ON THE SOCIAL RATE OF DISCOUNT AND PRICE OF CAPITAL IN COST-BENEFIT ANALYSIS

Various proposals have been made in recent years regarding investment planning in developing countries. In addition to the general project evaluation procedures published by the OECD and UNIDO, numerous papers on specific topics have appeared in the technical journals. This essay is intended as a critical survey of several specific aspects of public investment planning.

In this paper questions relating to the rate of time discount for use in evaluating the social profitability of an investment project are reviewed. Attention is paid to the underlying social welfare implications of cost-benefit analysis, including treatment of objectives other than aggregate efficiency.

The author shows that unless the economy is investing at the optimal rate, the social discount rate cannot be inferred from either the market rate of interest or the marginal productivity of capital. Two conclusions emerge from this analysis when the rate of investment is non-optimal. First, the social discount rate cannot be estimated and must be taken as given exogenously. Second, changes in the level of investment induced by the project, through wage payments and use of capital expenditure funds, must be evaluated at the shadow price of investment. This shadow price is shown to be a function of marginal savings rates, marginal productivity of capital, and the social discount rate.

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I. INTRODUCTION

The Articles of Agreement of the Bretton Woods Convention set forth the basic principles for World Bank lending. Most important among these are that loans should be for specific projects and that priority should be given to "urgently needed, productive projects". Correspondingly, project evaluation plays a key role in Bank operations. As an international lending agency and as a promoter of economic development and social change, the World Bank should carefully examine each project loan request in terms of that project's relative contribution to the recipient country's development program.

In this essay, we will review cost-benefit analysis as a general method for project evaluation. In particular, we will pay special attention to: 1) the specification of the relative weights which should be attached to costs and benefits occurring at different points of time; and, 2) the social value of current investment relative to current consumption.

In addition to the standard assumptions inherent with cost-benefit analysis, we shall also assume that at least during the first stage the Bank does not consider alternative projects in other countries in calculating the project's profitability. Such considerations would involve introducing criteria other than domestic social profitability into the analysis which would weight the benefits accruing to different
countries. This is an interesting and operationally important question for the Bank, but it is probably more usefully studied in a slightly different context than cost-benefit analysis. There are several reasons for this exclusion. First, we would need considerable amount of data on inter-country prices, demand function, etc. Second, such a system would be anything but partial equilibrium like cost-benefit analysis. Third, little is known about the constraints on Bank country lending and/or the relative weights which the Bank could attach to different objectives in different countries. For all these reasons, we shall examine the concept of social profit from the recipient countries viewpoint only.

This essay is intended as a critical survey of various proposals made in recent years regarding investment planning in developing countries. In particular, we will be commenting on the OECD's Manual of Industrial Project Analysis by I.M.D. Little and J.A. Mirrlees and UNIDO's Guidelines for Project Evaluation written primarily by P. Dasgupta, A.K. Sen, and S.A. Marglin.

We will review cost-benefit analysis paying special attention to: 1) underlying welfare implications and assumptions; 2) the specification of the relative weights which should be attached to costs and benefits occurring at different points of time; and, 3) the social value of current investment relative to current consumption.

In Section II, cost-benefit analysis as it pertains to developing countries will be discussed. In particular, attention is paid to specifying alternative social objectives and the limitations of market pricing in determining a project's contribution to any specific objective. The social rate of time discount is the topic
in Section III. Specific issues which are discussed include its
definition, estimation, and the use of alternative discount rates.
Section IV deals with the shadow price of investment and its use in
correcting the cost-benefit analysis to reflect the non-optimality
of the economy-wide rate of investment. A summary and conclusions
are given in Section V.
II. A BRIEF REVIEW OF COST-BENEFIT ANALYSIS

Cost-benefit analysis (alternatively called "investment planning" or "project appraisal") is a partial equilibrium technique for estimating the net contribution of a project to some set of objectives. It is a method which attempts to maximize the decision maker's "utility", subject to various economic and political constraints imposed on him. The set of objectives differs widely and depends on the particular decision maker. For example, an individual consumer or family might wish to maximize the total utility of their consumption of goods and services and their leisure time, while a private entrepreneur might measure his "utility" as the amount of profits his firm earns. In turn, the government, through its investment program, attempts to maximize its concept of social welfare.

Despite these differing objectives (or ways of defining "utility"), the methodology dictated by benefit-cost analysis is basically the same for each decision maker. Namely, add up all relevant costs incurred in using scarce resources (i.e., resources which can be used to contribute to the desired objectives) and the benefits from producing similarly scarce goods and services. The difference between the benefits and costs is the net profit of the project, in terms of the relevant objectives for that decision maker. The decision rule which emerges is simple—implement the project only if its "profit" is not negative.

