps would differ from country to country for each relation. Some generalizations however, can be stated with confidence that they will apply in most cases.

It is important to recognize that the specification of any abstract relations, such as might make up a price function, consumption function, or investment function, entails more than just making a list of explanatory variables for each dependent variable. Just as important is the form of algebraic or differential or integral equation or inequality that describes how the interaction takes place. The time lag structure is also important, especially when feedback adjustment mechanisms are involved. For some functions there is considerable evidence on the general forms of abstract relations; for others very little.

The dynamics of price changes have received relatively little attention from the builders of models intended to explain real-world observations. This is a somewhat surprising fact, inasmuch as the statistical offices of a number of countries have remarkably detailed price data for different goods in different places, often on a monthly basis, from which it might be possible to infer some sort of dynamic function. Whatever may be the reason, there is no generally accepted, empirically tested, theory of price response that could be "plugged in" to an abstract model and calibrated from available data. Moreover, data on explanatory variables is harder to find than that on
prices. Most likely, in any country for which a model is to be made, there will be a need for collecting data on potentially explanatory variables and then testing alternative price functions until observed dynamic behavior can be matched.

Consumption patterns have been the subject of a great deal of analysis as well as data collection in a good many countries, so that it is possible to specify the abstract equations with some confidence and have a reasonable chance of finding sample survey data from which to quantify them. It would probably even be safe to draw inferences from data on different countries to make up for any deficiencies in consumers' budget data in the country being modeled. The one point that still is probably weak is the dynamic response to a deviation from the accustomed rate of inflation, but that should not be important for regional and transportation studies.

Studies of investment inputs and of the timing of construction processes have been made in a few countries in microeconomic terms for selected industries. It is unlikely that directly usable data on this subject will be found in any chosen country other than those where specific studies have been made. However, the qualitative forms of equations should present no problem, and numerical coefficients for a given industry should be roughly transferable from country to country. Even "educated guesses" for the distribution of gestation lags would be better than the traditional assumption of a one-year step delay for all sectors and industries. It is hard to foresee how much the simulation results would be affected by the errors likely to remain in such guesses, and the question should be pursued in a series of sensitivity tests with the completed model to ascertain whether more research is needed.

Of the response mechanisms involved in translating transport cost changes into effects on the ultimate criteria of economic policy, the least well identified and perhaps most important is the behavior of investors.
Even without the locational decision to complicate matters, the art of forecasting investment behavior has not yet become reliable, and with the choice of location as an additional facet, there is still much to be discovered. It is not yet possible to be sure what all the relevant independent variables are, nor in what form the impacts of some variables on the decisions should be expressed if they are to simulate reality. It seems very unlikely, with knowledge in its present state, that any representation of decisions about the magnitude and location of investment can be sufficiently valid to be relied upon for generating the criteria with which to evaluate transportation projects or programs.

* A number of ingenious attempts have been made to model effects of transportation on regional investment and growth patterns by econometric methods. For example, Amano and Fujita, in "A Long Run Economic Effect Analysis of Alternative Transportation Facility Plans" (Journal of Regional Science, 1970) have formulated a regional model of Japan with which to study the effects of building bridges between certain major islands. Interesting though this model may be on first reading, it includes a number of very dubious functional specifications, the worst of which is probably that of the regional-sectoral investment function. Investment patterns are determined by lagged market shares and income shares. They do not respond to changes in transport costs until a year after production patterns have responded, and investment is not affected by capacity utilization. Profitability is a factor only in a rather indirect way and with time lag, through the effect of transport cost reductions on production patterns.
For studies of regional economic development, it may be necessary to make conjectural models of the functions for sensitivity testing, as well as to do extensive field research to learn more about them. For studying a transportation system, however, it would be better to settle for a less idealistic and more reliable basis of evaluation.

Several alternatives are open, less ambitious but still offering improvement over present methods of evaluation. One is to assume that all investments are based on the current rate of return in each sector and region, with due allowance for replacement and new capacity already under construction. The evolution of the economic system under this assumption might be taken as a potential which, even if not realized in actuality, would nevertheless measure the relative merits of different transport programs. With this approach, however, there would be a danger that the results would be misinterpreted as realistic forecasts rather than hypothetical possibilities.

Another alternative is to use an economic system model only to provide the frame of reference for transport studies but not to evaluate the effects of the alternative projects or programs. The economic model then would be what is known as a consistency model rather than a forecasting model. Its function would be to provide a consistent set of projections within which to study transport alternatives, but not to reflect the effects of those alternatives on economic criteria. More conventional criteria would be used for comparing transport plans, such as cost savings, or consumers' plus producers' surpluses. This is the alternative recommended on the basis of this study.

IIId. Field Research Required

To implement a model that included realistic feedback affects through investment and location decisions would, it appears, require a large and time-consuming program of research into the bases of those decisions and their response to various factors, especially changes in the pattern of transport
costs. Even with a rather large research program, success could not be assured. With the less ambitious approach of using a consistency model some of the uncertainty could be eliminated, but it should be recognized that a large amount of work would probably still be required in most countries to compile data that has already been gathered by various agencies but not published, to gather new data to supplement what is available, and to analyze whatever data can be obtained. This part of the operation is most often underestimated in model-building projects.

In the attempt to apply their model to Colombia the Harvard research group evidently underestimated the time and resources needed. Although the fact is not made clear in the book, in their final report, or in the report on data collection, the quantification of the Harvard economic system model to represent Colombia was far from successful. Many of the coefficients they used when running the simulation were "reasonable guesses" or purely arbitrary values. The empirical work that was actually done for the Colombia study was quite a bit less than sufficient, and less than could have been done with a little more time and manpower.

To see if, with a reasonable effort, enough more could be

1/ Kresge and Roberts, op. cit.
2/ Analysis of Investment Alternatives in the Colombian Transport System, op. cit.
done to make the model actually useful for the Colombian Planning Department, a group from the World Bank visited Colombia in 1969 and consulted with experts and officials in various agencies and collected and studied data that were obtainable. The conclusion reached from that visit and follow-up communications was that, given sufficient effort, a set of data could be compiled that would be just barely adequate for a consistency model -- a model that would provide consistent projections of economic system variables as the context for the transport system simulation but that would not attempt to represent the behavioral and decision-making elements involved in the responses of the economy to changes in transport costs. Even this limited objective would require filling in some gaps in the available data, either by new field research or, perhaps, from data on equivalent concepts in other countries.

The abstract model formulated by the Harvard group could be converted into a consistency model for use as described above, but, as will be seen in the course of the next chapter, that model has many flaws that would have to be corrected or improved upon. In the final chapter the outlines of a new formulation are presented, which it is believed would be more practical to use.
IV. Flaws in the Harvard Formulation

IVa. Summary

In this chapter the abstract formulation of the Harvard economic system model will be examined and criticized in some detail, both on a theoretical plane and in terms of mechanical matters like awkwardness and mistakes. Some readers may find the details of this particular model irrelevant to their interests; if so, they are invited to skip this chapter and go to Chapter V.

It was pointed out earlier that an abstract model is formulated on the basis of qualitative empirical knowledge, to the extent that such knowledge is available, combined with hypothetical relations based on theory or judgment. But in formulating this model, the designers did not always make full use of existing qualitative knowledge. Some functions had to be based on conjecture or reason, in the total absence of factual information, and their validity may be purely a matter of opinion. Some others, however, could clearly have been made more realistic on the basis of knowledge that is generally accessible. Examples are various relations involving dynamics (discussed in IVb), some elements of the investment function (IVc), and elasticities of consumers' demand (IVd).

In addition the formulation includes conceptual inconsistencies and what appear to be mistakes in keeping track of accounting definitions. Many of these errors can be attributed to the confusing formulation of price concepts and of relations between the values of goods at different stages of distribution. Because of the confusing definitions, similar errors are likely to be made by anyone attempting to make improvements or correct other errors in the model or trying to adapt it to a different application. Ensuing sections of
this chapter give specific details underlying these general statements.

IVb. Dynamics

The Harvard multiregional economic-system model embodies a mixture of modeling approaches, and that mixture is the source of a number of problems. Some major functions in the model are formulated as if they were intended to respond dynamically to endogenously-varying economic conditions. In other respects, however, the procedure is not a dynamic simulation but a series of static equilibrium solutions. Relations that are apparently intended to simulate behavioral responses to endogenous information ("feedbacks") are formulated with an unfortunate disregard for the effects of time lags on their dynamic characteristics.

The choice of one year as the time interval between calculations severely limited the possibility of representing dynamic feedback-determined responses. It made it impossible to represent lags shorter than a year or to distribute the impact of a one-year-lagged variable. To make matters worse, it seems that in the consumption and investment functions the feedback variables were lagged by a year for computational convenience without regard either to realism or to stability. The introduction of a large discrete lag in a feedback signal is one of the surest ways to create dynamic instability in any feedback adjustment mechanism. While a one-period lag might not make trouble in a simulation that operated in steps of a month or a week, a whole year's lag is another matter. Considering also that investors were assumed to ignore the prospective completion of new capacity under construction, it is almost certain that unstable oscillations of investment could only be avoided by making the

1/ The idea of using alternative time intervals is mentioned in the report, but the formulation (and computer programming) did not leave this option open.
feedback terms in the equation very weak relative to the other terms.

