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SHADOW PRICES FOR PROJECT EVALUATION UNDER ALTERNATIVE MACROECONOMIC SPECIFICATIONS*

Clive Bell and Shantayanan Devarajan

This paper takes the view that a project is a disturbance to an economy in equilibrium, and examines the shadow prices for project evaluation under alternative assumptions about how equilibrium is restored. When the government reacts by altering its foreign exchange reserves—a nondistortionary adjustment mechanism—the shadow prices coincide with those advocated in the manuals on social cost-benefit analysis. However, if the government adjusts its domestic expenditures or tariff rates, the shadow prices will differ from those of the manuals, except insofar as the relative shadow prices of tradeables remain their relative border prices.

For purposes of social cost-benefit analysis, a project may be viewed as a disturbance to the economy, displacing it from some initial equilibrium to a new one. But the new configuration will depend on which particular variables adjust to restore equilibrium. Since there may be more than one admissible form of adjustment, it is natural to ask how—if at all—the corresponding shadow prices for project evaluation depend on the nature of the adjustment. Now, the manner in which the economy equilibrates depends on how the government responds to the disturbance that the project generates. While some responses are distortionary, others are not—a distinction that underpins the results obtained for the various forms of adjustment analyzed below.

These equilibrating mechanisms are specified as different rules for “closing” a general equilibrium system that is initially specified so as not to be fully determined. By taking different combinations of variables to be fixed exogenously, one arrives at different formulations of the way in which the economy adjusts to the introduction of a project. It is then possible to solve for the different variables that enter into the social welfare function, the gradient of which yields the vector of shadow prices for the economy. Here, the introduction of a set of income and expenditure accounts for the government is essential if the results are to capture a central feature of social cost-benefit analysis: the distinction between public and private incomes when there is a premium on the former, usually because savings are sub-

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optimal. It is worth remarking also that our approach shares a good deal in common with Sen's [1963] critical examination of alternative macroeconomic theories, inasmuch as both emphasize that apparently minor changes in the choice of endogenous variable can lead to quite different results.

One more point must be emphasized at the outset. In this paper the output of nontraded goods is always determined endogenously, with inputs being drawn off from other uses if the introduction of a project leads to a rise in the demand for nontradeables. In those cases where the adjustment of the economy leaves domestic prices unchanged, the assumption of constant returns to scale implies that extra output will be produced at constant costs. But even if prices adjust too, choosing to make the output of nontradeables endogenous places the specification of the model squarely in the semi-input-output tradition.

The structure of the paper is as follows. Section I sets out the basic model, paying particular attention to certain national income identities and the system's degrees of freedom. With the foundations thus laid, Section II deals with two important cases in which, with parametrically given world prices for tradeables, domestic prices and wage rates do not vary because ad valorem tariffs are fixed. In the first, the government does not alter its existing outlays, so that all additional demands for goods stemming from a project are met ultimately by imports; in effect, the level of foreign borrowing is endogenous. It is shown that the shadow prices appropriate to this form of adjustment are exactly those advocated by Little and Mirrles [1974]. In the second case, the level of foreign borrowing is fixed exogenously, so that the government must "make room" for a project by altering its outlays elsewhere. This formulation is in spirit very close to that of Bhagwati and Srinivasan [1979] and Dasgupta and Stiglitz [1974]. But while we are able to reproduce their result that the ratio of the shadow prices of any pair of tradeables equals the ratio of their respective world prices, it turns out that the complete vector of shadow prices is identical (up to a scalar multiple) with that in Little and Mirrless only under special conditions. In Section III we consider a case in which

1. Blitzer, Dasgupta, and Stiglitz [forthcoming] also derive shadow prices under alternative equilibrating mechanisms. Their conclusions are similar to ours in spirit: the choice of adjustment mechanism crucially affects the nature of the shadow prices. In particular, the assumption of optimal borrowing is necessary for the "border price rule" to hold. However, their treatment differs from ours in a number of ways. First, and most important, they permit the government to distribute its trade surplus to consumers in a nondistortionary (lump sum) fashion, whereas we deal with both distortionary and nondistortionary responses. Second, when they introduce nontraded goods, they assume that the government's control variable is the market price of the nontradeable, whereas we treat three other equilibrating mechanisms. Finally, they do not consider the effects of factor market distortions, which appear in a central way in the results derived in Sections II and III below.
tariffs are endogenous, so that wage rates and the domestic prices of goods are likewise. The findings are very similar to those obtained in the second case in Section II. Section IV is devoted to some concluding remarks.

I. THE MODEL

The economy presented here is described by a three-sector, one-period model similar to those in Blitzer, Little, and Squire [1977], and Bell and Devarajan [1980]. The first two sectors produce tradeables, and the third a nontradeable by means of intermediate inputs and labor (the only nonproduced factor) under constant returns to scale, with no joint production. While there are neither private savings nor investment in this economy, the government raises taxes and makes outlays on goods and services. The only sources of tax revenue are ad valorem tariffs on traded goods.

Let

\[ X_j = \text{gross output in sector } j \quad j = 1,2,3 \]
\[ C_j = \text{private consumption of good } j \]
\[ E_j = \text{net exports of good } j \quad (E_3 = 0) \]
\[ G_j = \text{government consumption of good } j \]
\[ a_{ij} = \text{average input of good } i \text{ needed to produce a unit of good } j \]
\[ l_j = \text{average input of labor needed to produce a unit of good } j. \]

In general, the input coefficients \( a_{ij} \) and \( l_j \) will depend on domestic prices. However, if domestic prices stay constant following the introduction of a project, these coefficients will not change either. Constancy of domestic prices, in turn, is guaranteed if world prices and domestic tariffs are constant. Those forms of macromconomic adjustment to a project that leave domestic prices unchanged greatly simplify the task of deriving shadow prices, so it is no accident that the various manuals on social cost-benefit analysis implicitly embrace one of them. This matter will be taken up in detail in Sections II and III.