In the pre-appraisal part of the project analysis, data are gathered regarding the production and usage of scarce resources during the project's life. In producing its output of goods and services, factors of production (Such as labor, capital, intermediate
goods, raw materials, and land) are required. In addition, there are certain contractual financial flows which must be considered (borrowing, repayments, and interest). The problem of cost-benefit analysis is to convert these yearly flows of resources into a single aggregate representing the profitability of the project. In general, this involves setting up a three-way aggregation scheme to take account of different physical quantities, different objectives, and different points of time.

For example, let $X_{i,t}$ represent the input (if negative) or output (if positive) of good (or factor) $i$ during year $t$ associated with the project being considered. Let $P_{i,j,t}$ represent the marginal value to objective $j$ of one unit of good (or factor) $i$ during year $t$ of the project. With these marginal values, we can aggregate the physical and financial flows, $X_{i,t}$, into yearly vectors of net profit to each objective—one vector for each year of the project's duration. This aggregation is shown in equation (1).

\[
P_{j,t} = \sum_{i} P_{i,j,t} X_{i,t}
\]

In the second step of aggregation, we are interested in converting the yearly net profit vectors, $P_{j,t}$, into a single number reflecting yearly net profit, $P_t$. This requires knowledge of the marginal substitution rates between different objectives; let us refer to these weights as $w_{j,t}$. This aggregation is represented in equation (2).

\[
P_t = \sum_{j} w_{j,t} P_{j,t}
\]

The final step in the aggregation process is to weight the net profits at different points of time in order to derive a scalar representation of the net profit of the project. Typically this is done using the present
date as the numeraire. This is what is meant by a net present value calculation. Defining the net profitability as \( P \) and the time weights as \( v_t \), we have (3).

\[
P = \sum_t v_t P_t
\]

As mentioned in the beginning of this section, cost-benefit analysis is a partial equilibrium method. This means that it is best applied to marginal changes in the economy in which the project itself has little effect on the weights used for the various aggregations--relative marginal values for different physical units, weights for aggregating net benefits to different objectives, and the weights for time aggregation. When this is not a fair approximation of reality, then the weights themselves become part of the problem and general equilibrium methods are necessary.

Assuming for the moment that we are considering a private decision maker whose only objective is maximizing the present value of his resources, the various aggregation weights are easily determined as observed market prices and behavior. Prices for goods and services become the weights for equation (1) and the market interest rates are the weights for equation (3). The second aggregation step can be dropped since we are assuming only one objective. If the decision maker is not a price taker in all markets, then his actions will affect the weights used in deriving the present value of his investment opportunity. In this case, he can "correct" the prices he affects to take account of his actions; this correction involves estimating the relevant demand and supply schedules for that market.

While private cost-benefit analysis seems to reduce to simple profit and loss accounting guided by market prices, measuring benefits
and costs for public decisions is much more complicated and lies at the heart of the cost-benefit analysis literature. The reasons are quite simple. First, public investment projects, being larger and tending to account for a large portion of total investment in a sector, are more likely to affect prices significantly than private ones. Second, and more importantly, market prices may not be good reflections of the marginal social benefits (costs) of any scarce good. Although this is well known, let us briefly review the reasons for the divergence because it will be useful in helping derive better weights to use in public investment decision making.

First, there may be market imperfections of various kinds. These range from tariffs and other taxes, to quotas, to increasing returns to scale, to monopoly and monopsony power by various buyers and sellers, to a lack of necessary market institutions. All of these can be characterized as leading to inefficient allocations of resources and the corresponding price system no longer equates marginal costs and benefits as it should in a system without imperfections.

Despite the complexities introduced when different objectives and distributional problems are considered, "efficiency" prices derived from the market are of considerable use if we assume that aggregate consumption is one of the social objectives. As has been known since the seminal work of Dupuit (1844), the proper measurement of aggregate consumption benefits for a government is the consumer's willingness to pay. For marginal changes in the supply or demand for a particular good, this can be represented by the price. Non-marginal changes can be handled in a similar way as was suggested for private decision makers—taking into account supply and demand functions. Thus, using "corrected" prices,
which take into account all the standard types of market imperfections, as weights will yield estimates of a project's yearly contributions to aggregate consumption.

This is a useful point at which to compare some of the procedures which have been suggested for correcting market prices in the context of developing countries. The procedure which seems to be suggested by Little and Mirrlees (1969) and some proponents of effective protection methods 6/ involves estimating the market prices which would emerge if all market imperfections were eliminated. This procedure has one glaring deficiency which has been pointed out recently by the authors of the UNIDO (1972) guidelines and others 7/, namely—the government might not be able for various reasons to implement the "perfect" market situation. In this case, the public planner must take his environment as given and estimate only those corrections which might take place. This entails careful study of such issues as trade policy, rigidity of wages, and alternative uses of scarce resources.