Another strange treatment of a dynamic process, which also undoubtedly contributes to instability, concerns the gestation period for capital formation. In actuality, for a given capacity-creation project, investment is a time-distributed flow of inputs to the construction process during the whole gestation period. In this model, however, the entire investment in a project takes place in one step, and the corresponding increase in capacity occurs one gestation period later. If the gestation period is specified as two or more years, an investment project entails no construction or other inputs in the year or years after the first; there is merely a waiting period before the new capacity becomes available.

These odd formulations of time relations might not be so important if used for conventional projections of steady growth without feedback adjustment mechanisms. They do, however, make a big difference in a simulation where changes in transportation are supposed to influence decisions about the magnitudes of production and investment in different locations, so that different transport programs yield different growth rates and patterns. The peculiar time relations do not just bias or delay the impacts of the factors to which the functions are responding. They make the responses destabilizing, so that the Harvard group found it necessary to weaken the feedback effects and make supposedly endogenous variables largely exogenous. That, of course, defeats the intended functioning of the economic model as a criterion for transport system alternatives.
IVc. The Investment Function

The Final Report of the Harvard Transport Project introduces the investment function as follows: "The investment function . . . contains five separate factors which may play a role in determining the amount of gross fixed investment."

An examination of the equation, however, shows that two of the five "factors," are merely alternative ways of specifying investment exogenously, while a third is simply a term proportional to output of the sector in the region in question. Two terms are left to express the dynamics of a decision-making process, and they do it poorly.

As given in Chapter 3 of the report (but omitting the sectoral and regional subscripts for clarity), the equation for investment in any particular sector in a given region is:

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1/ An Analysis of Investment Alternatives ... op. cit., p. 62.

2/ The report contains two descriptions of the model, in Chapter 3 and Appendix I; there is considerable duplication (and occasional disagreement) between them. The former is a more basic general explanation, omitting some complicating details, while the latter is supposed to be a complete specification. For some parts of the model, neither explanation is complete, and both must be read for a better understanding. The presentation in this section is primarily based on Chapter 3, where most aspects of the investment function are more fully explained, but as will be seen, part of the story has to be found in the Appendix and part of it even in the computer program itself.
RINVST\textsubscript{t} = ACCEL \cdot (\frac{1}{CME}) \cdot (ROUTPT - RPCAP)_{t-1} \\
+ \text{CONST} \cdot (\text{RETAIN} + \text{DEPCST})_{t-1} \\
+ \text{EXOG} \cdot \text{ROUTPT}_{t-1} \\
+ \text{WTINVS} \cdot \text{RINVST}_{t-1} + \text{XINVST} \quad \text{(Eq. 3.4)}

subject to the constraint, RINVST\textsubscript{t} \geq 0,

where: RINVST is gross fixed real investment,
ROUTPT is gross real output,
RPCAP is nominal capacity,
RETAIN is retained net profits,
DEPCST is depreciation allowance,
XINVST is a purely exogenous component of investment,
WTINVS is an exogenously specified rate of growth of investment,
CME is the capacity-capital ratio

and ACCEL, CONST, and EXOG are parameters.

The first term is meant to represent a tendency of firms to adjust capacity to correspond to output. But it is hard to believe that entrepreneurs base current investment decisions on the previous year's capacity utilization rather than on current or expected future conditions. In fact, rational entrepreneurs, recognizing that their decisions will not affect capacity until a full gestation period for construction of new facilities has elapsed, should project demand and the decay of old capital at least a gestation period into the future and should take into account the completion, in the meantime, of facilities that are currently under construction. Entrepreneurs, of course, may not be fully informed or perfectly rational, but it seems unlikely that their misinformation or
irrationality would take the form of totally overlooking replacement requirements, projects in gestation, and expected growth of demand.

The second term in the equation given above (3.4) is actually deflated by an appropriate price index to make it harmonize with the real-terms concepts of the rest of the equation. As explained in Appendix I,

"This term is not designed to represent the total funds available to the investors, nor is it in any way used as a constraint on investment. Rather, it is designed to represent a greater propensity to invest among those industries (and regions) with higher profitability." 2/

It was quite right not to apply the term as a financial constraint on each sector and region or even on an economy-wide basis. Investable funds are easily transferred from place to place and from one sector to another, and credit can be used to supplement plowed-back earnings. But if the authors wanted a term to represent profitability, one wonders why they included depreciation and why they excluded profits that are distributed. Net profits are computed elsewhere in the model and could easily be reduced to rates of return for a proper measure of profitability.

The investment function, in determining the relative rates of growth in different sectors and regions as well as the overall rate of growth of the economy, is one of the most significant mechanisms through which differences in transportation costs (if translated into differences in charges) effect differences in the rate and pattern of development. In light of that important role, it is surprising that the function was not formulated on the basis

2/ Relations in Chapter 3 are simplified by omitting prices. Corresponding equations in the Appendix include price effects, but in this case (Equation 100) the deflation is done incorrectly. The computer program turns out to be correct, and that is what counts.

of more careful theoretical analysis.

Of course, as pointed out in the preceding chapter, there would be essentially no possibility of verifying the abstract relation or of quantifying any sector-and-location-specific investment function unless a vast, long-term research program were focused on that objective. Thus, rather than try to improve on the theoretical formulation it seems better -- for applied planning models, at least -- to abandon the effort to include this sort of feedback.

IVd. Consumers' Budgets

Although incomes are kept in two categories -- wage and non-wage income -- to permit ascribing different savings propensities and consumption patterns to the two groups, no effort appears to have been made to account for effects of changes in either group's per-capita income or in relative prices. For each group (and in each region) the budget is calculated in two steps. First, the group's overall consumption expenditures are computed from the following formula:

\[ C_{\text{EXP}} = (\alpha + A_{\text{INFLT}} (Y_{\text{INFLT}} - 1)) \cdot Y_{t-1} \]

where \( C_{\text{EXP}} \) is consumption expenditure,

\( Y_{t-1} \) is disposable income earned in the previous year,

\( Y_{\text{INFLT}} \) is the ratio between the GDP deflator for the current year and that for the previous year, and

\( \alpha \) and \( A_{\text{INFLT}} \) are parameters.

Considering first the operation of the formula in the absence of inflation, \( Y_{\text{INFLT}} \) would be equal to 1.0 and the equation would simplify to:

\[ C_{\text{EXP}} = \alpha \cdot Y_{t-1} \]

\(^{1/} \text{Analysis of Investment Alternatives ... op. cit. Equation 3.1 in Chapter 3 or Equation 8 in Appendix I. The regional subscript has been omitted here; both } Y \text{ and } C_{\text{EXP}} \text{ are region-specific and the equation is applied in each region.} \]
Thus ALPHA looks like the average propensity to consume; in fact that is what the report says it is when prices are constant. However, the one-year lag between $Y_{t-1}$ and CEXP has a significant effect on savings if income is growing. If all terms were simultaneous, household savings (SAV) would be

$$S_{AVt} = (1 - ALPHA) \cdot Y_t$$

However, the effect of the lag is to increase this by almost the full amount of the year's growth in income, $\Delta Y$:

$$S_{AVt} = Y_t - CEXP_t = Y_t - ALPHA \cdot Y_{t-1}$$

$$= (1 - ALPHA) \cdot Y_t + ALPHA \cdot \Delta Y$$

In terms of a ratio to disposable income the corresponding relation is:

$$\left(\frac{SAV}{Y_t}\right) = \left(1 - ALPHA\right) + \left(\frac{ALPHA \cdot R}{1 + R}\right)$$

where R is the proportionate rate of growth of Y. Thus the effect of the lag is to augment the static savings ratio, $(1 - ALPHA)$, by a term, $(\frac{ALPHA \cdot R}{1 + R})$ which may well be of the same order of magnitude. For example, if ALPHA = .95 and $R = .06$

then $(1 - ALPHA) = .05$

and $(SAV/Y) = .104$, an increase of .054 over the value it would have if R were zero.

Working against this effect of real growth is the effect of inflation, reducing savings by an amount proportional to the square of the inflation rate, "... to reflect what appeared empirically to be a nonlinear response to very

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1/ ALPHA being, normally, almost 1.0.

2/ Dividing the equation above by $Y_t$, $(\frac{SAV}{Y_t}) = (1 - ALPHA) + ALPHA \cdot (\frac{\Delta Y}{Y_t})$; since $\Delta Y = R \cdot Y_{t-1}$ and $Y_t = (1 + R) Y_{t-1}$, the last term becomes $\frac{ALPHA \cdot R}{1 + R}$, leading to the result shown.
high rates of inflation.\textsuperscript{1} No clue is offered to the empirical basis of this relation.

Once the total expenditure of each group in each region has been established it is allocated over the various goods in fixed proportions, regardless of prices or the level of per-capita income. In other words, both price and income elasticity for every category are unitary. Thus, there is no opportunity for increased per-capita income to change the ratio between subsistence-type expenditures and those that are more income-elastic, a shift that is well established as part of our empirical knowledge\textsuperscript{2} and that is sometimes considered very important in the development process. Neither can a reduction in the price of food, for example, release any income for spending on other goods. The Harvard report actually states that this fixed proportions allocation is unsatisfactory and suggests incorporating non-unitary elasticity where possible,\textsuperscript{3} but the unitary-elasticity assumptions are built in to the model and the computer program and cannot be changed without extensive reprogramming.