Returning to the "snapshot" picture of the economy, we see that the material balance equations are

\[ X_i = a_{i1}X_1 + a_{i2}X_2 + a_{i3}X_3 + C_i + E_i + G_i + Q_i, \quad i = 1,2,3. \]

There is only one household in the economy, which consumes all its income \( C \):

\[ C = w_1 l_1 X_1 + w_2 l_2 X_2 + w_3 l_3 X_3 + w_4 L, \]
where \( w_i \) is the market wage rate in sector \( i \). The variables \( Q_1, Q_2, Q_3, \) and \( L \) are the net direct demands by the project for goods 1–3 and labor, respectively. These are set equal to zero in the pre-project equilibrium.

Assume, for simplicity, that consumer behavior is represented by a linear expenditure system. Thus, if \( p_j \) is the domestic market price of good \( j \),

\[
C_j = d_j + b_j \left[ C - \sum_j p_j d_j \right] / p_j \quad j = 1, 2, 3,
\]

where \( d_j \) and \( b_j \) are, respectively, the intercept and slope terms of the linear expenditure curve for good \( j \). Note that \( \Sigma d_j = 0 \) and \( \Sigma b_j = 1 \).

Since this is a constant-returns-to-scale economy, the domestic prices of goods equal their respective unit costs:

\[
p_j = \sum_i p_i a_{ij} + w_i l_j \quad j = 1, 2, 3.
\]

Note that adding (1)–(7) and applying (8)–(10) gives

\[
\sum_{j=1}^3 p_j (E_j + G_j) = 0,
\]

where \( E_3 \) is zero by definition. That is, government consumption expenditure equals the value of the import surplus at domestic prices, there being no direct taxation or investment.

The world prices \( \tau_j \) of tradeable goods are assumed to be parametrically given. Hence, if \( \tau_j \) is the ad valorem tariff on (net) exports on the \( j \)th tradeable, then

\[
p_j = \pi_j (1 - \tau_j) \quad j = 1, 2.
\]

The government’s surplus in its domestic accounts, denoted by \( G \), is the excess of its tax revenues over its expenditures:

\[
G = \sum_{j=1}^3 [(\pi_j - p_j)E_j - p_j G_j].
\]

Combining this equation with (11) yields

\[
G = \sum_{j=1}^3 \pi_j E_j,
\]

which is the economy’s trade surplus, or the change in its foreign as-
sets. In other words, the government’s budget deficit or surplus registers an equal change in the government’s net foreign assets through foreign borrowing or lending. As a special case, \( G = 0 \) implies that a government which balances its “domestic books” also balances its “foreign books.”

The government’s expenditure functions for goods do not have to be specified precisely at this stage. Quite generally, we have

\[
G_j = G_j(p_1, p_2, p_3, \sum (\pi_j - p_j)E_j) \quad j = 1, 2, 3.
\]

To complete the system, some assumptions about aggregate labor use and supply are needed. For the present, we assume that the labor force is completely mobile and fully employed, although those assumptions will be relaxed for some of the shadow price derivations in Section II. Let \( L \) be the total supply of labor available to the economy, and \( L \) the labor employed by the project (set initially at zero). Then full employment requires that

\[
\sum_{j=1}^{3} l_jX_j + L = \bar{L}.
\]

Now the seventeen equations (1)–(10), (12)–(18) describe the general equilibrium interdependencies of our stylized economy. However, there are twenty-two variables of interest, namely, \( X_1, X_2, X_3, E_1, E_2, C, C_1, C_2, C_3, G_1, G_2, G_3, p_1, p_2, p_3, \tau_1, \tau_2, w_1, w_2, w_3, w_4, \) and \( G \). If, therefore, any five of the variables listed above are specified, the remaining seventeen are determined by the model. For example, suppose that \( w_1, w_2, w_3, w_4, \) and \( G \) were given. Then the entire set of prices and tariffs becomes determined, and these, in turn, will be the prices that equilibrate the quantity flows \((X_1, X_2, X_3, \text{etc.})\) in the economy. The lesson to be learned from this is that for a given budget deficit and set of wages in the economy, there exists a specific set of prices and equilibrium quantity flows. For different \( w_i \)'s and \( G \), a different set of \( p_i \)'s, \( X_i \)'s, etc. will obtain, so that conclusions from comparative static exercises, which involve variations in some exogenous variables or parameters, have to be interpreted cautiously.

II. MACROECONOMIC ADJUSTMENT WITH CONSTANT DOMESTIC PRICES

Now, if world prices and tariffs do not change when the project is introduced, the domestic prices of tradeables will not change either.

2. To be sure, tariff revenue is earned on gross imports or exports, whereas here we have defined \( E_j \) as net exports. Our assumptions on the technology ensure that there is full specialization in this economy, so that any particular tradeable commodity is either only exported or only imported.
Hence, if equilibrium is unique, wage rates and the price of the non-tradeable will be unaffected, too. As the technical coefficients depend only on prices, the model (1)-(18) may be stated in differential form, allowing only the quantities to change. We have

\[
\begin{align*}
(1')-(3') & \quad dX_i = \sum_{j=1}^{3} a_{ij} dX_j + dC_i + dE_i + dG_i + dQ_i \\
(4') & \quad dC = \sum_{j=1}^{3} w_j l_j dX_j + w_4 dL \\
(5')-(7') & \quad dC_i = b_i dC/p_i \\
(14') & \quad dG = \sum_{j=1}^{2} (\pi_j - p_j) dE_j - \sum_{j=1}^{3} p_j dG_j - \left(\sum_{j=1}^{3} p_j dQ_j + wdL\right) \\
(15')-(17') & \quad dG_i = dG_i(\cdot) \\
(18') & \quad \sum_{j=1}^{3} l_j dX_j + dL = 0,
\end{align*}
\]

which is a system of twelve linear equations in fourteen unknowns; namely, \((dX_i,dE_i,dC_i,dG_i)\); \(i = 1,2,3\). The differentials \((dQ_1, \ldots , dQ_3,dL)\) are exogenously specified, as they represent the project's inputs and outputs.