Secondly, market prices (even corrected for imperfections) correspond to an efficient allocation of resources for the given distribution of wealth and income and cannot be considered optimal if this distribution is non-optimal.8/ Rather, these prices can be associated with the marginal weights society puts on one very important social objective—economic efficiency or, equivalently, aggregate consumption over time. Market prices give little or no indication of the marginal values of resources to other objectives or the relative weights which society would like to attach to these conflicting objectives.
The project planner must directly estimate the contributions of the project to the objectives other than aggregate consumption. Usually this will not be too difficult. For example, if income going to a poor region is an objective, this can easily be calculated from project data. The difficulty is in choosing the weights, \( w_j \), which relate the various objectives. These are subjective numbers which in the last analysis must be decided by the political process rather than econometric estimation.

So far, we have discussed very briefly the first two steps in the aggregation process. In Section III, we will deal at greater length with the problem of time aggregation and the choice of the discount rate.
III. ON THE SOCIAL TIME RATE OF DISCOUNT

In Section II it was noted that in project evaluation it is necessary to specify weights, \( v_t \), in order to aggregate the net benefits which accrue in different time periods, \( P_t \), into a single aggregate, \( P \), reflecting the project's social profitability. Observing equation (3), it is easy to see that there are only two circumstances in which the \( v_t \) are not important. In the first instance, the project will be profitable no matter what weights are chosen if \( P_t \geq 0 \) for all \( t \). Similarly, if \( P_t \leq 0 \) for all \( t \), the project will not be profitable under any method of discounting. Secondly, if \( v_t \) are constant for all relevant \( t \), the undiscounted sum of \( P_t \) is all that matters. The first situation need not be further considered because, in general, investment projects incur net losses at first, at least while the plant is being constructed, and net profits in later periods. As long as this is the case, the signs of \( P_t \) varying, \( P \) will in general be positive for some values of \( v_t \) and negative for others. 10/1

In each of the three aggregation steps mentioned in Section II, one weight can be chosen as the numeraire. As we mentioned earlier, usually, the present is chosen as numeraire for equation (3). Similarly, and without loss of generality, we chose aggregate consumption as the numeraire for equation (2). The weights \( v_t \) then refer to relative values of aggregate consumption at different points of time, and net profit \( P \) is in terms of the present value of aggregate consumption.

There are strong reasons to assume that the weights, \( v_t \), do vary and, moreover, decline over time. First, the weights might reflect
the value from other investment opportunities which are available; this reason would be most important for an entrepreneur operating within a perfect capital market. Second, the \( v_t \) might represent some risk premium for savers. Third, it might represent pure time preference in the sense that consumption now is more desirable than in the future. Fourth, the decision maker may have diminishing marginal utility for the aggregate consumption objective and expect to be richer in the future and, therefore, derive less utility from additional increments.

The first two motivations can be characterized as reflecting supply considerations or alternative uses of resources. The other two, pure time preference and diminishing marginal utility of consumption, are directly related to demand or social preferences. It is these two latter motivations for declining \( v_t \) which we shall consider in this section. The reasons for this concentration will shortly become clear; they relate to an \textit{a priori} assumption of disequilibrium between supply and demand in the dynamic choice between consumption and investment. This disequilibrium will be the primary topic for discussion in Section IV. We, therefore, dichotomize the problem and include supply or marginal cost considerations in the calculation of the marginal benefits, \( P_t \), which are then discounted on the basis of society's time preference and marginal utility of consumption.\(^{11/} \)

We assume that \( P_t \) is measured in aggregate consumption units and already includes the aggregation of other objectives through the weights \( w_{jt} \) shown in equation (2). If total profit of the project, \( P \), is measured in terms of current aggregate consumption, then \( v_0 \) would be set at unity and \( P \) would be represented as follows:
\( (4) \ P = P_0 + v_1P_1 + v_2P_2 + \ldots + v_TP_T \)

where the project affects the economy from now until year T. A common working assumption, and one which will make the interpretation and selection of \( v_t \) easier, is that the weights decline at a constant rate. That is,

\[
\frac{v_t - v_{t+1}}{v_{t+1}} = \text{constant} = i.
\]

Using this simplification, equation (4) can be transformed into the following:

\( (5) \ P = \sum_{t=0}^{T} \frac{P_t}{(1+i)^t} \)

In this formulation, \( i \) can be called the social rate of discount and is the factor determining the weights, \( v_t \). In this section, we shall discuss the meaning of \( i \) and various methods for its estimation.

