Graham Pyatt

Marginal Costs, Prices, and Storage
MARGINAL COSTS, PRICES AND STORAGE*

1. INTRODUCTION

Much of the recent literature on public utility pricing can be traced back to Williamson's important 1966 article on peak load pricing when the production of a commodity is subject to a capacity constraint. Subsequent work has extended this analysis in various ways and, in particular, a recent paper by Gravelle (1976) considers the modifications which are introduced by allowing that output of the commodity can be stored. This is also the subject of the present paper. However, the formulation differs from that of Gravelle in a number of respects so that the implications of storage possibilities emerge in a somewhat different light.

The first important aspect of the present formulation is to represent seasonality in demand and supply in continuous time. In common with Williamson and Gravelle, demand is assumed to be stationary from one year to the next and subject to a fixed seasonal pattern within the year. Production is also assumed to have a fixed seasonal pattern and to be constrained by fixed or exogeneously determined capital. This latter assumption implies that fixed investment decisions are not to be discussed here, so that attention is focused exclusively on production scheduling over time and the pricing problem. Within this frame of reference, the adoption of a continuous time formulation is in contrast with Williamson. Williamson (and Gravelle) assume that the year is divided into discrete time intervals so that demand (and supply) move in stepwise fashion from one level to another at some specific dates within the calendar year. The present paper attempts to show that this simplification can mask some important considerations when storage possibilities are allowed, especially when (a) the year is divided into only two intervals, and (b) the analysis does not question whether intervals defined by demand shift have a one to one correspondence with intervals defined by supply response.

Secondly, the present paper attempts to bring out the crucial importance of the shape of cost curves. With respect to production, marginal costs are taken as being constant by Williamson up to a level of output defined by capacity. This is contrasted in Fig. 1 with a curve which increases monotonically with output and tends asymptotically to infinity as output approaches some finite level. In this latter case, markets can always be cleared by setting price equal to marginal cost, irrespective of the level of demand. This is not so in the Williamson formulation: indeed the peak load pricing problem derives from the possibility that the demand curve may lie above the point B in Fig. 1 and so preclude market clearing with a price/output combination corresponding
questions depend crucially on the shape of production cost curves as will be discussed at a later point in the argument.

In addition to the treatment of time and cost curves, a third feature of the present approach is the separation of the issue of satisfying given demand at minimum cost from the question of what price should be set, and hence what quantity demands might be. The first of these problems is discussed in section 3 below, following the introduction of notation and definitions in section 2. In section 3 it is shown that any one of three roots to an equation can determine the time path of output, and hence of stocks, when demand is taken as given. This shows that a variety of solution regimes is possible, with the alternatives depending in part on the shape of the production cost curve. A necessary and sufficient condition for storage to be used for some interval of time is provided. This depends on the rate of increase of demand and not its level. Hence the role of storage emerges as regulating the rate at which output increases over time. Section 4 then explores some of the variety of situations which can be analysed given the general results of section 3. The discussion of production scheduling is completed in section 5 by considering a form of production cost curve proposed by Gravelle (1976), which is different from both the alternatives shown in Fig. 1. This section also discusses the relationship between the optimal solution for a given year derived in section 3, and the cost-minimising solution for a series of years.

![Fig. 1. The shape of cost curves.](image)

Questions of pricing policy are addressed in section 6. Efficiency prices are shown to imply that storage should be regulated by gains from stock appreciation. In a special case where storage costs are determined entirely by in-
four possible cases given by answers to the questions: is output at capacity?; and are stocks being held?