Before choosing the set of variables whose values are to be fixed exogenously, we need to make specific assumptions about the workings of the labor market. Suppose, first of all, that there is full employment, so that equation (18') holds. Even though the output of nontradeables will, in general, change with the introduction of the project, this implies that the output of tradeables is also endogenous. Here, we assume that the output of good 1 does not change \((dX_1 = 0)\), so that \(dX_2\) is the endogenous variable. This assumption amounts to treating sector 2 as a source of labor to the rest of the economy. Hence, output in this sector contracts as labor is drawn off in response to project-induced expansion in other sectors. It is also natural in the context of this paper to assume that the wage rate in the source sector is lower than those ruling elsewhere, which will be taken to be identical.\(^3\) These assumptions are close to the spirit of Little and Mirrlees, and also have the merit of simplicity.

\(^3\) As noted in Section 1, since the set of initial conditions defines the economy, a specific set of \(w_i\)'s, such as that just formulated, amounts to choosing a particular initial equilibrium. Choosing another set—for example, a uniform wage everywhere—will therefore define a different economy, so that comparisons of the two must be made cautiously.
The second labor market regime considered below can be formulated as a restatement of (18'):

\[(18^\prime) \quad \sum_{j=1}^{3} l_j dX_j + dL + L_p = 0.\]

Here, there exists a limited pool of unemployed labor \( L_p \), all of which is drawn into employment following the introduction of the project. However, if the additional demand for labor induced by the project is less than \( L_p \), there will be no contraction of employment and output in the source sector \( dX_2 = 0 \), and in this event, \( (18^\prime) \) is dropped. This case is analyzed later in this section.

We note that, by definition, \( dE_3 = 0 \), and the system has the same number of unknowns as equations. We shall now derive shadow prices for two cases in which the government's response to the introduction of the project leaves domestic prices and wage rates unchanged. For the first of these cases, we also analyze different labor market regimes.

**Little and Mirrlees: Endogenous Foreign Borrowing**

In Little and Mirrlees the numéraire is uncommitted government income, freely convertible into foreign exchange. It will now be shown below that adopting this numéraire is equivalent to assuming that when a new project is introduced, the government does not change the bundle of goods it was already purchasing. Hence, when all other adjustments in response to the project are complete, the resulting changes in the net demand for resources in the economy are financed by a change in net foreign borrowing. If the project is sufficiently small, the marginal cost of foreign borrowing will be unaltered. Furthermore, if the level of foreign borrowing were optimal, a small change would not disturb the conditions for optimality. Moreover, in this case, the government's response introduces no new distortions into the economy; for resource allocation is unaffected by its response.

The change in national income in the economy described by \( (1') - (18') \) is

\[(19) \quad dY = dC + \sum_{j=1}^{2} (\pi_j - p_j) dE_j - \left( \sum_{j=1}^{3} p_j dQ_j + wdL \right),\]

the change in government income being the change in tariff revenue.

---

4. By "optimal" foreign borrowing we mean a level that maximizes an intertemporal objective function. This objective function is necessarily distinct from the one-period social welfare function \( U(\cdot) \) in equation (21). For the questions addressed in this paper, it is sufficient to confine the analysis to a one-period world. In order to calculate a complete set of shadow prices, the intertemporal problem must be solved too.
plus the profits generated by the project, which is a public sector undertaking. By a fundamental national income accounting identity, we know that this is equal to the change in total expenditures by domestic institutions,

\[
dC + \sum_{j=1}^{3} p_j \, dG_j,
\]

net of foreign borrowing,

\[
- \sum_{j=1}^{2} \pi_j \, dE_j.
\]

Thus,

\[
dY = dC + \sum_{j=1}^{3} p_j \, dG_j + \sum_{j=1}^{2} \pi_j \, dE_j.
\]

If \(dG_j = 0, j = 1,2,3\), it is seen at once from (19) and (20) that the change in (uncommitted) government income is indeed the change in its foreign assets, which is endogenous.

Now a unit of extra income accruing to households and government, respectively, may not be equally valuable in the sense of making the same contribution to social welfare. Hence, looking at the effect of a project on national income alone is inadequate; we need a social welfare function \(U(-)\), which gives appropriate weight to the components of national income. For small changes, \(U(-)\) may be linearized in the neighborhood of the initial equilibrium, so that the social valuation of the change in national income may be written as

\[
dU = dG + (1/s) \, dC,
\]

since

\[
dG = \sum_{j=1}^{2} \pi_j \, dE_j
\]

is also the change in government income. This is equivalent to assuming that \(U\) has two arguments, \(G\) and \(C\). With \(G\) as numéraire, \(\partial U/\partial G\) is set equal to unity and \(\partial U/\partial C = 1/s\), so that a unit of \(G\) is \(s\) times as valuable as a unit of \(C\). Moreover, in the neighborhood of the initial equilibrium, \(s\) is constant.

As \(dG_j = 0\), the system (1')-(18') reduces to nine linear equations in nine unknowns \((dX_2, dX_3, dC_1, dC_2, dC_3, dE_1, dE_2, dG)\), from which expressions can be obtained for \((dE_i, dC)\) in terms of \((dQ_i, dL)\).