\textsuperscript{1} ibid. p. 60.

IVe. Price Definitions and Accounting

Perhaps nothing has been more confusing for those trying to understand and use the economic model than the definition of price indexes for delivered products (PCIF) independently of the indexes for the same products at the factory, farm, or other point of production (P). By defining both indexes to equal 1.0 for any product in the base year, the formulators of the model made it extremely awkward to account for the split-up of end-users' expenditures between the factory cost of the goods, their transportation, distributors' margins (actually not accounted for explicitly), and sales or excise taxes. That made it difficult to trace the effects on prices due to changes in transportation costs, to understand some of the equations in the model, and to check the model for consistency.

It might be supposed that these difficulties would only be encountered by people who did not make enough of an effort to understand the model. However, it appears that even the authors of the model were confused by their own definitions. A careful re-derivation of some of the equations \(^1\) has revealed that a number of them are wrong, presumably because of that confusion. For examples:

The price of a product including transportation to the user's location is incorrectly related to its price at the factory in Equation 117 \(^2\).

A parallel formula for imported goods is similarly wrong in Equation 118.

Another formula supposedly specifying how the tariff-paid price of imports is affected by changes in the pre-tariff price and in the tariff rate is incorrect in Equation 116.

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\(^1\) Presented in detail in Appendix II. of this report.

\(^2\) Equation numbers are those of the Harvard report's Appendix I.
The value of imports, for balance-of-trade accounting, is incorrectly priced in Equation 90.

Thus, the delivered prices of all goods are incorrectly specified with respect to changes in transport costs, the prices of imports are misspecified, both at the port and at the point of use, and the value of imports, which imposes a basic constraint on the system, is wrong. There are other errors, too, but these should be enough to demonstrate that the approach used in this formulation was tricky and confusing, even to the formulators of the model. Many important errors might have been avoided by choosing concepts more suitable for relating "real" and current-priced variables and for relating values with and without transportation. Use of appropriate concepts together with a consistent accounting framework would also have assured that flow continuity conditions were satisfied.

IVf. Conclusions About the Abstract Formulation

In terms of the theoretical or abstract formulation and apart from problems of quantification, the economic system model is defective in three principal respects:
1. in the formulation of dynamic relations,
2. in the specification of behavioral or decision functions, and
3. in the lack of a consistent system of concepts and accounts.

If a model is to be a dynamic simulation including adjustment mechanisms that respond to feedback information, then it is essential to formulate those mechanisms with great care about the time relations involved, the information that constitutes the feedbacks, and the form of the response of the adjustment mechanism to the feedback signals. The behavioral or decision functions should represent the best available knowledge of the qualitative nature of what they represent, even if good quantitative information is lacking. Otherwise they may introduce spurious instabilities or produce results contrary to their real-world counterparts. And finally the need for well-chosen concepts and a systematic accounting system should be obvious in the light of the mistakes cited above.
V. Design for a New Model

Va. Why a New Model

The foregoing chapters have described several kinds of difficulties that stand in the way of using Harvard's economic system model for either of its two main objectives - comparing the economy-wide effects of alternative transport system plans or projecting supplies and demands of goods to be transported. It was concluded that the absence of empirical knowledge about investment and plant location decisions made it impossible to evaluate benefits in terms of the effects of transportation on the growth of national product or its regional pattern. This conclusion is probably valid regardless of the specific country considered. Moreover, it does not depend on the model formulation; no model can compensate for lack of knowledge about phenomena that crucially affect the results.

Despite that negative conclusion, however, information on Colombia that is available or obtainable with a reasonable effort appears to be just about adequate for the less ambitious objective of projecting consistent and feasible growth patterns as the context for transportation-system analyses. In effect, it has been concluded that it would be feasible to make a "consistency model" but not one with dynamic feedbacks. Undoubtedly this conclusion would apply to a number of other countries, but not (without a good deal of primary data collection) to countries whose data base is weaker than Colombia's.

Making projections of the regionally-distributed production and use of various categories of goods as a basis for projecting traffic may seem pedestrian compared with the idea of simulating the dynamic interactions of the transport and economic systems. Yet, to make such projections on a fully-consistent basis, with a computerized model that facilitates multiple solutions, would be a significant improvement over present methods. Like all other forecasts, these projections would be subject to uncertainty, but
several alternative projections can be generated for the purpose of sensitivity analysis by using different assumptions. Different transport development plans would be simulated by the transportation model, in the context of first one economic-system growth pattern and then another. In this way, for any proposed set of regional development objectives, or any forecast of the regional pattern of growth, a suitable and efficient transportation plan could be devised.

Within each projected pattern of economic growth, alternative transportation plans would be compared, not in terms of their uncertain effects on growth of GDP, but in terms of resource savings and consumers' surplus, as in more conventional analyses. It would also be possible to observe what links in the system became overloaded, and thus infer where changes in either the economic pattern or the transport system might be worth exploring.

An economic-system model to be used as just described does not have to fulfill the more difficult requirements of one designed to simulate the dynamic interactions of the economic and transportation systems. Instead of a dynamic behavioral simulation model, what is required is a consistency or projection model. Its purpose is to assure consistency among the projections for various sectors with respect to:

- interindustry demands and supplies
- consumption patterns as related to income
- capacity and production
- investment, replacement, and capacity
- import requirements and balance-of-payments feasibility
For this purpose, feedback characteristics and the distributions of time lags are less critical. A year is suitable as a time interval. And some very elusive behavioral relations can be left out.

There is no doubt that the Harvard model could be used in this way, if certain errors were corrected and modifications made. The feedback effects from the transportation model can easily be cut out, and the questionable investment and location decision formulae can be supplanted by trends or exogenous series without any change in the computer program. The one-year lag in consumption$^{1/}$ would be a problem for which the solution is not clear, however, and its effects may be significant. All the errors and inconsistencies described in Chapter IV would have to be corrected, and some fundamental changes ought to be made in the treatment of prices and the definitions of units to reduce confusion and give the user a better chance of avoiding mistakes. And at that point, the computer program would still contain much complexity that had been put in for purposes no longer relevant.

$^{1/}$ See Section IV d
It seems likely that correcting each of the flaws in the existing model individually would entail more effort than making a whole new formulation, without yielding as good a result. Therefore, it is recommended that a new formulation be carried out. A first approximation for a new design has already been made and is presented below. Among the ways in which developing the new design offers a better result are the following:

(1) The whole system of prices and the corresponding accounting relations are defined on a consistent and more comprehensible basis.

(2) A basic change is made in the treatment of consumption, which eliminates the one-year time lag and at the same time permits specifying demand elasticities different from 1.0.

(3) A simpler method is used for the treatment of import constraints (quotas, in the Harvard report's frame of reference; planning constraints in the present context).

(4) There are a number of simplifications in the formulation, some of them quite significant; thus a new computer program can be made simpler than a modified version of the existing program.

1/ To realize these advantages it is not actually necessary to discard everything in the computer program of the Harvard economic model. The best approach may be to replace some subroutines in it completely while keeping other subroutines intact except for limited corrections.
Vb. Overall Approach

In the remainder of this chapter, a preliminary formulation of a new model is presented. It is not complete, and what has been done may require modification before it is entirely consistent, compatible with available data, and programmable for the computer. Most of it is described as if it were already finished, as a matter of convenience in exposition, but the reader is asked to remember that it is offered as a first approximation and not as a finished product.

The general scheme of the proposed model is described and compared with that of the old model in the "Sequence of Operations" Table 2, which follows. On the left side, the chart shows the sequence of steps in the solution of the Harvard model\(^1\) On the right are brief comments indicating how some of the operations would be formulated differently in the new program while others would be omitted altogether. Some of the steps shown in the chart are believed to be clear without further explanation. Other portions of the new formulation are further explained in subsequent sections, as follows:

Vc. Units, Prices, and Accounting
Vd. Relation of Capacity to Investment
Ve. Investment Programs
Vf. Capacity and Demand
Vg. Consumption
Vh. Imports and Import Constraints
Vi. Input-Output and Income Relations
Vj. Regional Patterns

\(^1\) As shown before, in Table 1, Chapter II.
Table 2
Sequence of Operations: Old and New Models

Old Model (METS)

1. Compute new wages, unit costs, and prices by sector and region.

2. Gross Fixed Investment by sector and region:
   - optional combination of cash flow, accelerator, ratio, trend, exogenous.

3. Inventory increase proportional to output.

4. Consumption:
   - by sector and region; function of last year's income, inflation, and fixed APC's.


6. Intermediate goods:
   - Invert (I-A) matrix.
   - (National aggregates)

7. Imports (National and Regional)
   - Consumption: API's
   - Intermediate: AIMP's
   - Capital: BIMP's

8. Adjust for Imp. Restrictions
   - Shift from AIMP to A coefficient
   - " " BIMP to B " " CIMP to C (flow)
   - Then: Reduce investment
   - Shift some more consumption
   - Return to Step 6 and reiterate until restrictions are satisfied.