2. NOTATION, DEFINITIONS AND SOME PRELIMINARY INFERENCES

The quantity of a commodity demanded at time \( t \), and the amount produced are to be denoted by \( q \) and \( x \) respectively. Both are functions of time which it will be convenient to write explicitly as \( q(t) \) and \( x(t) \) on occasion. Similarly, \( s \) or \( s(t) \) denotes stocks held at time \( t \). Its derivative is given by definition as

\[
\dot{s} = x - q. \tag{1}
\]

At all points in time stocks must be non-negative, i.e.

\[
s \geq 0. \tag{2}
\]

Output, \( x \), is also subject to a non-negativity constraint. More importantly it is constrained to lie at or below a fixed capacity level, \( \bar{x} \), so that

\[
x \leq \bar{x}. \tag{3}
\]

For output levels within this constraint, a total production cost curve \( A(x) \) is defined such that marginal production costs, \( a(x) \), are non-negative and non-decreasing as a function of \( x \). Hence

\[
\frac{d}{dx} A(x) = a(x) \geq 0; \quad \frac{d}{dx} a(x) \geq 0 \text{ for all } x; \quad 0 < x < \bar{x}. \tag{4}
\]

From these convexity assumptions it follows that provided \( x, q < \bar{x}, \) then \( A(q) \) and \( a(q) \) are defined and satisfy the inequality

\[
\dot{s}a(q) = (x - q) a(q) \leq A(x) - A(q) \leq (x - q) a(x) = \dot{s}a(x). \tag{5}
\]

It is convenient to assume that time is measured in units of a year. The assumption that demand follows a fixed seasonal pattern can then be expressed as

\[
q(t) = q(t + 1) \text{ for all } t. \tag{6}
\]

An annual pattern of demand will be feasible, in the sense that it can be met by production within the year, provided that

\[
\int_{t}^{t+1} q(t) \, dt < \bar{x}. \tag{7}
\]

The strict inequality here implies that there will be excess capacity at some point during the year: an equality in place of (7) produces an obviously trivial problem for production scheduling since the only possible solution is to produce at full capacity at all moments. It should be noted, however, that (7) does not preclude \( q > \bar{x} \) for some \( t \) i.e. demand in excess of capacity at some points in
holding can vary seasonally as well as in consequence of changes in $s$. Secular changes in $B$ are excluded by specifying that

$$B(s, t) = B(s, t + 1) \quad \text{for all} \quad t.$$  

(8)

Stocks are here defined to refer to stocks of finished products, so that raw material stocks and work in progress are excluded. The cost function $B(s, t)$ is assumed to satisfy

$$B(0, t) = 0; \quad \frac{\partial}{\partial s} B(s, t) = b(s, t) > 0 \quad \text{for all} \quad s \geq 0 \text{ and all} \quad t.$$  

(9)

There are no fixed costs of stock holding; and the marginal cost of holding stock is assumed to be always positive. Together these assumptions imply that (increasing) stock holding always involves (increasing) costs. In addition, economies of scale in stockholding need to be excluded, i.e.

$$\frac{\partial b(s, t)}{\partial s} \geq 0 \quad \text{for all} \quad s, t.$$  

(10)

The above conditions are sufficient to ensure that under a cost minimising policy, there will be no net increase in stocks from one year to the next and that at some point(s) within every year, stocks will be at zero level. They reduce the problem of production scheduling over time to one of determining an optimal annual production schedule. This restricted problem is addressed in the next section.

3. ANNUAL PRODUCTION SCHEDULING

Given an annual time path for demand, $q$, the relation between production and stocks over any particular interval of time $(t_1, t_2)$ is given by

$$\int_{t_1}^{t_2} x \, dt = \int_{t_1}^{t_2} q \, dt + s(t_2) - s(t_1)$$  

(11)

where $s(t_1)$ and $s(t_2)$ are opening and closing stocks. These can be set at any level provided the right-hand side of (11) is less than $(t_2 - t_1) \bar{x}$, implying that the required output can be met by available capacity.

Satisfying exogenous demand, $q$, over the interval $t_1$ to $t_2$ now implies finding an output path $x$ and hence, from (1), a path for stock changes, $\dot{s}$, which satisfies (11) without violating either of the constraints (2) and (3). The optimum path for $x$ is the one which does this at minimum cost. It can be found by minimising total costs over the period $(t_1, t_2)$ in the form

$$\int_{t_1}^{t_2} [A(x) + B(s, t) + \alpha(\dot{s} + q - x) + \beta s + \gamma(x - \bar{x}) - cx] \, dt,$$  

(12)

where $\alpha$ is a Lagrangian multiplier associated with the equation constraint (1); $\beta$ and $\gamma$ are multipliers associated with the inequality constraints (2) and (3); and $c$ is a constant associated with the integral constraint (11).