Following the notation in the UNIDO Guidelines, if we define

\[ \Delta v_{t+\delta} = v_{t+\delta} - v_t \]

equation (4) can be rewritten as

\[ i = \frac{\Delta v_{t+\delta}}{v_t} \]

For simplicity, let us derive \( v_t \) assuming continuous time.

Taking the limit as \( \delta \) goes to zero,

\( (6) \ i = -\frac{v_t}{V_t} \), where \( V_t \) is the time derivative of \( v_t \).

From our earlier discussion, it is clear what \( v_t \) refers to. Namely, \( v_t \) is the marginal utility of aggregate consumption in year \( t \), in terms of aggregate consumption in year 0. Assuming that this value depends on both society's preference for the present and the instantaneous utility.
of per capita consumption, we can formulate the total contribution of aggregate consumption to social welfare, \( W \), as follows:

\[
(7) \quad W = \int_{t=0}^{t} e^{-pt} U(c) \, dt,
\]

where \( c \) is per capita consumption, and \( p \) reflects social time preference. From this definition, it is clear that the marginal contribution to \( W \) of an increment in aggregate consumption at a given point in time, \( t \), is a function of \( U, c, t, \) and \( p \). In particular,

\[
(8) \quad v_t = W_c (c,t) = e^{-pt} U_c (c),
\]

where \( W_c \) and \( U_c \) represent the partial derivatives with respect to \( c \).

Taking the time derivative of (8) and dividing by (8) yields the following equation for the social rate of discount:

\[
(9) \quad i = - \frac{v_t}{W_t} = - \frac{W_c}{W} = \rho - \frac{U_c}{U}.
\]

The first term of (9) reflects the social rate of time preference. In the literature, it is usually assumed that \( \rho > 0 \), reflecting a preference for the present generation over future generations. There is no theoretical reason for this always to be the case regardless of the political system under which a country operates. Surely, however, a negative \( \rho \) would be considered highly unusual and somewhat perverse under most political philosophies.

The second term in equation (9) can be re-written as:

\[
(10) \quad \frac{U_c}{U_c} = \left[ \begin{array}{c} U_c \\ U_c \\ \frac{U_c}{U_c} \\ \frac{c}{c} \end{array} \right] \left[ \begin{array}{c} c \\ c \end{array} \right].
\]
where \( c \) is the time derivative of per capita consumption. The second term on the right half of (10) is the growth rate of per capita consumption. The other term in parentheses is the elasticity of marginal utility with respect to per capita consumption. This elasticity will always be negative if the instantaneous utility function, \( U(c) \), reflects diminishing marginal utility; i.e., \( U_{cc} < 0 \). Returning to equation (9), the second term will, in general, be negative and represents the elasticity of marginal utility of per capita consumption times the growth rate of per capita consumption.

Of the terms in equation (9) which determine the social rate of discount, \( i \), only the projected growth rate of per capita consumption is generally available as a datum. In a very real sense, the other terms represent parameters of social welfare which cannot be determined easily. Now that we have a clearer idea of the meaning of the social rate of time discount, there are two remaining questions which must be answered. First, can \( i \) be inferred from market behavior in an analogous way with the way weights were determined for contributions to aggregate consumption using correct market prices? Second, if we cannot easily determine the social discount rate, can any other number, which might be revealed in the econometric sense, substitute for \( i \) in equation (5)?

Neoclassical economics seems to suggest that the equilibrium market rate of interest could be interpreted as the social rate of time preference. Irving Fisher's seminal work *The Theory of Interest* (1930) presents a theory of individual decision making in which each consumer chooses a division of income between savings and consumption which equates his own marginal time preference with the market rate of interest in much
the same way that the consumer allocates between different current consumption goods. Indeed, some sort of market rate of interest is conventionally used as the discount rate in cost-benefit analysis.\textsuperscript{13} 

There are reasons for rejecting this perfect-competition neoclassical approach. In the first place, there is rarely a perfect capital market which could be counted on to generate efficient prices (in the Pareto sense of efficiency). The imperfections are similar to those noted in Section II while discussing the markets for factors, goods, and services. In particular, there appears to be a very wide gap, at least in the typical developing country, between borrowing and lending rates which cannot be equated with the marginal value added by the financial institutions. Secondly, there is a lack of easily accessible capital markets which leads to widely varying interest rates for different firms and individuals. Similarly, these divergent interest rates cannot be explained completely as risk premiums for different investments. Finally, government intervention in the capital markets is quite common and often leads to further inefficiencies.