9. Compute "depreciation" and new level of nominal capacity by sector and region.

10. Allocate production to regions on basis of:
   - a. past shares of output.
   - b. shares of capacity.
   - c. shares of profits.

New Model (HIBRD)

1. Use exogenous trends for prices, and trends or simple adjustment mechanisms for wages.

2a. Determine investment to match planned sectoral growth rates, with accelerator to adjust plans when appropriate, or

b. Program investment exogenously (from known plans or assumptions), or

3. Combine a and b.

l. Inventory increase proportional to increment in capacity.

4. Nationwide consumption by sectors, based on current income with R. Stone functions to give elasticities $\neq 1.0$. (See Item 6.)

5. No change.

6. Similar, but include part of consumption and value added in matrix to solve simultaneously. (See Section Vi)

7. Imports (National only)
   - Consumption: as in Item 4
   - Intermediate: AIMP's (not regional)
   - Capital: BIMP's (not regional)
   - Extra: as needed for excess demand

8. Assume coefficients already reflect all convenient substitution. Apply restrictions only to Investment. Build analysis into iteration process to reduce number of iterations. (See Section Vh)

Note: Steps 4 through 8 are on a nationally-aggregated basis -- no regional allocation yet.


10. Allocate production in proportion to regional capacities. (No reallocation necessary as in former Step 11.)
Old Model (METS)

11. Reallocate production if, in any sector-region combination, it exceeds point of diminishing profits (PRODMX).

12. Determine intermediate demands by region and total demand and supply by region, for each product. Recompute wages and costs.

13. Pass demands, supplies, and other economic information to Transportation Model and stand by for feedback from that model.

14. First time around each year: When information comes back on transportation costs, use it to revise input-output coefficients, etc. and repeat entire program from step 6 through step 13.

15. On second time around (each year) adjust flow discrepancies, and calculate transport cost per unit, taxes, profits, national aggregates of regional variables etc.

16. Go back and start another year.

New Model (HIBRD)

11. Allocate consumption to regions on basis of incomes and exogenous coefficients.

12. No change.

13. Prepare information in form that transportation Model can use independently. Do not wait for feedback.


15. This model only goes around once each year. Much of this stage is now unnecessary. Calculate national aggregates and miscellaneous items.

16. Go back and start another year.
Vc. Units, Prices, and Accounting

Quantities of goods produced and transported have to be measured in some sort of "real" or quasi-physical units in order to relate them meaningfully to capacities of production facilities and to convert them into transportation-system traffic. The only practical way to define "real" units for the heterogeneous aggregates of the macroeconomic model is in terms of values at some standard set of prices. (Actual physical units could have been used for coffee, but for the other nine categories in the Colombia model it would have been impractical.) The Harvard group, accordingly, used "producers' values at base-year prices" to measure "real" quantities, and we do the same. The conventional definition of "producers' values" excludes transportation and trade margins but includes indirect taxes (e.g. excise and sales taxes). Although we do not propose any change in the measure of real quantity used by the Harvard group, we use a different word in referring to it, in order to avoid the confusion that always arises when both current-priced values and base-year-priced values are called by the same name, such as dollars or pesos. For each aggregate category of products, one million pesos worth at base-year producers' prices is defined as a quant. Production, supply, and demand, being flows, are measured in quants per year.

Real Capital is another concept for which it is convenient (even if not absolutely necessary) to define a special unit. A cap is the capital resulting from an investment project that would cost one million pesos if all its inputs were priced at base-year prices. Real (i.e. deflated) investment, being a flow, is measured in caps per year.

1/ There are good reasons to prefer a system of valuation that excludes the taxes (see: United Nations, A System of National Accounts, ST/STAT/SER.F/2/Rev. 3. United Nations, N.Y. 1968, Chapter IV), but most available data includes them, and the advantages of making the conversion do not seem sufficient, in the present application, to offset the complications of doing so.
Prices are defined in terms of millions of current pesos per quant of product. Thus, by the definition of a quant, all producers' prices must be equal to 1.0 in the base year, and a producers' price is the same as a conventional price index. Delivered prices refer to the same quants and include transportation charges in addition. (This is where we depart from Harvard's approach.) Thus, although they, too, are indexes of a sort, they are not equal to 1.0 in the base year. It is really more convenient to think of prices in this system not as indexes but as ordinary prices, treating a "quant" as if it were a physical unit of an individual product, although of course it is a composite. The definitions of prices and quantity units are such that the "delivered price" of a product equals its "producers' price" plus the transportation charges per quant:

\[ P_{DEL_{i,m}} = P_{i,m} + TCSTPU_{i,m} \]

Distribution and selling markups are not included in PDEL. They must be included as complementary services in the final demand vectors. If desired, however, the model could be reformulated to treat these services in a similar way to transport cost.

As the model is presented, it has no mechanisms for simulating the dynamics of price behavior. Trends or other variations in both producers' prices and transport costs are assumed to be programmed exogenously. But if price functions were to be postulated, they could readily be added without major reformulation. Also, if this model were to be run in parallel with a transport simulation, values of TCSTPU from the latter could be fed in instead of using exogenous values.

Even with producers' prices and transport costs exogenous, however, there are several advantages to expressing all prices explicitly as follows:
1. Any expected differences in price trends for different sectors, or between domestic and world prices, may be explicitly represented (by exogenous trends, of course).

2. Any expected changes in overall average transportation costs may be properly accounted for (but not changes in part of the system relative to other parts).

3. If workable price functions are developed later on, they can be added without a major reprogramming of the basic model.

4. Some actual relations really involve current-priced variables (profits, taxes, etc.) and others are basically physical (production functions), and the only way to handle both types properly in an inflationary economy is by keeping prices explicit.

If, however, the user thinks in "real" terms and wants to use the model that way, he can specify that all producers' prices are constant at 1.0. Delivered prices will be higher than that, because they will include transportation, but all "real" variables will be properly accounted for.
The term capacity is used, as in the Harvard model, to refer to a normal level of production and not a ceiling. Production in any sector and region may exceed or fall short of capacity. Changes in capacity result from investment, from attrition of old facilities, and from changes in the productivity of existing facilities. Instead of a discrete "point-in-point-out" relation between investment and its effect on capacity, we assume a distributed lag for gestation periods that are longer than a year. For example, with a three-year overall period, a certain proportion, $E_1$, of each unit of investment (in a particular region and sector) in year $t$ affects capacity in the next year ($t+1$); another portion, $E_2$, affects it in the second year ($t+2$), and the remainder, $E_3$, which is equal to $1-E_1-E_2$, has its effect in the third year ($t+3$). A certain proportion, $\delta$, of existing capacity wears out each year, and the productivity of the remaining capital increases by the proportion $\pi$. The combination of all these assumptions is expressed by the following equation:

$$QK_t = QK_{t-1} \cdot (1-\delta) \cdot (1+\pi) + \text{CMES} \cdot E_1 \cdot H_{t-1} + E_2 \cdot H_{t-2} + E_3 \cdot H_{t-3}.$$

where the sectoral and regional indexes have been omitted for clarity. The symbols are defined as follows:
QK = nominal real output capacity (in a specific sector and region), measured in quants per year. (RPOCAP in Harvard model)

σ = proportion of capacity lost by wearing out of facilities in any year

π = proportionate increase in capital productivity in one year — proportion/year

CME = output-capital ratio — quants/year/cap

H = gross fixed real investment (in the specified sector and region) — caps/year

(The letter I is used for investment expenditures at current prices.)

E1, E2, E3 = proportions of investment that yield new capacity after 1 year, 2 years, and 3 years, respectively.

All of the concepts above are sector-specific. In addition QK and H refer to the sector within a specified region.
Ve. Investment Programs

Real investment for any sector in any region may be programmed in any of three different ways, none of which represents a behavioral function. As in the Harvard formulation, the equation includes all the options additively, but it is assumed that two of the three coefficients will be set to zero at any time. One of the options is simply to specify an arbitrary exogenous time series. The second is to specify a level of investment for the initial year and a proportion by which it is to be increased each year thereafter. The third alternative is to specify a planned growth rate for capacity of the sector in the region, and the program will calculate the investment required to attain it.

The investment required for a specified growth rate, $R$, if the entire effect of an investment took place in the following year, would be:

$$H_t = \frac{QK_t}{CME} \times \frac{R}{\zeta - \tau(1-\delta)}$$

where the symbols are the same as defined above. The assumption of a distributed lag complicates matters slightly, requiring the above expression to be divided by

$$\frac{E_1}{(1+R)} + \frac{E_2}{(1+R)^2} + \frac{E_3}{(1+R)^3}$$

to allow for the fact that the absolute increment in capacity has to be larger each year to keep up with the growth of capacity itself. The
resulting expression -- entailing assumptions that CME is constant and that \( \frac{H}{QK} \) is the same for all years -- is:

\[
H_t = \frac{QK_t}{CME} \times \left[ \frac{R + \delta - \gamma*(1-\delta)}{E2 \times \frac{E3}{E1 + (1+R) + (1+R)^2}} \right]
\]

or, defining the expression in square brackets as GROW

\[
H_t = \frac{QK_t}{CME} + GROW
\]

where GROW for any sector and region will normally not have to be recomputed after the first year unless a change in growth rate is specified.