Applying Euler’s condition to (12) yields necessary conditions for an optimum which can be written as

$$\alpha(t) = \frac{b(s, t) - \beta(t) - \gamma(t)}{\bar{x} - s(t, \bar{x})} = 0.$$  

(13)
i.e. a single equation in \( x \) and \( s \) which has three roots or solutions. It follows that, corresponding to different roots, the optimal solution will be characterised by different regimes. Moreover alternative regimes may hold within different subperiods of the interval \((t_1, t_2)\). The variety of solution paths may therefore be quite complex.

To find some pattern in this variety, it can be noted that for the Williamson production function in Fig. 1, it must always be the case that

\[ x = \bar{x} \text{ or } a(x) = \text{constant.} \quad (14) \]

Now, by assumption in the present analysis, \( b(s, t) > 0 \). Hence the root \( b(s, t) = a'(\bar{x}) \bar{x} \) cannot be satisfied for the Williamson production function. It therefore follows that in this case we must have

\[ s(x - \bar{x}) = 0, \quad (15) \]

i.e. stocks are zero or production is at the capacity level.

The simplified result (15) still allows that during a year there may be \( n \) periods when output is at capacity, and in principle \( n \) can be any positive integer. But essentially this is a trivial case which can be worked out in detail for any given demand path \( q \). In outline, the character of a solution will be such that if \( q \) is always less than \( \bar{x} \), then there will be no stock-holding at any time: demand will always be met by current production.\(^4\) Only if \( q \) exceeds \( \bar{x} \) for some \( t \) will stocks be held. They must be held for all periods in which \( q \) exceeds \( \bar{x} \), since these are times at which stocks must be running down \((\ddot{s} < 0)\), and to permit such run-downs, stocks must be built up in prior periods. The solution (15) tells us that these prior periods will be of minimal necessary length, i.e. if stocks are being built up so that \( s > 0 \), then output will be at the capacity level even though demand is below it. Finally, since these rules determine total production costs during a year, overall cost minimisation will be achieved by minimising the costs of stock-holding, i.e. by having periods of stock accumulation immediately prior to periods of stock decumulation.

Having disposed of the Williamson cost curve in this admittedly cursory fashion, it is useful to focus attention on the case where the optimal solution is characterised throughout by \( x < \bar{x} \). This is not the most general situation, but it covers the most interesting cases for present purposes and affords a basis for discussion of exceptions. Accordingly, the argument now proceeds under the assumption \( x < \bar{x} \) so that the cost minimising solution for period \((t_1, t_2)\) is given from (13) by

\[ s[b(s, t) - a'(x) \dot{x}] = 0. \quad (16) \]

Equation (16) can be described as the characteristic equation of a storage problem. It states that either stocks are zero or changes in output and the level of stocks are linked by the relationship

\[ a'(x) \dot{x} = b(s, t). \quad (17) \]
which implies

\[ a[x(\tau + \theta)] = a[x(\tau)] + \int_{\tau}^{\tau + \theta} b(s, t) \, dt. \]  

(18)

An interpretation of this result is as follows. The amount \( a[x(\tau)] \) is the marginal cost of producing an extra unit at time \( \tau \). The integral on the right-hand side of (18) is the marginal cost of storing this extra unit from time \( \tau \) to \( \tau + \theta \). Hence the right-hand side of equation (18) is the marginal cost of producing an extra unit at time \( \tau \) and holding it in store until time \( \tau + \theta \). The left-hand side of (18) is simply the marginal cost of producing an extra unit at time \( \tau + \theta \). Since equation (18) is to hold for all \( \theta \), provided stocks are being held, the result (18) states that optimal production scheduling over an interval of time in which stocks are held results in the marginal cost of supplying an extra unit at a particular point in time being independent of the point in time at which it is produced. An important point to note from (17) or (18) is that since the marginal costs of holding stocks are positive, marginal production costs must be rising over a period in which stocks are held.