Pigou, and more recently, Marglin and others have shown that the problem is more severe than just market imperfection and involves the applicability of the principle of consumer sovereignty as it applies to the consumption-savings decision.\textsuperscript{14} In the first place, the choice intimately involves time, and time flows in only one direction. The decision maker cannot change his overall consumption-investment choice as easily and as often as he does for current consumption decisions such as between meat and potatoes. Also, for the Fisherian view to be consistent, the savings-consumption choice must be made with some idea of lifetime income.\textsuperscript{15} In addition, there may be externalities in individuals savings behavior regarding the willingness to transfer resources to future generations.\textsuperscript{16}
The determination of the individuals' rates of savings has been a topic of serious economic thought since at least the time of Adam Smith. The counterweight to the neoclassical theory of savings can be called the classical theory and is associated with Ricardo, Marx, Keynes, Robinson, Sraffa, Lewis, and their disciples including most of economic development thought. In the classical theory of savings, individuals consume a fixed proportion of their income (either average or marginal), with the proportions varying with the income class of the individual. Much of recent theory on the economic growth process in developing countries has assumed that aggregate private savings is determined by the division of aggregate value added between workers and owners of capital, with workers consuming their entire income and capitalists saving almost all of their profits.\footnote{17} If the classical theory is accepted, then the market interest rate does not have the same normative implications for public decisions regarding the social rate of time discount as it would if the neoclassical theory were valid.

If these hypotheses make sense, then even if there were perfect capital markets or approximations thereof, the equilibrium rate of interest might not represent the social discount rate of aggregate consumption because the associated rate of investment might be less than optimal. The market rate of interest would be correct only if the government were free to choose what it views as the optimal level of investment. If the government's fiscal and monetary authority is limited, as is usually the case, the choice of discount rate cannot be separated from the savings-consumption choice, as it is for the Fisherian individual.\footnote{18}
We are now faced with a related question—namely, not knowing the actual value of the social discount rate, is there any other factor (which is econometrically determinant) that can serve as a useful substitute in discounting for project appraisal?

The marginal productivity of capital in the private sector has also been frequently suggested as an appropriate discount rate for public investment projects. This line of reasoning says that if the marginal investment can earn q percent in the private sector, no public investment project should be allowed to earn less, and vice-versa. The first problem with this approach is the measurement of the marginal productivity of capital. In the real world, capital is not a homogeneous good and the marginal products, or "own" rates of interest, from different capital goods may differ. This means that we are forced to find the marginal productivity of a capital good whose price relative to aggregate consumption can be expected to remain constant. However, as is indicated in footnote 20, the marginal productivity of such a capital good would equal the private rate of profit on investment, viz., the market rate of interest. If this rate were used in discounting, we would be approximating marginal benefits (social discount rate) by a marginal resource cost. But, as we have seen, this is correct only if the rate of investment in the economy is socially optimal.

So far, our conclusions are quite pessimistic and almost all negative. We have no clear way of determining the social rate of discount and we are not able to substitute anything which we do know for it. How, then, can we discount net benefits to derive the profitability of a public investment project? The rate of discount should be taken
as an unknown for the project and the cost-benefit discounting
should be done either with a number of alternative discount rates, or with a single rate set exogenously by political authority. As we will show in the next section, however, it may be possible to determine upper and lower bounds for the social discount rate from some knowledge of a national economic plan.

So, we must conclude that the social rate of discount, i is not, in general, revealed through the market or econometric data and its determination implicitly involves political judgments. The choice of discount rate, therefore, is the responsibility of those vested with political responsibility, rather than the economist.
IV. ON THE SHADOW PRICE OF CAPITAL

In Section III, we derived the social rate of time preference, \( i \), in terms of the value society places on incremental aggregate consumption. We stated that the social rate of discount could not be estimated from the opportunity costs of investment because it was thought that there were constraints which prevented the government from equating the marginal costs and marginal benefits of foregone consumption.

Optimal levels of investment imply that at the margin, increments in consumption and investment are equally valued to society. However, if the rate of investment is less than is desired, then investment is marginally more valuable than consumption. This is extremely important in project evaluation, because the social cost of the project is partially dependent now on its effect on the distribution of income between consumption and savings. In this section, we will discuss the motivations for assuming that projects affect the rate of investment, derive some expressions for the price of investment relative to consumption, and discuss methods for estimating this relative price which is commonly called the shadow price of investment or capital.

Whenever we assume a classical savings function and the government is unable to directly increase investment to the desired level, the rate of investment will be at least partially determined through project selection. This is because the rate of savings is determined by the functional distribution of income between various classes of wage and profit earners and the government, and this distribution is affected by project selection. For example, suppose that we are considering two projects designed to manufacture textiles; they both have comparable costs per unit of gross output. If the value added in one goes largely to workers as wages, while
the value added for the other is paid mostly to profit earners as return for their invested (physical or human) capital, then acceptance of the first project would imply less future savings than the second, as long as workers have a higher marginal propensity to consume than profit earners. (Note that any extra value to society of workers' consumption should be included as part of the calculations for objectives other than aggregate consumption.)