If CME, the output-capital ratio, were made variable (to represent "embodied" technical progress), \( H/QK \) would also have to vary for a given growth rate, and the expression above -- having been derived on the assumption of a constant \( H/QK \) -- would be only approximately right. However, it is believed to be a good enough approximation for present purposes.
Vf. Capacity and Demand

Some national development plans are based on projections of sectoral production capacities with little or no serious concern about supply-and-demand balances. The Harvard model and the new model described here both project demands as well as capacities, and equate the output of each sector with total demand for its product. Thus it is possible to observe whether the capacities of particular sectors are growing too fast or too slowly to match the evolution of demand. In the Harvard model, moreover, the capacity-adjustment term of the investment decision function might have made appropriate corrections to the growth of capacity, had it not been made unusable by a built-in dynamic instability.

For the present formulation, we are not trying to simulate investors' behavior. The object is to produce projections that are feasible, consistent, and suitable for development planning, assuming that the projected levels and patterns of investment will be achieved by means that are not represented in the model. But projections suitable for planning would not include those that provide a great deal too much or too little capacity in any sector. Hence it can be useful, even in a consistency model, to include an automatic mechanism to adjust investment that is creating too much or too little capacity.
With a relation already established for making investment in any sector correspond to a specified rate of growth of capacity, a convenient way to change the level and trend of investment in the sector is simply to re-specify the target rate of growth. The primary criterion of the need for adjustment is the capacity utilization ratio of the sector:

\[
\text{CAPUTL}_i = \frac{Q_i}{Q_{Ki}}
\]

which, ideally, should remain close to 1.0. In order to avoid recomputing GROW for all sectors and regions every year, some deviation from the ideal is tolerated. No adjustment occurs if

\[
-\xi < (\text{CAPUTL} - 1.0) < +\xi
\]

where \(\xi\) might be specified as .05 or .10. If the deviation exceeds this tolerance, a two-part reaction occurs, one part to respecify the trend of capacity and the other to offset the discrepancy that has already built up. The first part depends on prior measurements of the trend of the sector's capacity and of its output.1/ The following symbols are used:

- \(R_{i,m}\) the target rate of growth specified for Sector I in Region M as a proportion per year.
- \(DQK_i\) the measured rate of increase of nationwide capacity of the sector, in quantity units -- quants per year, per year.
- \(DQ_i\) the measured rate of increase of nationwide production in the sector, in quantity units -- quants per year, per year.
- \(DDIFF = DQ_i - DQK_i\) the difference between the two rates of increase -- quants per year, per year.

1/ These measurements have to be "smoothed", but to avoid complicating the exposition, the smoothing function is not specified here.
The adjustment to the target growth rate of the sector in each region is made as follows:

\[(R_{i,m})_{\text{new}} = (R_{i,m})_{\text{old}} \times (1 + \text{DDIFF}_i/DQK_i)\]

Since DDIFF may be positive or negative, the new target growth rates may be higher or lower than the old ones. The new rates remain in effect until another correction is found necessary. Whenever the R's are adjusted, an additional increase or cutback in investment, for one year only, is used to help bring the utilization ratio back within bounds:

\[\Delta H_{i,m} = \theta \times \frac{QK_i}{\text{CME}_i} \times (\text{CAPUT}_i - 1)\]

where \(\theta\) is a coefficient, \(\leq 1.0\), that permits adjustment if necessary for smooth operation.

Once this adjustment mechanism has operated on a sector's investment, it is prevented from repeating its action on the same sector until a full gestation period has passed, to avoid correcting more than once for the same deviation.

As explained later\(^1\), changes in investment levels may also be made in order to avoid excessive import requirements. Those changes will be superimposed on the results of the adjustment described above, and could, in some cases, prevent the output-capacity balance from being achieved.

---

\(^1\) In Section Vh.
Econometric studies of consumer demand patterns by Stone and others have led to the formulation of expenditure patterns in a form that is linear but not equiproportional, so that price- and income-elasticities may differ from 1.0. It turns out that this formulation can be conveniently combined with the input-output matrix, so that the time lag between income and consumption can be eliminated. (If the model were operating with short time periods, it might be appropriate to consider whether there should be a small time lag here, but since it is an annual model the choice is between a minimum lag of one year and no lag at all. The latter is clearly more realistic for consumers' spending.) Thus two of the shortcomings mentioned in Section III concerning consumption are overcome -- the unitary elasticities and the one-year lag with its anomalous effect on saving.

The Stone formulation makes the expenditure component for each category in the consumers' aggregate budget linear with total expenditure, in such a way that some components may expand relatively faster than others, but the components always add up to the total. The relation may be expressed as follows:

$$QC_i = \frac{AV_i}{PDE_i} (V - \sum QB_j PDE_j) + QB_i$$

where $$QC_i$$ is the quantity of good $$i$$ consumed

$$QB_i$$ is a parameter of the same dimensions as $$QC_i$$ (quants per year)

$$AV_i$$ is a nondimensional parameter, the marginal propensity to consume good $$i$$, relative to total consumption expenditure

$$PDE_i$$ is the delivered price of good $$i$$ and

$$V$$ is the total expenditure in value terms.

The summation over \( j \) includes \( i \). Consistency requires that the \( AV_i \) parameters add up to 1.0. Both imports and domestic goods are included.

It is possible to interpret the set of \( QB \)'s as a basic set of requirements that consumers find necessary to provide for before considering other possibilities, and the other term in the formula as the consumers' allocation of what they find they have left after satisfying their minimum needs. This is perhaps too rigid an interpretation. All that is necessary, actually, is to know that the formula has given a satisfactory approximation to the observed changes in consumers' budgets over fairly long periods in England and several other countries, and that its parameters can be chosen to give whatever income elasticities of demand fit empirical data or seem plausible to the user (subject to consistency conditions, and to the fact that the elasticities change as total expenditure changes).

Our formulation departs very slightly from Stone's, in that we replace total expenditure by total disposable income, and let consumers' savings be determined as another "good," by the same budgeting procedure.

We postulate that the consumer budget is basically a per-capita affair, and specify the dimensional parameter on a per-capita basis, as \( QN_1 \), such that

\[
QB_i = NPOP * QN_1
\]

where \( NPOP \) is the population at the particular time.
Vh. Imports and Import Constraints

In an adaptation of a scheme used by Ragnar Frisch, imports are treated as two sets of components, "substitutable" and "nonsubstitutable". Each of the several aggregates in the former category is treated as being perfectly equivalent to the product of one of the domestic sectors. Those in the latter category are treated as completely non-substitutable in the short run, although a gradual substitution process may be programmed, as described below.

This formulation is not meant to imply that the goods actually imported could be sorted out into these two extreme categories; a great many imports probably have imperfect domestic substitutes. The formulation is merely a convenient way of permitting imports to make up for shortages of domestic supplies, while preventing certain basic import requirements from being phased out without deliberate planning for their replacement. These relations, it should be remembered, are to be used in making consistent projections rather than in simulating behavioral decisions and reactions. As in most consistency models, the policy measures that would be required to make the system follow the projected path are not necessarily all explicit.

Imports that are "nonsubstitutable" (in the short run) include consumers' goods, intermediate goods, and capital goods. The demands for consumers' imports - of three types - are determined jointly with the demands for domestic goods in the consumers' demand function. (Actually, this does allow some substitution if relative prices change.) The demand for non-substitutable intermediate goods is the sum of individual sector demands computed with a set of input-output coefficients:
The demand for capital goods imports comes from the investment vector and a set of input-capital coefficients

\[ Q_{Mj} = \sum_{j} A_{mj} * Q_j \]

Long-run substitution for any of these import components can be programmed by progressively reducing the appropriate \( A_{mj} \) or \( B_{mj} \) coefficients -- provided that domestic capacity is built up sufficiently to meet the shift in demand. (Otherwise it will merely be shifted to substitutable imports.)

In addition to the nonsubstitutable imports, another set of import flows may be generated to supplement domestic supplies from certain designated sectors. When demand for the product of any of these sectors exceeds the sector's nominal capacity, the balance is made up by increased imports:

\[ Q_{Mi} = Q_i - Q_{Ki} \quad \text{for designated sectors.} \]

In the Colombian application, the sectors in which imports could be directly added in this way to domestic supplies would be agriculture, foodstuffs, and the two other manufacturing sectors. The relation would not apply to export sectors, construction, or services.

If unlimited foreign exchange were available - or if the purpose were to compare alternatives in terms of their requirements for foreign capital, then a balance-of-payments account could be generated with imports determined as above, and with the required capital inflow determined as the residual. However, if the available inflow of foreign capital is given as a constraint, as it sometimes will be, then imports may prove excessive on first calculation, and it may be necessary to alter some other assumptions in order to bring them down to the limit of available finance.
The Harvard model included an extremely elaborate and roundabout set of adjustments to be made for reducing imports to match a given total quota or a set of sectoral quotas. Domestic goods were first substituted for imports without change in the total quantities. Then some consumer-good substitution was made with adjustment of the quantities to compensate for any difference between domestic- and imported-goods prices. Finally, investment could be reduced to diminish the requirement for foreign capital goods.