Equations (1) and (17) provide two first-order differential equations in \( x \) and \( s \) for given \( q \). To obtain a complete determination of these variables it is necessary to fix their initial and terminal values. With respect to \( s \), such values are taken as exogenously given in equation (11) at levels \( s(t_1) \) and \( s(t_2) \) which are arbitrary subject to the requirement that the right-hand side of (11) is less than \( (t_2 - t_1) \bar{x} \).

If the period \( (t_1, t_2) \) is to be such that stocks are held throughout, with no stockholding prior to \( t_1 \) or after \( t_2 \), then at times \( t_1 \) and \( t_2 \), stocks must be zero, i.e.

\[ s(t) = 0 \quad \text{for} \quad t = t_1, t_2. \]  

(19)

Prior to \( t_1 \) stocks are zero by assumption and demand must be met by current production. Thus at each moment prior to \( t_1 \), say for the interval \( t_0 \) to \( t_1 \), total costs are given by \( A(q) \). This must also be the case for some interval subsequent to \( t_2 \), say \( t_2 \) to \( t_3 \). Hence total costs from \( t_0 \) to \( t_3 \) are given by

\[ \int_{t_0}^{t_1} A(q) \, dt + \int_{t_1}^{t_2} [A(x) + B(s, t)] \, dt + \int_{t_2}^{t_3} A(q) \, dt. \]  

(20)

The conditions for an optimal determination of \( t_1 \) and \( t_2 \) can now be derived by differentiating (20) with respect to \( t_1 \) and \( t_2 \). This yields the conditions

\[ A(x) + B(s, t) = A(q) \quad \text{for} \quad t = t_1, t_2. \]  

(21)

But by assumption, stocks are to be zero at times \( t_1 \) and \( t_2 \), and from (9), storage costs \( B(s, t) \) will be zero in both instances. Hence the conditions (21) can be met only by equating \( A(x) \) and \( A(q) \), i.e., by having
solution to the storage problem under diminishing returns. Fig. 2 illustrates
the solution in a particular case.

In Fig. 2 demand is assumed to follow an annual cycle which is reflected
in the time path for $a(q)$. Until time $t_1$ no stocks are held. Then at time $t_1$
stock accumulation begins with stocks rising throughout the period $t_1$ to $t^*$. During this period output must exceed demand to allow stock accumulation, and output must itself continue to rise as required by equation (17). At some point stocks must start to fall, and this moment in time is denoted $t^*$ in Fig. 2. Subsequent to it, output continues to rise as required by (17), but is less than demand. Hence stocks are falling and are reduced to zero by time $t_2$. Thereafter demand is met by current production until time $t_1 + 1$.

In generating a solution in a particular case with $q$ given, the crucial ques-
tion is to find the moment, $t_1$, at which stock holding begins. For if $t_1$ is given,
then the starting point for $a(x)$ is known from (22) while the initial value of $s$
is also given (as zero) by definition. The differential equations (1) and (17)
now determine complete time paths for $x$ and $s$, so that the system is fully
determined once $t_1$ is given.

If the correct choice of $t_1$ is made, then the time paths for $x$ and $s$ will meet
two conditions. First, they will satisfy $s \geq 0$ for all $t$: $t_1 < t < t_2$, and secondly,
they will lead to a point in time at which stocks are reduced to zero once more and output is equal to current demand. This is the moment $t_2$. If the time
and sufficient condition for some storage to be optimal at some time in the year is that
\[ qa'(q) > b(o, t) \text{ for some } t. \] (23)

This brings out the point that the question of whether it is efficient to use storage does not depend on the existence of peaks in demand but rather on how rapidly demand increases. The role of storage is to regulate the rate at which production costs increase, not their level.