5. If the distribution of income between households and government is thought to be optimal (or no business of the analyst), then \(s\) is by definition equal to unity. The results that follow can be trivially specialized to this case.
Now, the shadow prices of good $i$ and labor, respectively, are given by

$$(-\frac{\partial U}{\partial Q_i}, -\frac{\partial U}{\partial L}).$$

(The theoretical rationale for this is presented in the Appendix.) Thus, substituting the expressions for $(dE_i, dC)$ in terms of $(dQ_i, dL)$ into (21), some tedious manipulation yields

\begin{align*}
(22) & \quad p^*_1 = \pi_1 \\
(23) & \quad p^*_2 = \pi_2 \\
(24) & \quad p^*_3 = \mu \left\{ \pi_1 a_{13} + \pi_2 a_{23} \right\} + \frac{l_3}{l_2} \left[ \pi_1 \left( \frac{b_1}{p_1} \right) + \pi_2 \left( \frac{b_2}{p_2} \right) - \frac{1}{s} \right] \Delta w \\
& \quad + \frac{l_3}{l_2} \left[ \pi_2 - (\pi_1 a_{12} + \pi_2 a_{22}) \right];
\end{align*}

\begin{align*}
(25) & \quad w^* = \mu (1 - a_{33}) \left[ \left\{ \pi_1 \left( \frac{b_1}{p_1} \right) + \pi_2 \left( \frac{b_2}{p_2} \right) \right\} + \frac{(\pi_1 a_{13} + \pi_2 a_{23}) (b_3/p_3) - 1}{s} \right] \Delta w \\
& \quad + \frac{1}{l_2} \left[ \pi_2 - (\pi_1 a_{12} + \pi_2 a_{22}) - \frac{(\pi_1 a_{13} + \pi_2 a_{23}) a_{33}}{1 - a_{33}} \right],
\end{align*}

where $\Delta w$ is the wage difference between the source sector and the rest of the economy, $(b_i/p_i)$ is the quantity of good $i$ consumed out of an extra unit of household income, and $\mu = 1/[1 - a_{33} - \Delta w l_3 (b_3/p_3) + a_{32} l_3/l_2]$.

Here, the shadow prices of traded goods are indeed their respective world prices. For if the government purchases a unit of a tradeable good, the nature of the economy's adjustments thereto under the assumptions adopted above imply that the government's net income will fall by the world price of that good, while leaving private income (consumption) unchanged.

Turning to $p^*_3$ and $w^*$, we see that it is necessary first to interpret $\mu$. Suppose that the final demand for the nontraded good rises by one unit (output in this sector is wholly demand-driven). At the first round, the own-intermediate demand for this good rises by $a_{33}$ units, and the $l_3$ workers drawn off from sector 2 spend a fraction $b_3$ of their additional earnings ($\Delta w l_3$) on nontradeables. However, there is an attendant fall in output in the "source" sector, so that its intermediate demand for good 3 also falls—by $a_{32} l_3/l_2$ to be exact. Subsequent rounds follow in train, and $\mu$ is simply the value of the multiplier corresponding to this adjustment process. Next, consider the term $\mu (1 - a_{33})$, which may be rewritten as $[1 + \mu (\Delta w l_3 (b_3/p_3) - a_{32} l_3/l_2)]$. Suppose that the final demand for labor, i.e., the project's demand
for labor, rises by one unit. Initially, this will cause the demand for nontraded goods to rise by 
\((\Delta w(b_3/p_3) - a_{32}/l_2)\), with an associated rise in employment in sector 3 of 
\((\Delta w l_3(b_3/p_3) - a_{32} l_3/l_2)\). Hence, 
\(\mu(1 - a_{32})\) is the total (direct and indirect) labor drawn off from sector 2 for each unit of labor required directly by the project.

As we shall now show, the terms in braces in (25) all refer to the direct consequences of drawing off one worker from sector 2. That being so, the reason for scaling those terms by the appropriate “employment multiplier” is at once clear. Recall that any net use of foreign exchange implies an identical fall in uncommitted government income, which is the numéraire. Thus, the first term in braces is the net direct opportunity cost (in terms of the numéraire) of employing an extra man in the project arising out of his gain in consumption \(\Delta w\). The second term is the direct loss of foreign exchange resulting from the fall in output in sector 2 brought about by shifting one worker out of that sector. Equivalently, it is value added per worker, where inputs and outputs are valued at world prices.

The various terms entering into the expression for the shadow price of the nontraded good may be explained in a similar vein. But it is also worth noting that this shadow price is equal to the unit shadow cost of production, which is as it should be, given our assumption that average and marginal costs are equal.

At this point, it is natural to ask whether the shadow prices for the nontradeable and labor are the same as their counterparts in Little and Mirrlees. First, a small change in definition is needed. L-M write, concerning the source sector: “If inputs . . . are important, one should subtract them before calculating average productivity” [p. 277]. But if market prices are constant, drawing off a worker will also lead to an appropriate fall in intermediate inputs, which must therefore appear in the calculation. Hence, if what L-M term the “marginal productivity of the wage earner” [p. 271] is defined to be

\[ m = \pi_2 - (\pi_1 a_{12} + \pi_2 a_{22} + p_3^* a_{32}) \]

where \(p_3^*\) is the shadow price of the nontradeable as formulated by L-M, their shadow wage rate is given by

\[(25') \quad w^{**} = \frac{1}{l_2} \left[ \pi_2 - (\pi_1 a_{12} + \pi_2 a_{22} + p_3^* a_{32}) \right] + \left[ \pi_1 \left( \frac{b_1}{p_1} \right) + \pi_2 \left( \frac{b_2}{p_2} \right) + p_3^* \left( \frac{b_3}{p_3} \right) - \frac{1}{s} \right] \Delta w. \]

6. Note that only multiplier effects due to interindustry linkages appear here, employment effects being captured by the scalar \(\mu(1 - a_{32})\).