Development theorists have long recognized this problem and both the UNICO Guidelines and Little and Mirrlees' Manual place great importance on the effects of project selection on the rate of investment. In particular, these writers point out the government's inability to lower real consumption among industrial workers through taxation. This is why the shadow wage of unskilled labor proposed by them is greater than the alternative marginal product in agriculture.\footnote{22}

Let us dig a bit deeper and ask when we have to correct our estimates of net benefits, \( P_t \), to reflect the indirect effects on investment of a particular project. Recall equation (5) and assume that the \( P_t \) reflect net contributions (positive or negative) to aggregate consumption of the project in year \( t \). If the project's costs are always financed out of current consumption and the project's benefits are always currently consumed, then no correction is needed because everything is denominated in terms of consumption when it is foregone, and by definition the price of consumption in any period is unity. But, if part of the costs come from funds which would have been expanded for other investments or part of the benefits are saved for investment purposes rather than immediately consumed, then these portions of the costs and benefits
must be valued at the shadow price of investment rather than at the price of consumption.\textsuperscript{23} 

This requires that each component of net benefits in each time period be broken down into a functional distribution between government, profit earners, and workers. Then, applying their respective marginal savings rates, we can derive a breakdown between these costs and benefits which are from (or to) current consumption and those from (or to) investments. In practice, this breakdown will be most important for wages, government expenditures, and items on the project's capital account.

It is now necessary to derive an expression for the shadow price of investment relative to consumption; we shall call this price $\lambda_t$. After defining $\lambda_t$, we will compare the formulas for the shadow price of capital derived by Little and Mirrlees and the UNIDO Guidelines. The shadow price of capital represents the discounted value of the consumption increments generated by the alternative uses of investment relative to the value of consumption. Assume that a unit of investment in year $\gamma$ yield aggregate consumption benefits, defined as $\frac{\partial C}{\partial K}$, from years $\gamma+1$ to $T$. Then, $\lambda$ is given by the following expression:

\begin{equation}
\lambda_t = \frac{1}{(1+i)^t} \sum_{t=\gamma+1}^{T} \frac{\partial C_t}{\partial K} \frac{x}{1+i}^t
\end{equation}

Therefore to derive $\lambda_t$, it is necessary to determine the marginal productivity of the investment, previously defined as $q$ in Section III, the division of these returns between immediate consumption and reinvestment, the value of $T$, and the social rate of discount.
Both the UNIDO Guidelines and Little-Mirrlees assume that throughout the relevant period, both the productivity of capital and the reinvestment rate remain constant. With these assumptions, and a constant value for $i$, it is possible to easily determine

$$\frac{\partial C_t}{\partial K_Y}$$

If we define the reinvestment rate as $s$, then:

$$y_t = (1 - s)q (1 + sq)^t - 1 - \gamma$$

where all variables are defined as before. Combining (11) and (12) gives an expression for $\lambda_t$ as a function of $i$, $s$, $q$, and $T$. It is easy to prove that if $q = i$, then $\lambda_t = 1$ for all $t$.

The most important difference between the two approaches is in the definition of $T$. Little and Mirrlees assume that by year $T$, marginal increments in investment and consumption will be equally valuable and, therefore, $\lambda_t = 1$. Therefore, they take the increment in aggregate consumption in year $T$ as the value of the terminal capital stock. With this formulation, $\lambda_t$ varies, declining to unity by year $T$. The authors of the Guidelines, however, assume that investment will always be more valuable than consumption and set $T$ at infinity. This allows them to have $\lambda_t = \lambda$, as well as a simpler expression for the shadow price than that of Little and Mirrlees.

Both formulas are subject to similar criticisms. The rigidity of the assumptions (constant $i$, $s$, and $q$) will, in general, lead to overestimates of the shadow price of capital, because as the economy develops it is quite natural for the marginal productivity of capital to decline while savings rates might increase. This is especially true of the Guidelines formulation where $\lambda$ always exceeds unity and is constant.
However, Little and Mirrlees give very little guidance as to the determination of $T$ and in their case studies they assume a constant shadow price of capital which does not change in the manner recommended in the text.\(^{27}\) Note that this is not a theoretical problem but rather a data and computational problem involving the time paths of $q$ and $s$, for once these parameters are known, the shadow price of capital becomes a function of the social discount rate.\(^{28}\)

It is at this point that economy-wide general equilibrium models become useful. These models usually pay very careful attention to the inter-related supply functions which determine the time path of $q$, the parameter of equation (12) which is most likely to vary over time. Also, parametric testing of economy-wide models with varying utility functions might give the best possible indication of the date when investment and consumption will be equally valued. That is, the year $T$ when the rate of investment is optimal.