4. ALTERNATIVE SCENARIOS AND THE CONDITIONS FOR STORAGE USE

The solution to the storage problem set out in the previous section provides some general rules which can be applied in a variety of actual situations. In particular it is important to demonstrate that the solution is not dependent on assuming a single-peaked seasonal demand pattern as shown in Fig. 2. Fig. 3 illustrates some alternatives.

In Fig. 3(a) demand has two seasonal peaks and an optimal solution is shown as involving two separate periods of stock holding during the year. This illustrates the fact that different solution regimes for equation (16) can recur in the annual cycle, so that a dichotomy of the year into a period of stock holding and a period with zero stocks is potentially inadequate. But multiple peaks in seasonal demand do not necessarily imply multiple periods of stock holding. Fig. 3(b) shows a single period of stock holding even though demand has two peaks. In this case the policy illustrated involves zero stocks at time \( t_1 \). Stocks then grow from \( t_1 \) to \( t_2 \) and decline from \( t_2 \) to \( t_3 \), but they are still positive at \( t_3 \), and grow subsequently from \( t_3 \) to \( t_4 \). Thereafter they decline once more and reach zero at time \( t_5 \). There is no reason why such a scenario cannot correspond to optimal policy. Nor is the occurrence of a peak sufficient for stockholding to be optimal if the demand curve is relatively flat in the sense that the condition (23) is never satisfied. This is illustrated by Fig. 3(c).

Fig. 3(a)-(c) suggests that the number of storage periods must be less than or equal to the number of demand peaks. A counter-example to this proposition is illustrated in Fig. 3(d). In this example, two steep segments of \( a(q) \) are separated by a relatively flat one. The figure illustrates the possibility that one peak of demand calls for two distinct periods of storage during the year. There is therefore no simple relationship between the number of demand peaks and storage periods. Moreover, the dependence of storage on the slope rather than the level of \( q(t) \) means that storage may not take place at the peak, and therefore does not necessarily reduce peak output. This is illustrated in Fig. 3(e) and (f).

Finally, Fig. 3(g) illustrates the fact that if demand exceeds capacity for some interval of time (implying that \( a(q) \) is not defined), or if, as is shown in Fig. 3(h), (e) in which some segments disappear, there may be a transition to the
5. SOME REMAINING ISSUES

The previous section has set out rules for production scheduling in the case of diminishing (or constant) returns to stock holding and a smooth production cost curve of the type illustrated in Fig. 1 as an alternative to the Williamson formulation. However, other forms of cost curve are obviously possible even

Fig. 3. Alternative scenarios for $a(x)$ in solving a storage problem.
The Gravelle formulation of the cost curve satisfies all the conditions needed for the result (13) to maintain. The rules for optimal production scheduling in this case may therefore be derived from it. They will lead to a production scheduling solution which may be no different from that discussed in section 3 under the assumption that \( x < \bar{x} \) at all \( t \). Clearly, \( x < \bar{x} \) for all \( t \) is a feasible solution, given the condition on annual demand set in (7). It will characterise an optimal solution if the rules of section 3 above can be applied, that is if equations (1) and (17) are consistent with terminal conditions as specified by (19) and (22).

Some characteristics of a solution path assuming the Gravelle cost function can be pieced together from previous results. Corresponding to the alternative solutions of (13), there may be periods of each of three types: (i) zero stocks, so that \( x = q \); (ii) optimal storage with \( x < \bar{x} \) on the lines of section 3 above; and (iii) periods of capacity production, \( x = \bar{x} \). In cases where \( q < \bar{x} \) throughout the year, capacity production will never take place. A solution according to the rules of section 3 is always feasible in this case, and is therefore the optimal solution.\(^1\) If \( q > \bar{x} \) for some part of the year, then periods of type (ii) and/or (iii) above must be part of the annual solution, but in what combination or relationship is an open question. All that is certain is that the final solution path of \( x \) will not satisfy the sufficient condition (23) for further storage to take place. There will therefore be no discontinuous increases in \( x \) in the final solution. This is because, in all cases, a solution path for \( x \), and hence for \( a(x) \), cannot contain any further opportunities for cost saving through storage if the solution is optimal. Hence, substituting \( x \) for \( q \) in (23), it is necessary for an optimal solution that the sufficient condition for storage given by (23) should not be met for any interval of time. In this sense an optimal solution path cannot involve any upward discontinuities in output.