7. This requires that \(p_3^*(1 - a_{32}) = \pi_1 a_{13} + \pi_2 a_{23} + w^* l_3\). From (25) and the fact that \(\mu(1 - a_{32}) = 1 + \mu(\Delta w l_3(b_3/p_3) - a_{32} l_3/l_2)\), it is easily checked that this indeed holds.
It is readily shown that \( w^{**} = w^* \) if and only if \( \pi_3^* = \pi_3^* \). Now since
\[
p_3^*(1 - a_{33}) = \pi_1 a_{13} + \pi_2 a_{23} + w^{**} l_3,
\]
substitution into (25') reveals that \( w^{**} = w^* \). Hence (24) and (25) are the “closed form” solutions for the shadow prices of the nontradable and labor as formulated, but not actually derived, by L-M. These solutions also make it transparent why L-M’s claim that their “methods ... take care of the multiplier effect” [p. 272] is correct.

We turn now to an examination of how alternative labor market regimes affect these shadow prices, assuming that the government adjusts to the project solely by varying its foreign exchange balances. First, suppose that there is no wage gap (\( \Delta w = 0 \)). Then household income does not change, so that the shadow price of labor is simply the total fall in value added (measured at world prices) resulting from the employment of one worker in the project. (This is seen by setting \( \Delta w = 0 \), in (24) and (25).) Clearly, if value added in the source sector measured at world prices is negative, then so too is the shadow price of labor. This is a specific instance of a proposition in Bhagwati, Srinivasan, and Wan [1978], although it should be pointed out that their proof assumes equal numbers of primary factors and goods. Naturally, if output in the source sector is produced by unassisted labor \( (a_{12} = 0) \), the shadow price of labor is always positive. However, if the marginal productivity of labor in the source sector is zero \( (l_2 = 0) \), then the absence of a wage gap implies that the shadow price of labor is also zero. In the more general case of a positive wage gap and positive value added per worker (measured at world prices) in the source sector, the shadow wage rate may still be negative if the marginal social valuation of private consumption relative to the numéraire is sufficiently high.

Now consider what is probably the polar case to the one considered above: where the project-induced demand for labor is met entirely by a pool of unemployed workers. To say these workers were previously unemployed is equivalent to saying that the “source sector” suffers no loss in output when they are drawn off by the project. In terms of (27) this implies that \( 1/l_2 = 0 \), so that the shadow wage rate in this regime is given by
\[
(25'') \quad w^* = \mu(1 - a_{33}) \left[ \pi_1 \left( \frac{b_1}{p_1} \right) + \pi_2 \left( \frac{b_2}{p_2} \right) \right] + \frac{(\pi_1 a_{13} + \pi_2 a_{23})}{1 - a_{33}} \left( \frac{b_3}{p_3} - \frac{1}{s} \right) \Delta w.
\]
This is, in fact, the shadow wage derived by Blitzer, Little, and Squire [1977], although they do not state explicitly their assumptions about
the labor market. As would be expected, the shadow wage in this case simply reflects the increased foreign exchange costs and consumption benefits of employing on the project one previously idle laborer.

It may be tempting to assert that, since the absorption of an extra worker has no effect on output in this case, the shadow wage must be lower than in the basic case. While this may be so, two words of caution are warranted. First, as already noted, the shadow wage rate in the basic case will be negative if value added in sector 2 is negative at border prices and there is no wage gap.

However, there is a more fundamental problem in comparing shadow wage rates in the L-M and “unemployed pool” cases. As emphasized in Section I, the initial equilibrium of the economy is given by the solution to a set of simultaneous equations, which determine market prices, outputs, and interindustry coefficients. Now an economy with “pool labor” starts from an equilibrium that is derived on the assumption that there is not full employment, so that equation (18) is dropped. Hence, the initial conditions of this economy will be different from those of the L-M case, for they are specified by a different set of equations. Thus, market prices, technical coefficients and, most important, the social valuation of an additional unit of private consumption are not the same in (25”) as they are in (25). Any comparison between the two shadow wage rates requires, therefore a comparison involving the initial equilibria of the two economies, a task that is beyond the scope of this analysis.

No Change in Foreign Borrowing

For various reasons, it may happen that the economy faces a foreign borrowing constraint in the sense that, ex ante, the trade deficit cannot exceed a certain level. If the constraint binds—it must do so for the case to be interesting—the introduction of a project must leave the level of the trade deficit unchanged when valued at world prices. Hence, if the government cannot “make room” for the project by levying lump sum taxes, it must do so by altering the level or composition of its existing outlays. As one would expect, it turns out

8. The system is still determined because although there is one equation fewer, \( X_2 \) is now set exogenously.
9. A third possibility is that the project-induced demand for labor is so large that it exhausts the pool of unemployed workers, and then draws extra workers from the source sector to satisfy the residual demand. It is intuitively plausible that the shadow wage rate will not be independent of the scale of the project. As this case is a combination of the basic case and the “pool only” case described just above, the shadow wage rate will be a weighted average of the shadow wages derived in these other two regimes, with the weights depending on the scale of the project relative to the size of the pool. This nonstationarity of shadow prices is of interest because the literature appears to focus on situations where the project is so large that its output affects prices in the markets where it is sold. Here we are considering the case where the size of the project’s demands for inputs affects their average opportunity costs.
that the binding constraint on foreign borrowing is associated with a shadow exchange rate.

In terms of the above model, leaving foreign borrowing unchanged amounts to setting \( dG = 0 \), so that some combination of the \( dG_j \) must be endogenous, while leaving domestic prices unchanged. From (14'), (19), and (20), the change in government income equals the change in government outlays, namely, \( \sum_j p_j dG_j \), so that the social valuation of the change in national income is

\[
(26) \quad dU = \sum_j p_j dG_j + \frac{1}{s} dC.
\]

It is important to note that since both the initial equilibrium quantity flows and the set of prices are the same as those in the L-M case analyzed above, the value of \( s \) is also the same, even though the macroeconomic adjustments to the introduction of a project differ in the two economies.