But such models are costly to construct, require rather complete and sophisticated data, and are available for only a few countries. Therefore, the shadow price of capital must be estimated using the more restrictive assumptions regarding $q$, $s$, and $T$ which are compatible with the very limited data which is usually available to a country economist.

Although both the Little-Mirrlees Manual and the UNIDO Guidelines have almost identical formulas for the shadow price of capital, their data requirements are quite different. The authors of the Guidelines suggest that the parameters $q$ and $s$ be directly estimated. The parameter $q$, the marginal productivity of investment, can perhaps be estimated by econometric techniques. Alternatively, if we assume a labor surplus economy, then the incremental output-capital ratio becomes
the marginal product of capital, \( q \), and perhaps can be derived from examination of data from a large sample of projects or from some national economic plan. Similar procedures can be used in the estimate of the marginal savings rate, \( s \).

In the Manual, Little and Mirrlees suggest that \( q \) and \( s \) be indirectly estimated from alternative project data. First, they determine the reinvestment rate per unit capital. Clearly, this is nothing but the proportion of marginal product which is saved, or \( sq \) in our notation. A formula is then provided which relates the increase in consumption per unit capital, \((1-s)q\), to the industrial wage rate, the marginal product of labor in traditional agriculture, and the project's labor-capital ratio. Once \( sq \) and \((1-s)q\) are known, \( s \) and \( q \) are determined and can be inserted into equation (12). The most serious flaw in this approach is that we do not know which project is "marginal" and this is not something which should be left for each project planner to determine. The shadow price of capital should be a national parameter which insures that every project has to face the same profitability test before acceptance.

What can we conclude from this discussion of the shadow price of investment? First, for countries with limited fiscal opportunities for increasing savings, project selection can affect the rate of savings and, therefore, this aspect should be considered in evaluating public investment projects. Second, this does not imply discounting a project's benefit-cost stream by the marginal product of capital in other uses. Rather, changes in overall savings generated by a project should be evaluated using a shadow price of capital. Third, while there is no ideal method for deriving this shadow price, except perhaps through use of very
reliable economy-wide models, both the Little-Mirrlees Manual and the UNIDO Guidelines suggest consistent and estimable formulas for its calculation. In choosing between the two, it seems as if the estimation procedure of the Guidelines is more direct and reasonable since it can be determined by examining economy-wide parameters.
V. SUMMARY AND CONCLUSIONS

We now return to the questions raised in the Introduction and ask what have we learned about how to evaluate public projects. In the first place, we have limited our survey to partial equilibrium techniques for project appraisal which seek to measure the impact of a project only on those economic variables it most affects, rather than general equilibrium techniques which measure the impact of a decision on all other aspects of the economy. As long as the partial equilibrium method is broad enough to allow all relevant changes to be included, the results will be approximately the same as from the more general method, as well as much easier to compute with far less data.

The criteria for public project selection should not be monetary profit, but social profit; i.e., accept projects which have a positive present value of social profits. This implies that the cost-benefit calculations should take account of the project's net contributions to all relevant social objectives. It is recommended that this be done explicitly in the manner suggested by equations (1) and (2), which allow the project planner to present the decision maker with the implications of accepting or rejecting the project. This is especially true of the income redistribution objective which is not adequately represented by measurement of aggregate consumption.

In estimating the net contribution of a project to the various objectives, it is crucial that the planner take account of the "environment" of the decision, or, in other words, the relevant constraints on government action. For example, the project analyst should not evaluate a potentially tradeable item at the world price if the government will be unable to resist pressures to domestically produce the item. On the
other hand, it would be useful if project planners should show decision makers the negative implications of such inefficient policies.

We have seen that time enters the cost-benefit calculations in a crucial way, and, therefore, it is necessary to specify time weights for each objective to properly aggregate all contributions into a present value. Projects should be evaluated at the social discount rate, which is the time rate of change in the present value of the marginal utility of consumption. This value depends on the social rate of time preference, the growth rate of per capita consumption, and the elasticity of the marginal utility of per capita consumption. We have concluded that the social discount rate is not, in general, revealed either in free market interest rates or in the marginal productivity of capital, unless the rate of investment is optimal. In any case, the determination of the optimal rate of investment is a social choice and must be made in reference to the social discount rate. Even if the project economist does not know the social rate of discount, he should evaluate projects at several rates falling between the economic growth rate and the estimated marginal product of capital.