A final issue on annual production scheduling concerns the assumption that in every year there will be a point of time at which stocks are zero, so that net stock changes over a 12-month period are zero also. This must be the case for optimal policy under increasing marginal costs since net stock accumulation in one year adds more to the costs of that year than can be saved in the following year through use of an inherited stock. More formally, previous arguments have shown that it is optimal to hold stock only when production is increasing: and that a period of stock holding is terminated optimally at a point when \( x = q \). Since demand is stationary from year to year in our formulation, this must mean that, starting from a position of zero stocks, any accumulated stocks must be used up within one year of the initial stockpiling: no other type of solution can be consistent simultaneously with the restriction on demand and the monotonic increasing output level which is a necessary characteristic of stock holding.
6. Pricing Policy

The general rule for efficiency pricing is that marginal cost should equal price if this allows markets to be cleared. The analysis presented above therefore suggests that when stock holding is feasible, prices, $p$, should be given by

$$p = a(x) \quad \text{for all } t,$$

(24)

irrespective of whether the stock holding capability is actually being used or not. Hence the alternative scenarios illustrated in Fig. 3 can be interpreted as solutions for prices as well as for output levels.

While this is the simplest implication of the analysis for price behaviour, it is both incomplete and of limited relevance. If demand at a point in time is not independent of prices at other points in time, then efficiency pricing as defined by equation (24) is not necessarily the most attractive solution. However, the questions for pricing policy now raised are not peculiar to the storage feature which is the central concern of this paper. These problems are therefore set aside. Similarly, it is not proposed to pursue here the problems which arise when costs are incurred in changing price, or when only a limited number of different prices are allowed during the year. These may be important aspects of particular cases, but treatment of them can add little to the discussion of main principles.

However, there are two aspects of demand structure and prices which are of some interest within the present frame of reference. The first concerns the possibility that consumers may have their own storage capabilities. The second concerns the role of stock appreciation.

With respect to consumer storage possibilities, the obvious implication is that consumers will have some flexibility in scheduling their demands over time, just as producers have flexibility in scheduling production. Under constant returns for the storage activity, the total of producer and consumer storage costs is independent of the volume stored and who stores it. Results in this case will not be influenced by the fact that consumers may have their own storage capability. But under decreasing returns, the implication of high marginal storage costs at peak storage periods is one of a relatively high rate of increase of prices under a centralised storage system. Consumers with their own storage capability can potentially save money by rescheduling their demands in such circumstances, and therefore are likely to do so.

On the second question of stock appreciation, it can be noted that when storage is optimal, equations (24) and (17) imply that

$$\dot{p} = b(s, t),$$

i.e. that the rate of price increase is equal to the marginal cost of storage. This result in turn implies that
be interpreted as the stock appreciation on such a unit. Hence equation (26) implies that stocks will be held up to the point where the marginal cost of holding an extra unit is equal to the stock appreciation on that unit. This then is an efficiency pricing rule in the stock holding context.

This last result can be brought closer to familiar results in capital theory by considering the case where storage costs are proportional to the amount stored, as when they depend simply on the opportunity cost, \( \lambda \), of working capital. Thus

\[
B(s, t) = \lambda ps.
\]

In this case, equation (25) reduces to

\[
\dot{p}/p = \lambda,
\]

i.e. the growth rate of price is equal to the rate of interest. In the simplified cases which allow equations (24) and (27), the result (28) defines the optimal time path for prices when storage is taking place.

The combination of the efficiency pricing rules (24) and (25) with the results of section 3 on optimal production scheduling permits a simple systematic presentation of the conclusions reached in this paper. In general there are four variables to be determined, namely demand, \( q \); output, \( x \); stocks, \( s \); and price, \( p \). Also there are four possible solution regimes depending on (i) whether output, \( x \), is at or below the capacity level, \( \bar{x} \); and (ii) whether stocks are being held or not. With four variables to be determined, four equations are needed to define a solution, and since there are four alternative solution regimes, there must be four alternative sets of four equations defining the solution.