By way of illustration, let the government’s marginal expenditure decisions be as follows: \( dG_2 = 0 \), and marginal outlays on goods 1 and 3 are made in fixed proportions. We have

\[
(27) \quad \frac{p_1 dG_1}{p_3 dG_3} = \frac{\phi}{1 - \phi'}
\]

a formulation that captures the basic point that any variations in government outlays will fall partly on the demand for nontradables.\(^{10}\) Proceeding as before, further manipulation yields the following shadow prices:

\[
(28) \quad \hat{\beta}_1 = \theta \pi_1
\]

\[
(29) \quad \hat{\beta}_2 = \theta \pi_2
\]

\[
(30) \quad \hat{\beta}_3 = \theta p_3 + (\theta - 1) \mu \Delta w / s
\]

\[
(31) \quad w^* = \theta w^* + (\theta - 1) \mu (1 - a_{33}) \Delta w / s,
\]

where

\[
\theta = \left[ 1 + \left( \frac{1 - \phi}{p_3} \right) \frac{\mu \Delta w l_3}{s} \right] \left( \frac{\phi \pi_1}{p_1} + \frac{1 - \phi}{p_3} \right) \mu \left( \pi_1 a_{13} + \pi_2 a_{23} \right) l_3 + \frac{1}{l_2} \Delta w l_3.
\]

Clearly, these shadow prices closely resemble their counterparts in the L-M case, the sole difference being that in each case the “border

\(^{10}\) By adding (1')-(4') and using (5')-(7') and (27), it is easily shown that \( \Sigma_{j=1}^n \pi_j dE_j = 0 \).
price content" is scaled by the parameter $\theta$, whereas the terms dealing with the social value of additional household income is not. Hence, it appears that $\theta$ is a shadow exchange rate, and we shall now argue that this is indeed the case. Whenever the government makes an outlay for a new project, it must cut its existing expenditures to maintain balance of payments equilibrium without resorting to new foreign borrowing. Now, the denominator in the expression for $\theta$ is the direct and indirect savings of foreign exchange from a unit fall in existing outlays. Similarly, the numerator is the corresponding fall in social welfare (measured in terms of the numéraire). Hence, $\theta$ is the increase in social welfare made possible by a unit increase in the foreign exchange available to the economy. In effect, it is the dual variable associated with the balance of payments constraint.

Although the ratio of the shadow prices of traded goods is equal to the ratio of their respective world prices, the two sets of shadow prices will be identical (up to a scalar multiple) if and only if either there is no wage gap, or $\theta$ is unity. Thus, in general, a project that passes the test of social profitability in the L-M case may not do so in the present one, and conversely. In the absence of a wage gap, private consumption will not be affected by the introduction of a project. Hence, the government's response does not change the force of existing distortions in the economy, and the L-M rules continue to hold.

Where the shadow exchange rate is concerned, one of the special cases of some interest is that in which $\phi$ is unity. Here, $\theta$ equals $(p_1/\pi_1)$, so that the shadow price of good 1 is its domestic price. This is as one would expect, since if the government adjusts its purchases of one tradeable alone, that will be the numéraire good. If, further, the numéraire good is not subject to a tariff, so that variations in government outlays are exactly equivalent to changes in foreign borrowing, $\theta$ equals unity, and the shadow wage rate is indeed given by (25). Second, if all adjustments are borne by changes in the governments' outlays on the nontraded good, the shadow exchange rate is the ratio of (i) the net supply available from an extra unit of gross output of the nontraded good (after allowing for interindustry adjustments and private spending out of the extra value added thus generated) plus the social value of the extra value added to (ii) the foreign exchange content of that extra output and consumption.

A comparison of these two macroeconomic adjustments, i.e., endogenous and fixed foreign borrowing, respectively, sheds some light on the exchange between Balassa [1974] and Scott [1974]. Scott, a distinguished practitioner of the L-M approach, argued that there are, in effect, as many shadow exchange rates as nontraded goods: each
nontradeable has its own shadow price. In this connection, we have shown why Balassa's argument [p. 161], that "the" shadow exchange rate need not be estimated when deriving shadow prices through the L-M approach, under the assumptions employed here, is correct: the shadow exchange rate is unity. However, if no additional foreign borrowing is allowed, which seems to be the case Balassa has in mind, a shadow exchange rate is certainly needed, unless there is no wage gap or the marginal social valuation of private consumption is zero. Thus, it would appear that the main source of the dispute is that the two authors were making different assumptions about the way in which the economy adjusts to a project.

III. MACROECONOMIC ADJUSTMENT WITH ENDOGENOUS TARIFFS

If changes in foreign borrowing and the existing bundle of goods purchased by government are both ruled out, then the introduction of a project must be accompanied by changes in tariffs or quantitative restrictions on trade. In either case, there will be changes, not only in the domestic prices of the tradeables in question, but also in the prices of nontradeables and wage rates. The case of quantitative restrictions will not be pursued here, there being a full analysis in Bhagwati and Srinivasan [1979]. Instead, it will be assumed that the ad valorem tariff on good 2 is constant, implying that its domestic price is likewise, while the tariff on good 1 is determined endogenously. This is, in effect, a dual exchange rate system, in which one rate adjusts to maintain balance of trade equilibrium in the face of changes in the level of domestic absorption.

To keep the formulation close to that in Section II, the output of good 1 will be held fixed. Moreover, for simplicity, we assume that the set of technical coefficients of production \( a_{ij}, d_j \) are fixed. However, the demand system certainly permits substitution effects, so that following the introduction of a project, equilibrium is reestablished through substitution in private consumption as well as changes in real private incomes.