In the new formulation the first two of these processes, entailing increased production from domestic sectors, are not considered appropriate unless those sectors have excess capacity before the adjustment. Import-substitution is not to be treated as a short run response to disequilibrium but as something for which capacity has to be planned and created in advance. Of course, if one or more sectors do have idle capacity in the initial calculation, it may be possible to take advantage of the fact, but, even though capacity, by our definition, is a nominal concept and could be exceeded in a real situation, it is not appropriate to exceed it in a planning exercise or consistency projection.

To meet a binding foreign-exchange constraint, there are two possibilities. One is to replan investment in earlier periods of the projection in such ways as to expand the capacity of a sector whose product can either augment exports or substitute for imports. This has to be worked out by the investigator on a cut-and-try basis, and it may not always be feasible. The other possibility is to reduce final demand in the period that requires adjustment.

We assume, however, that consumption at a given income level cannot be arbitrarily reduced (except by increasing taxes, which may or may
not be appropriate for the experiments for which the model is being used). We assume, also, that government non-investment expenditures have been carefully budgeted and cannot be further cut, as far as our long run projection is concerned. The only final demand that can be readily changed is investment.  

In effect, the assumption is made that investment may be constrained by insufficient capacity to finance imports of capital goods.

This assumption is embodied in a mechanism for reducing investment automatically if necessary to adjust the balance of payments. (Use of the mechanism is optional, so that experiments can also be run without it, in order to estimate foreign capital requirements for a given plan.) When the mechanism is to be used, the user stipulates the relative allocation of cutbacks among different sectors by specifying a vector of coefficients, $\mathbf{CCUT}_i$, such that for all sectors together,

$$\sum_i CCUT_i = 1.00$$

A single coefficient, CUTHM, indicates the total number of units by which real

1/ No provision is made for throttling the inflow of non-substitutable intermediate goods, thus making it impossible to use some of the capacity in existence, while the construction of additional capacity continues. Although this situation does occur in the real world, it was regarded as an inappropriate basis for planning studies.
investments are to be cut back for each unit of excess imports that it is desired to eliminate.

\[ \text{HCUT}_i = \text{CUTHM} \times \text{CCUT}_i \times \frac{(\text{WM} - \text{WMAX})}{\text{PMKW}} \]

Where \( \text{HCUT}_i \) is the amount by which real investment in Sector \( i \) is to be cut; \( \text{CUTHM} \) and \( \text{CCUT}_i \) are coefficients, explained above; \( \text{WM} \) and \( \text{WMAX} \) are respectively the tentatively calculated value and the maximum permissible value of total imports, in world currency, Mn$/year; and \( \text{PMKW} \) is the price of imported capital goods in world units, Mn$/quant.

The adjustment coefficient, \( \text{CUTHM} \), will be set initially by the user, but whenever an adjustment is made, the program will monitor the effectiveness of the change and will automatically alter the value of \( \text{CUTHM} \) in order to make the next adjustment come as close as possible to the right magnitude and reduce the number of iterations required. Thus, if the first adjustment accomplished only half of the desired result, \( \text{CUTHM} \) would be doubled before the next try.
VI. Input-Output and Income Relations

It was stated above that the consumption budget is based on current income. Current income, of course, depends on current output, which in turn depends partly on consumers' demand. Thus, a simultaneous solution of consumption and production of the various categories of goods is called for. Interindustry (input-output) demands must be taken into account, and it is desirable, at the same time, to constrain the solution by the limits of domestic capacities and to ascertain the implied import requirements.

In Table 3, the various relations are shown as a set of simultaneous equations. If the equations that are in "real", or quantity terms were multiplied by appropriate prices, the set of equations would be an accounting matrix. A typical row across the table spells out the demands for the products of one sector. Demands from other sectors (in real terms) are related to the other sectors' \((\text{real})\) outputs by input-output coefficients. Thus,

\[ Q_{ij} = A_{ij} * Q_j \]

To the right of these intersectoral demands appear two unfamiliar terms:

\[ (AY_i/PDEL_i) * YA + QB_i \]

Reference to Section Vg reveals that this is a slight rearrangement of the typical equation for consumers' demand, once the YA is identified as disposable income minus the "basic" consumption expenditures for the QB_i's. Formulae for incomes and for the YA concept in terms of other variables in the system appear in the lowest three rows of the table, labeled "labor", "gross profits", and "income available after 'basic' expenditures".

Returning to the upper rows of the table, the QG_i's are government non-capital-forming expenditures, and the QX_i's are exports. As in most
Table 3.

| Intermediate flows | Consumers' demand | Other final demands | Imports | Domestic output | Exports | Input-Output Demand
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>( A_{11} Q_1 + \ldots + A_{1n} Q_n + (AY_1/PDEL_1) * YA + QB_1 + QG_1 + X_1 + \sum_{j} B_{1j} H_j - QM_1 = Q_1 )</td>
<td>( \leq QK_1 )</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

| 2 | \( A_{21} Q_1 + \ldots + A_{2n} Q_n + (AY_2/PDEL_2) * YA + QB_2 + QG_2 + X_2 + \sum_{j} B_{2j} H_j - 0 = Q_2 \) | \( \text{no imports} \) |

| n | \( A_{n1} Q_1 + \ldots + A_{nn} Q_n + (AY_n/PDEL_n) * YA + QB_n + QG_n + X_n + \sum_{j} B_{nj} H_j - QM_n = Q_n \) | \( \leq QK_n \) |

Imports: \( AIMP_1 Q_1 + \ldots + AIMP_n Q_n + (AY_m/PDEL_m) * YA + QB_m + QG_m + 0 + \sum_{j} BIMP_{m,j} H_m - QM_m = Q_m \) \( \text{QM}_m \text{ dependent} \)

Labor: \( P_w A_{w1} Q_1 + \ldots + P_w A_{wn} Q_n = YW \)

Gross: \( \frac{P_1 Q_1}{(1 + TX_{ndr,1})} - \sum_{i} P_i A_{i1} Q_1 + \ldots + \frac{P_n Q_n}{(1 + TX_{ndr,n})} - \sum_{i} P_i A_{i1} Q_n = YP \)

Income: \( YW * (1 - TX_{dir,w}) + YP * (1 - TX_{dir,p}) * (1 - CSAV_p) - \sum_{i} P_{DEL} * QB_i = YA \)

Notes:
1. Row 1 illustrates a sector in which capacity-limited domestic supply is supplemented by imports. If \( Q_1 < QK_1 \), \( QM_1 = 0 \); otherwise \( QM_1 \) adjusts so that \( Q_1 = QK_1 \).
2. Row 2 illustrates a sector without supplemental imports; \( Q_2 \) cannot be directly related to \( QK_2 \) unless some component of \( Q_2 \) is flexible.
3. The row labeled "imports" could, if desired, be split into several rows.
4. Labor could be divided into categories if enough information were available.
5. \( CSAV_p \) is the fraction of gross after-tax profits that is saved as depreciation allowances and undistributed profits.

Symbols: All symbols beginning with \( A, B, C, \text{ or } TX \) are coefficient, the \( TX's \) being tax coefficients; the initial \( Q \) indicates real quantity flows; \( P \) is for producer's price (\( P_w \) for effective wage rate); \( PDEL \) is price including delivery to the user (but not commercial markup); \( H \) is real investment; the \( Y's \) are income at current prices.
input-output analyses, both of these sets of final demands are exogenous (unless some special assumptions are added). The next column is the set of demands due to investment; the formula shows how they are derived through use of the matrix of input-capital coefficients, $B_{ij}$. The symbol $H_j$ stands for real investment for sector $j$.

The final term before the equality sign is either $-QM_i$ or 0, depending upon whether it is assumed that imports may be used to supplement domestic output. Row 1 corresponds to a sector whose limited domestic output is supplemented by importing equivalent goods to the extent necessary to meet all demands without letting domestic output exceed capacity.

Either: $Q_1 < QK_1$ with $QM_1 = 0$

Or: $Q_1 = QK_1$ and $QM_1 \geq 0$

Row 2 corresponds to a sector without supplemental imports. If it were to be assumed that output equaled capacity, one or more components of demand would have to be flexible to satisfy the equation. The alternative is to allow $Q_1$ at any time to exceed or be less than $QK_1$; this will normally call into play the capital stock adjustment mechanism.¹

¹ See Section Vf.
with the object of revising the investment plan to match capacity more closely to demand. Or, if the user prefers, he can switch off the adjustment mechanism and merely record the magnitude of the output-capacity discrepancy.

It is to be noted that there is an import row, in addition to the import column. It is in the row that "nonsubstitutable" imports are determined from technical coefficients and the consumption terms. At its intersection with the import column, the total of other terms in the row is entered, with a minus sign. This serves to put the equation in the same form as those above it, showing that domestic production of imports is nil, while bringing imports of all kinds together in one column.

The entire set of equations shown in Table 3 are to be satisfied simultaneously, but this should not be much more complex than dealing with the basic input-output relations, and should simplify some other parts of the program considerably.