The alternative sets of equations which define a solution are set out in Table 1. Of the four equations required for a solution, two are common to all cases, namely the accounting identity (1) and the inverse demand equation

\[
p = \rho(q, t).
\]

\[\text{Table 1}
\]

\text{Alternative equation systems for determining } x, q, p \text{ and } s

<table>
<thead>
<tr>
<th>Use of storage</th>
<th>Production level</th>
<th>Common row equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>No storage</td>
<td>( p = a(q) = p(q, t) )</td>
<td>( s = 0 ) (implying ( \dot{s} = 0 ) so that ( x = q ))</td>
</tr>
<tr>
<td>Storage</td>
<td>( p = a(x) = p(q, t) )</td>
<td>( \dot{p} = b(s, t) ) (( s &gt; 0 ))</td>
</tr>
<tr>
<td>Common column</td>
<td>( \dot{s} = x - q )</td>
<td>( \dot{s} = \bar{x} - q )</td>
</tr>
</tbody>
</table>

\( \rho = a(x) \)

\( (x < \bar{x}) \)

\( x = \bar{x} \)

\( \dot{s} = x - q \)
on the answer) – and the appropriate column – is production at or below capacity? (hence \( x = \bar{x} \) or \( p = a(x) \)). Thus a set of four equations is obtained for each of the four cases depending on the answers to the two basic questions. In the body of Table 1 these four equations are presented in condensed format by eliminating equations in \( s \) when \( s = 0 \) and eliminating \( x \) when \( x = \bar{x} \) or \( q \). From the first row, the results correspond to the solution of the peak-load pricing problem in the absence of a storage capability, i.e. price is set to equate demand and marginal cost if this implies output less than capacity: otherwise price is set to restrict demand to the full capacity level of output. The second row of the table then gives the results obtained in this paper for the case when a storage facility is being used.

The fact that the four equations defining a solution can be set out as in Table 1 has a simple implication. It is that optimal behaviour implies (i) that price should be set equal to marginal cost if output is below capacity and irrespective of whether a storage facility exists or is being used; and (ii) the rate of increase of price should be equal to the marginal cost of storage whenever storage is being used and irrespective of whether output is at or below the capacity level.

7. CONCLUSIONS

The analysis in this paper does not attempt to describe any particular example of the use, or potential use, of a storage capability. Rather it is concerned with the general principles of production scheduling and pricing when a storage capability exists. The main result is to demonstrate that such a capability results in a restriction on the rate at which output increases over time. This may or may not imply a reduction in the peak rate of output, so no strong result is available on whether a storage capability reduces the maximum rate of output that would otherwise be required. Obviously if output is limited by capacity and demand exceeds this limit for some period, then a storage capability can be a substitute for pricing policy in lowering demand. But the basic mechanism through which storage works is to reschedule production over time in such a way that marginal production costs increase continuously when stocks are being held, and at a rate which is controlled by the marginal cost of storage. This is the consequence of a storage capability. Its role is essentially to allow production at different points in time to be substitutes in providing the supply of goods at a single point in time. The articulation of a storage problem therefore depends on equating (at the margin) the total resource costs of production and storage which would be involved in producing at alternative dates to generate marketed supplies at a particular time.

Given that storage regulates rates of increase of output through marginal production costs, it follows that it also regulates the rate of increase of prices when efficiency prices are adopted. Storage may or may not affect the maximum
Reference to efficiency pricing as a basis for price determination is one option among several for treating the price aspect of the storage problem. Another is in terms of consumer surplus, and for some this will be the preferred approach. Either will suffice to derive the results set out in Table 1 above. But these results have been obtained under strict convexity assumptions with respect to both production and storage costs. Introduction of scale economies implies the possibility of multiple local optima in a storage problem. For that reason they have been excluded here.

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