11. This adjustment mechanism resembles those employed by Bacha and Taylor [1971] in their case of an "equilibrium (without tariff) exchange rate" and by Dornbusch [1974]. Bacha and Taylor have only tradeables in their model, so that finding the equilibrating tariff is akin to pinpointing their equilibrium exchange rate. Dornbusch's model is more general, since it allows for nontradeables as well as cross-price effects in consumption. However, he simply asserts that the price of the nontradeable will adjust to equilibrium; in our case, we break down this change in price into two parts, both of which affect unit costs of producing the nontraded good; namely, (i) a change in the wage rate, and (ii) a change in the tariff on one of the tradeable inputs.

12. If there is no change in foreign borrowing, of course, then this latter condition is redundant.
The complete system has the following form. Equations (1')–(3') are retained, with \( dG_i = 1 \), \( 2 \), \( 3 \) = 0 in this characterization and \( dX_1 = 0 \) as a specific assumption. However, there are now changes in wage rates, so that

\[
(4') \quad dC = w_2 l_2 dX_2 + w(l_3 dX_3 + dL) + (dw_2 l_2 X_2 + dw l_3 X_3),
\]

since \( L \) is initially zero. Similarly, we have

\[
(5')-(7') \quad dC_i = \left[ \frac{b_i}{p_i} \right] \left[ \left( \frac{dC - \sum_j dp_j d_j}{C - \sum_j p_j d_j} \right) \left( \frac{dp_i}{p_i} \right) \right], \quad i = 1,2,3.
\]

Also,

\[
(8')-(10') \quad dp'[I - A] = (dw, dw_2, dw_3)' \hat{l},
\]

where \( A = \|a_{ij}\| \) and \( \hat{l} = \text{diag}(l_j) \). Since, by assumption, \( dp_2 = 0 \), (8') and (10') may be solved to yield \( dp_i = \lambda_i dw, i = 1,3 \). Hence, from (9'), \( dw_2 = -\nu dw \), where \( \lambda_i \) and \( \nu \) are positive constants depending only on \( |a_{ij}, l_j| \). The wage gap between the "source" sector and the rest of the economy is, therefore, endogenous; in the presence of the project, it may grow wider or narrow. There is also balanced trade:

\[
\sum_{j=1}^{2} \pi_j dE_j = 0,
\]

and full employment, expressed by equation (18'). Hence, we have a system of twelve linear equations in twelve unknowns \( (dX_2, dX_3, dC_1, dC_2, dC_3, dC, dE_1, dE_2, dw, dw_2, dp_1, dp_3) \).

Given that the physical bundle of commodities purchased by the government is fixed, the change in social welfare brought about by the project is equal to the change in the welfare of households arising from its effects on prices and nominal incomes. And since prices change, albeit only infinitesimally for "small" projects, it is prudent to start with the direct utility function for households. With our specialization to a linear expenditure system, the social "profit" from a project is

\[
(32) \quad dU = \sum_i b_i dC_i, \quad C - \sum_i p_i d_i,
\]

where, by virtue of the linearity of the above system, the \( dC_j \) are linear in the exogenous variables that represent the project, namely, \( (dQ, dL) \). Now, for small changes in prices, the terms \( b_i/(C - \sum_j p_j d_j) \) are locally constant. It follows at once that the shadow prices for the
system \(-\partial U/\partial Q_i, -\partial U/\partial L\) are independent of \((dQ_1,dL)\); as in the L-M case, they depend only on technical and consumer demand parameters and world prices.

In order to arrive at a more precise characterization, it is necessary, as before, to wade through some dreary algebra. As \(dw_2 = -\nu dw\), and recalling (18'), we have

\[
dC = -\Delta w l_2 dX_2 + (l_3 X_3 - \nu l_2 X_2) dw,
\]

so that

\[
dC_i = -(b_i/p_i)[\Delta w l_2 dX_2 + \xi_i dw] \quad i = 1,2,3,
\]

where

\[
\xi_i = (\nu l_2 X_2 - l_3 X_3) + \sum_j \lambda_j d_j + \left(C - \sum_j p_j d_j\right) \frac{\lambda_j}{p_i}.
\]

The term \(\xi_i\) captures the effects of changes in incomes and prices on the demand for good \(i\) arising from a unit change in the wage rate in sectors 1 and 3.

Using (3') and (18') to get an expression for \(dX_2\) in terms of \(dw\), \(dQ_3\), and \(dL\), then substituting for \(dX_2\) and \(dX_3\) in (3') and (4'), and noting (11'), we obtain, at length,

\[
dw = \rho_p \left[ \pi_1 dQ_1 + \pi_2 dQ_2 + \gamma_3 dQ_3 + \gamma_4 dL \right],
\]

where

\[
\rho = 1/[\xi_1 \pi_1 (b_1/p_1) + \xi_2 \pi_2 (b_2/p_2) + \xi_3 \gamma_3 (b_3/p_3)],
\]

\[
\gamma_3 = \rho \gamma_3 + \mu \Delta w/s,
\]

\[
\gamma_4 = w^* + \mu (1 - a_{33}) \Delta w/s,
\]

\(\rho\) and \(w^*\) being given by (24) and (25), respectively. Thus, \(\gamma_3\) and \(\gamma_4\) are the "border price contents" of the nontradeable and labor, so that \(1/\rho\) is the "border price content" of a unit change in the wage rate in sectors 1 and 3.