Vj. Regional Patterns

The geographical pattern of growth in this model is exogenous. The expansion rates of sectoral capacities in each region are either specified directly or derived from exogenous investment programs. Although investment
in any sector may be modified by the accelerator adjustment or by the adjustment to reduce imports, these modifications are proportionately distributed among regions and do not alter the relative geographic pattern.

Inasmuch as the objective has been redefined as projection in the spirit of planning, rather than simulation of behavior, no attempt is made to account for regional differences in production cost and profitability as a basis for deciding how much will be produced where. Production of each sector is allocated among regions in direct proportion to the sector's regional capacities.

The analysis of interindustry demands, consumption expenditures, and imports has been carried out on a nationally aggregated basis up to this point. The locations of production determine the locations of use of intermediate goods, domestic and imported. What still remains to be determined is the geographical pattern of consumers' demand. For this the model-user has to make further assumptions. The formulation offers two approaches, of which the user may choose one or use a combination.

One approach is to assume that income is earned and spent where the production activity is located. This would be expressed in the following formula for the consumption of good i in region m:

$$Q_{CR_{i, m}} = \left(\frac{Y_{DR_m}}{Y_{D}}\right) \times Q_{C_i}$$

where QCR and QC are regional and nationwide consumption of good i and YDR and YD are regional and nationwide disposable income.
Several considerations cast doubt on this simple assumption. Part of disposable income is distributed business profits, which accrue to the shareholder, wherever he may live. Shareholders probably tend to be more densely located near major cities, both because the cities offer more amenities and because they are financial centers. Thus the region that contains Bogotá, for example, might have considerably more income than the value added in productive activities located there. Moreover, even people who don't live in big cities often go to them for certain kinds of shopping. This, of course, may or may not involve crossing a regional boundary.

If our income data really distinguished shareholders' profits from workers' wages, it might be useful to make different assumptions for the two categories. Unfortunately, the data thus far available on Colombia distinguishes between wage and nonwage income, including in the latter all income from owner-operated enterprises and thus aggregating most of the country's farmers and small-businessmen with the corporation shareholders. Whether further breakdown of these categories could be made, and whether it would be worth while are not presently known.

In any case, the user might wish to make some hypothesis about the regional distribution of consumption expenditures, other than that expressed in the foregoing formula. A completely exogenous specification could be applied through the following equation:

\[ Q_{CR,i,m} = CCR_{i,m} * QC_i \]

where \( CCR_{i,m} \) is one of a vector of regional distribution coefficients for sector \( i \), so that different distributions can, if desired, be specified for different sectors.

The two approaches are combined in the following formula, which permits using different combinations in different sectors:

\[ Q_{CR,i,m} = \phi_i * CCR_{i,m} * QC_i + (1-\phi_i) * (YDR_m/YD) * QC_i \]
APPENDIX I

Data on the Colombian Economy

A. Data Reconnaissance by IBRD Staff and Colombian Planners

In July and August 1969 a group from the World Bank visited Colombia in order to discuss various aspects of both the Harvard Transport Model and the associated economic model with members of the planning department staff (DNP) and to investigate possible sources of data beyond those already recognized and in use. Concerning the economic model, the group consulted not only with the DNP staff but also with officials of the coffee-growers association (Federación de Cafeteros), the industrial manufacturers association (ANDI), the national-accounts office of the central bank (Banco de la República), and the national statistical department (DANE). Several meetings were held with various experts in DANE, which was the most promising source of information and a locus of interest in and understanding of the model-building approach. The discussions led to the drafting of a 19-page request from the World Bank and DNP to DANE for specific types of data that would be useful in quantifying a model of the sort contemplated.

The request for data provided a focus for discussions of specific items and later formed the frame of reference for an evaluation by DNP of the feasibility of obtaining the various pieces of information and estimates of the work.
This formal request did not ask for all information that would in fact be required to estimate all of the parameters of Harvard's formulation, in as much as it was quickly determined that much of the data required to model the dynamic interactions of the transport sector and the rest of the economy did not exist. In particular, the functions for determining regional allocation of each sector's production and the magnitude and location of investment for each sector could neither be verified nor quantified. Therefore, the objective underlying the data request was to obtain the empirical basis for a consistency model rather than one that would simulate all of the behavioral responses.

In order to give a more specific idea of the types of data needed and to indicate the difficulties of finding them, the two communications referred to above are summarized in the following section. Disparities are evident between information "requested" and "available" in almost every category. Many of the disparities are minor enough to be overcome by using estimates based on related information. Some, however, represent real difficulties. Probably most important are the absence of data for sectoral investment, which would have been used to estimate capital-capacity ratios, and for the composition of investment inputs in various sectors, which would have been analyzed for coefficients of the domestic and imported input-to-capital matrixes ("B" and "BIMP")\(^1\). As will be seen, there are some clues, in terms of more aggregate information, but important details are lacking. Thus, even when the behavioral relations are eliminated and the

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\(^1\) Values were assigned to those coefficients for the tests run at Harvard, but they were evidently based more on guesswork than on data. Burns's report on data collection says, "There are no figures on the industry of origin and destination of domestically produced capital goods. The procedure here was to estimate the split between purchase of domestic and imported goods, for which some figures were available and allocate domestic purchases in a reasonable and consistent manner...coefficients in the domestic flow matrix had to be adjusted to converge on reasonable values." Data Collection Methods, op. cit.
Purpose of the model is correspondingly reduced to the making of consistent projections, there will still be doubts about some of the important numerical inputs.

Nevertheless, apart from the deficiency of information on investment and capital goods inputs, it seems that a fair data base for a consistency model like the one specified in Chapter V could be collected in Colombia. Capital-output ratios and capital goods input coefficients might be based on data from other countries, and varied in sensitivity tests to see how the results would be affected by different estimates. It must be recognized, however, that the compilation of the data that are "available" in Colombia would require considerable skilled labor and that analyzing the data, estimating coefficients, and perhaps revising parts of the formulation would also constitute a large job.
B. Data "Requested" and "Available"

This section summarizes the data request drafted by the IBRD group, and the assessment of data availability (and time for its compilation) prepared by the DNP group, which were referred to in the previous section. The six headings that were used to categorize the information in both of those communications are used here as well.

1. Production and Inputs Used.

   a. Information requested:

      Time series, by sector and region, of physical (or "real") outputs, gross values, value added, taxes, quantities and values of domestic and imported inputs. For agricultural activities, data on areas sown and harvested were also requested and the request was for either quarterly or half-yearly data on all items. The desirability of a finer breakdown for some sectors (where the original data are more detailed in the first place) was indicated. Breakdown of value added into wages and profits was requested where possible.

   b. Information available:

      Time series for real output, gross output, and value added, for many sectors by regions and for the rest in nationwide totals. Tax data, nationally, for some sectors. Inputs available for some sectors, nationwide, not by regions,

[1] "Information Requested by World Bank and D.N.P. from DANE...", op. cit. and Memorandum to Holland and de Weille from Peña, Caijao, and Mora, op. cit.
and not distinguishing imports from domestic inputs. A 1956 input-output table (37 x 37 with each item divided into domestic and imported supply) exists, but is open to some doubt, and there is none more recent.

For agriculture, estimates based on sample surveys are available by geographical Departamentos (states), half-yearly starting from 1964, including arable area, areas planted and harvested for various crops, and volume of each crop harvested. Older series exist, but in less detail. Nothing seems to be available on inputs or value added (except the sectoral value added in the national income accounts). Detailed information on livestock population and slaughtering are available. Coffee production is quite well documented.

Since some of the data, although available, have not been published in the desired detail, it was estimated that about 18 man-months would be required to compile the unpublished data from original records. Half of that time is for data on revenues from direct taxes on agriculture, livestock, and manufacturing. For other sectors it is not certain whether tax data are available.

2. Investment
   a. Information requested:

      Time series for changes in stocks, by products (or sectors), and location.

      Time series for real and current-priced gross fixed investment in each sector and geographical location, broken down into construction, domestic machinery and equipment, imported machinery and equipment, and other components, if any. Estimates of what part of each gross investment figure is for replacement.

      Information on gestation lags.
b. Information available:

Time series for nationwide changes in stocks of coffee, livestock, manufactured goods, and building materials.

Estimated time series for gross national fixed investment (not sectoral or regional) broken down as construction and machinery-and-equipment. Time series of fixed investment by the national government by 13 economic sectors, but without distinction between construction and machinery, without regional distribution, and with no similar data for the private sector. "Basic data on depreciation". Nothing on lags.

No estimate was made of the time required to compile data, since so little is available.

3. Wages and Prices

a. Information requested:

Wages and salaries in absolute figures (not indexes) time series of annual averages, by cities for each economic sector and national average for each sector.

Retail prices for various consumer goods in different locations, and nationally; producers', wholesale, and retail prices (or indexes), showing the differences due to transportation costs, distribution costs, indirect taxes, and markups, for various products and sector aggregates in different locations.

b. Information available:

Wages and salaries are available in time series for various sectors, some by states, some for particular cities, and some only at the nationally averaged level.