Recalling (32), we note that the terms \(b_j/(C - \Sigma_j p_j d_j)\) are equal to \(p_j\), respectively, in the equilibrium ruling before the project was introduced. Hence, if the project is sufficiently small, we have, on substituting for \(dX_2\),

\[
dU = \sum_j p_j dC_j = -\left[\xi_1 b_1 + \xi_2 b_2 + \xi_3 b_3 \left(1 + \mu \Delta w l_3 / p_3\right)\right] dw
\]

\[
+ \mu \Delta w l_3 dQ_3 + \mu (1 - a_{33}) \Delta w dL.
\]
This yields the following shadow prices:

\( \hat{p}_1 = \psi \pi_1 \)  
\( \hat{p}_2 = \psi \pi_2 \)  
\( \hat{p}_3 = \psi p_3 + (\psi/s - 1)\mu \Delta w \)  
\( \hat{w}^* = \psi w_3 + (\psi/s - 1)\mu(1 - a_{33})\Delta w, \)

where \( \psi = \rho[\zeta_1 b_1 + \zeta_2 b_2 + \zeta_3 b_3(1 + \mu \Delta w/l_3/p_3)] \). Like \( \theta \) in Section III, \( \psi \) is a shadow exchange rate. However, under these circumstances, it features the demand parameters for households rather than government, since the pre-existing bundle of goods and factors consumed by the government does not change. As before, if there is no wage gap \( (\Delta w = 0) \), each of these shadow prices is a scalar multiple of its counterpart in L-M. However, if there is a wage gap, then the two sets of shadow prices are identical (up to a scalar multiple) if and only if \( \psi = s \), which will hold only by a fluke. Nevertheless, it remains the case that the ratio of the shadow prices of the two tradeables is equal to the ratio of their border prices, even though domestic prices shift (slightly) in response to a project.

IV. Conclusions

Although our analysis has not dealt with all possible cases, two general conclusions may be drawn with some confidence.

If the only distortion is the presence of ad valorem tariffs, the L-M rules for deriving shadow prices are always correct, for sufficiently small projects, even if equilibrium following the introduction of a project is not reestablished through additional foreign borrowing. For under these circumstances, the shadow exchange rate will simply scale the whole vector of L-M shadow prices. What is more, if tariffs do not alter in the face of a project, so that domestic prices stay constant, then these rules also hold good for “large” projects provided that the marginal cost of foreign borrowing is constant over the relevant range. This follows from the Rybczynski-line properties of the model when adjustment takes this general form, a point emphasized by Srinivasan and Bhagwati [1978] in connection with a somewhat different model.

But what if there is also a distortion in a factor market, which we have represented in the form of a wage gap between the source sector and the rest of the economy, the presence of which implies that private consumption will change with the introduction of a project? Then it
certainly matters how the economy adjusts to a project, in the sense that the vector of shadow prices now depends on the form of the adjustment. If additional foreign borrowing is possible, then the L-M rules stand. This may well be the case, especially if the government has been pursuing what potential creditors perceive to be prudent policies. But instances of “recklessness” or just plain bad luck are by no means uncommon, and in such cases the level of foreign borrowing will be given exogenously. This being so, the L-M rules will still hold only if the government is able to bring about appropriate changes in private consumption by means that are not distortionary, such as lump sum taxes.

Yet our analysis of different forms of distortionary adjustment leads us to two conclusions. First, the principles involved in the derivation of appropriate shadow prices do not depart in any radical way from those used in the L-M case. Second, as a purely practical matter, the computation of shadow prices for the various cases requires virtually the same information and the same tool, namely, matrix inversion.

In general, since the rationale for using shadow prices is to decentralize decision-making in a way that preserves macroeconomic consistency, it is impossible to estimate such prices without making judgments about macroeconomic policy, even though individual projects may be small. In this context the particular point we have emphasized here is that the manner in which the government attempts to restore balance of payments equilibrium determines the appropriate set of shadow prices. As governments are frequently faced with the need to restore balance of payments equilibrium, a careful scrutiny of past behavior will usually give the analyst enough clues as to what is the appropriate assumption for deriving shadow prices.

APPENDIX

The purpose of this appendix is to lay out the theoretical foundations for the technique, used in the body of the paper, for estimating shadow prices when the equilibrium flows in the economy are specified.

Using a general social welfare function, we first establish an appropriate definition of the concept of a shadow price. We then show why a project’s social profit, i.e., the value of its direct outputs minus the value of its direct inputs (all calculated at shadow prices), measures the project’s contribution to social welfare.

Assume that there exists a differentiable function $U: \mathbb{R}^n \rightarrow \mathbb{R}$ which represents the government’s preferences over a set of variables.
denoted by the $n$-dimensional vector $Z = (Z_1, \ldots, Z_n)$. A project is described by small changes in the components of another ($m$-dimensional) vector $Q$, these changes being the physical inputs and outputs of the project. In principle, each of the $Z_i$'s depends on $Q$. Thus, we may write $Z$ as $Z = (Z_1(Q), \ldots, Z_n(Q))$, although in some cases, $\partial Z_j/\partial Q_i = 0$. If a project is denoted by $dQ = (dQ_1, \ldots, dQ_m)$, where inputs have a negative sign, then the change in social welfare due to a project $dQ$ is

$$dU(Z_1(dQ), \ldots, Z_n(dQ)) = \sum_{i=1}^{n} \frac{\partial U}{\partial Z_i} \frac{\partial Z_i}{\partial Q_j} dQ_j.$$

We can then define the shadow price of good $j$, $p_j$, as

$$(A2) \quad p_j = \sum_{i=1}^{n} \frac{\partial U}{\partial Z_i} \frac{\partial Z_i}{\partial Q_j},$$

so that the project's effect on social welfare is, indeed,

$$dU = \sum_{j=1}^{m} p_j dQ_j.$$

In words, the change in social welfare due to a project is the value of its outputs minus the value of its inputs, all valued at their shadow prices. One way of calculating shadow prices, therefore, is to solve for the changes in the $Z_i$'s induced by the project $(dQ)$ and then, using $(A2)$, set $p_j = \partial U/\partial Q_j$. Once these $p_j$'s are known, the government can evaluate projects simply by looking at each project's direct inputs and outputs $(dQ_1, \ldots, dQ_m)$ and calculating its social profit, namely,

$$p_j dQ_j.$$

When adopting this decentralization procedure for public investment decisions, the government can rest assured that a project which has a positive social profit will yield an increase in social welfare, as defined by $U(\cdot)$.

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