Retail prices are available for 16 principal consumption goods in the seven most important cities since 1955, annually or, for some of the information,
monthly. Wholesale and retail prices for all food products in 11 cities, and national average wholesale prices for 9 types of product are also available.

The time estimate for compiling all of the wage and price information was five man-months.

4. Household Consumption
   a. Information requested:

      Standard household budget survey data, or equivalent, for urban and rural families in different regions. Also data to relate total expenditures to income (or savings to income), if this is not explicit in the budget data.
   
   b. Information available:

      Household budget survey data for the four biggest cities and a rural survey, but without explicit identification of savings.

5. Foreign Trade and Payments
   a. Information requested:

      Export time series in physical units, peso values, and dollar values, with explicit information on exchange rates and taxes, for significant products and total, through each port. Locations of production if possible.

      Import time series in dollars and current and deflated pesos, classified as to type and use (e.g. machinery, production inputs, etc.), with sector of use when possible to identify it. Chronology of exchange rates, quotas, duties, etc.

      Time series of various invisibles (pesos and dollars), including factor incomes, debt interest, etc.

   b. Information available:
Essentially all of the desired data are available, but 4 man-months would be required for identifying the sector of use for imports, and 5 man-months for compilation of all other information (9 man-months total).

6. **Public Sector** (all levels of government)
   a. Information requested:

   Time histories, by state of source, of revenues from personal income taxes, business taxes, indirect taxes, import duties (by broad category of goods), with chronology of changes in tax rates and tariffs.

   Time histories of transfer payments and current account expenditures by functional category, and of capital expenditures by category and location, in current and deflated values.

   b. Information available:

   Essentially all of the desired data are available. To compile the revenues on a regional basis would require 5 man-months, and at least 3 more man-months would be needed to reconstruct the history of the structure of import duties (at least 8 man-months total).
Appendix II

Derivation of Some Price Relations

In Chapter IV (Section IVe) it was asserted that a number of important equations in the Harvard Model were wrong, apparently because of confusion about the price indexes involved in them. This Appendix shows how that conclusion was reached.

In the Harvard formulation, the price of any product at the producer's location and the price of the same product delivered to the user are represented by indexes with independent bases; i.e. both are defined to be equal to 1.0 in the base year, for all products and locations. With the indexes defined in that way, it is difficult to account properly for the effects of changes in the relative cost of transportation of the products, and it was not clear whether the accounting had been done correctly in the model. Therefore, two other price concepts were defined here and used to re-derive the relations in question as follows:

- pcif is the price, in pesos per "real unit" of product, including transportation.
- p is the price without transportation (f.o.b.) in pesos per "real unit."

When the subscript o is attached, the concepts are those for the base year.

The indexes corresponding to the Harvard formulation are derived from the above concepts as follows:

- PCIF is the delivered price index, pcif/pcif_o
- P is the "f.o.b." price index, p/p_o

Also, as in the Harvard report:

- TCSTPU is the cost (at current prices) of transporting a unit of the product to the user's location.
- TCSBAS is the value of TCSTPU in the base year.

Subscripts for product and region have been omitted. It is
understood that each concept and each formula derived applies to various goods in various regions.

Since the "products" in question are heterogeneous aggregates, their "real" units of measurement are defined in terms of value at base-year f.o.b. prices. A result of that is that all of the $p_o$'s are equal to 1.0, and, since for each product and region $P = p/p_o$, it follows that $P = p$ in every case, in spite of the conceptual difference in their definitions. When transportation costs are added, the delivered price for each product (in any region) is:

$$pcif = p + TCSTPU = P + TSCTPU$$

In the base year,

$$pcif_o = p_o + TCSBAS = 1 + TCSBAS$$

Then, from the definition of PCIF,

$$PCIF = \frac{pcif}{pcif_o} = \frac{P + TCSTPU}{1 + TCSBAS}$$

The corresponding equation in the Harvard report is complicated by the inclusion of a factor $1/\text{COIMLT}$ to adjust for differences in the composition of aggregates in different demand categories, a problem that is not related to the present discussion. Let us assume the adjustment factor is equal to 1.0 as it would be with uniform product mixes. Then the Harvard equation corresponding to the one derived above would be simplified to read:

\[1/\text{COIMLT}, \text{for "column multiplier"}, \text{or its weighted average, AVGMLT.}\]
(117) \[ PCIF = P + (TCSTFU - TCSBAS) \]

Comparison with the derivation above shows that this equation is in error. Quantitatively, the error can, under quite plausible circumstances, be greater than the effect of the change in transport costs that is being studied. The same error appears in Equation 118, a parallel equation for the delivered price of imported goods. Thus, all prices with transportation included are misspecified.

Other price equations that are evidently wrong are Equations 90 and 116, for prices applicable to imports, POE and PIMPRT. The difficulty with these is compounded by confusion over definitions. POE is described in the Dictionary of Terms as "initial cost of imports at port of entry." The word "initial" clearly does not refer to time, and presumably means prior to payment of tariff. On pages I-43 and I-44 of the Appendix, POE is described first as "producers price" and then as the port-of-entry price including an ad-valorem tariff; one equation where POE is used has a term to remove the tariff, in order to value imports in terms of the payments leaving the country:

\[ (90) \quad VALIMP = SUPPLY \times POE \times (1 -\text{TARIFF}) \]

Where VALIMP is the value of imports (of any class of goods designated by an appropriate subscript), SUPPLY is the quantity of imports of the same goods, and TARIFF is the applicable ad-valorem tariff rate.

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1/ Equation numbers correspond to those in Appendix I of An Analysis of Investment Alternatives in the Colombian Transport System, Harvard Transport Research Program, 1968. Symbols indicating sector and region have been omitted for simplification.

2/ At the end of Appendix I of the Final Report.
If, indeed, POE were the price with tariff included, the price before tariff would be POE/(1 + TARIFF), and the value of imports would be

$$\text{VALIMP} = \text{SUPPLY} \times \frac{\text{POE}}{1 + \text{TARIFF}}$$

instead of as given by Equation 90. However, in the explanation of PIMPRT, (page I-61) it appears that POE is meant to represent the price Before tariff and that PIMPRT is the price with tariff included. On that basis, the value of imports would be simply:

$$\text{VALIMP} = \text{SUPPLY} \times \text{POE}$$

Using these concepts of POE and PIMPRT, and assuming further that they are both indexes, equal to 1.0 in the base year, let us derive the relation between them and try to verify the equations in which they are used.

As before, we define symbols for the prices in non-index form:

- \(p_{im}\) the price in pesos per unit, with import duty paid, but without transportation away from port of entry.
- \(poe\) the price in pesos per unit at the port of entry before payment of duties.

When the subscript \(o\) is attached, these refer to the base year.

Assume that "quantity" units of imports are defined in terms of values at base-year prices, including tariffs (because this basis makes it convenient to treat them just like domestic products in other relations in the model).

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1/ The Dictionary of Terms defines PIMPRT \(_{i,m}\) as "price of import \(i\) in region \(m\) -- f.o.b. prices." Presumably "f.o.b." in this context means the price at the port of entry with import tariff paid but without delivery to the user. It is not clear why that concept would require a subscript for the region of delivery.
Then, for each imported good, $p_{im}$ is equal to 1.0 and

$$P_{IMPRT} = \frac{p_{im}}{p_{im}} = p_{im}$$

According to the definitions and assumptions already stated,

$$p_{im} = p_{oe} \cdot (1 + \text{TARIFF}) = P_{IMPRT}$$

$$p_{im} = p_{oe} \cdot (1 + \text{TARIFF}) = 1.0$$

Hence

$$p_{oe} = \frac{1}{1 + \text{TARIFF}}$$

$$POE = \frac{p_{oe}}{p_{oe}} = p_{oe} \cdot (1 + \text{TARIFF})$$

$$P_{IMPRT} = p_{im} = p_{oe} \cdot (1 + \text{TARIFF}) = POE \cdot (1 + \text{TARIFF})$$

Instead of using an expression of this sort, the Harvard report approaches $P_{IMPRT}$ in terms of its change from the previous year, using the letter $D$ as a prefix for the change in any variable. For comparison with their result, the relation above was used to derive one in terms of differences, as follows:

$$P_{IMPRT_t} = P_{IMPRT_{t-1}} + DPOE \cdot (1 + \text{TARIFF}) + POE_{t-1} + DTARIFF$$

This bears little resemblance to the equation in the report:

$$P_{IMPRT_t} = P_{IMPRT_{t-1}} + DPOE + FIMPRT \cdot \text{TARIFF} \cdot DTARIFF$$

In checking the computer program it was found that it did not correspond to Equation 116, which may have been transcribed incorrectly in typing. The formulation in the computer program, nevertheless, is also erroneous. It is:

$$P_{IMPRT_t} = P_{IMPRT_{t-1}} + DPOE + DTARIFF \cdot POE_{t-1}$$

The fact that relations among price indexes in the Harvard model could not be checked without auxiliary derivations, and the discovery of errors in so many of the relations that were checked, leads to the conclusion that the price concepts as defined were not well suited to
The purpose. They were, in fact, confusing. Some major errors might have been avoided by defining prices with special concern for keeping relations between the various concepts direct and simple.