If the rate of investment in the economy is not optimal, the project must be compared with the alternative uses of investment funds. This implies valuing the project's impact on investment funds at the price of capital. Both the Little-Mirrlees Manual and the UNIDO Guidelines suggest similar formulas for calculating the shadow price as a function of the social discount rate and econometric data (such as the marginal saving rate and the incremental capital-output ratio).
These are procedures which are relatively easy to implement. Most of the data necessary for the cost-benefit calculation are in the project reports. Judgment must be exercised in the conversion of market prices to shadow prices and in the estimation of the contributions to objectives other than aggregate consumption. Estimates of the shadow price of capital are relatively easy to determine since they can be based on economic aggregates. Finally, the choice of the social discount rate is a parameter of the project appraisal and it is easy to recalculate the present value at a number of alternative discount rates, remembering also to revalue the shadow price of capital with each iteration.
FOOTNOTES


2. The seminal work in appraising public investment projects is Dupuit (1844). Cost-benefit analysis was first popularized in the 1930's in the United States in the context of water resource planning. For an example of this literature, see United States Government (1932). Among the many theoretical studies on cost benefit analysis are Mishan (1971), Prest and Turvey (1966), and Eckstein (1958). For examples of the application of benefit-cost analysis in the context of developing countries, see Marglin (1967a), Little and Mirrlees (1969), and United Nations Industrial Development Organization (1972).

3. In fact, cost-benefit analysis is often referred to as public project evaluation. We have not done that here in order to emphasize the similarity in technique between public and private decision making. The dissimilarities involve using different criteria and different prices--not different methodology.

4. Inefficiency here is meant in the sense of Pareto. An economic situation is Pareto-inefficient if it is possible to reallocate resources in such a way that no one is made worse off and at least one person's situation is improved. For discussion of welfare economics, the concept of Pareto efficiency, and the role of prices, see Koopmans (1957) or Graaf (1957).
5. For a lucid explanation of how to account for non-marginal changes (i.e. increments which affect price) in supply or demand for goods, see the UNIDO Guidelines, Chapter 5. See Marglin (1967a) and Little and Mirrlees (1969).

6. For a sampling of the effective rate of protection literature, see Balassa (1965), Corden (1966), Johnson (1964).

7. In theory, a government should be able to implement any economic reforms it deems suitable. However, in practice this is not the case. Governments are limited in their fiscal and monetary authority; in addition, they often face irresistible pressures from special interest groups. In particular, it may be impossible to redistribute the benefits from even a Pareto-optimal move so that no individual is made worse off. These problems seem even more acute for the developing country and any project planner should recognize them. If decisions are made on the basis of unrealistic assumptions about possible government action, resources will often be wasted. For an excellent discussion of this point, see Sen (1972). Also see Balassa and Schydowsky (1972) and Bruno (1972).

8. In general, each Pareto-optimality situation corresponds to a particular set(s) of wealth and income distribution. In choosing between different points on the Pareto-optimal frontier, a social welfare function is implicit. Therefore, we can say nothing about the optimality, as opposed to efficiency, of a Pareto-optimal situation. For more on this topic, see Koopmans (1957) and Bator (1956).
9. See Marglin (1967a) and the UNIDO Guidelines. Little and Mirrlees mention alternative objectives but never explicitly include them in their recommended procedures or in their case studies.

10. The internal rate of return is defined as the discount rate at which \( P = 0 \). For discussions of using internal rates of return as a method for project selection, see Harberger (1965) and Seagraves (1970), and Hirshleifer (1958). Most evidence seems to indicate the superiority of the present value criteria, which cost-benefit analysis proposes, over the internal rate of return.

11. The social utility of consumption and time preference have been hotly debated topics in economic doctrine for over 100 years. See Fisher (1930), Ramsey (1928), Samuelson (1948), and Mishan (1966) for discussion of these issues. We have included both in this formulation for purposes of completeness, not to make welfare judgments.

12. In the UNIDO Guidelines, it is assumed that there is no pure time preference and \( p \) is set equal to zero. This is the standard Ramsey assumption. Little and Mirrlees do acknowledge both elements explicitly in their determination of the consumption rate of interest. However, in the example they cite on pages 173-176, it is assumed that the elasticity of the marginal utility of consumption is greater than one. This is clearly incorrect unless they are introducing income distribution through the back door. It is interesting to compare this formulation with that used in numerical multi-sector planning models. These models typically assume that the elasticity is zero, which implies constant
marginal utility of per capita consumption. Time preference, then, is the only determinant of the social rate of discount. For examples of this kind of planning model, see Eckaus and Parikh (1968), and Bruno, Fraenkel, and Dougherty (1969). The gradualist consumption function proposed by Manne (1970, 1972) is based on a constant elasticity of marginal utility of consumption rather than time preference. For a proof, see Chakraverty and Manne (1968).

13. In a World Bank paper on the choice of discount rate, Schmedtje (1965), recommended using a market interest rate and proposes a technique for deriving the "correct" rate from the multitude of observed rates. Market interest rates are also proposed by Tinbergen (1958), and Walters (1970). Also, the guidelines recommended for United States government projects involve using the market rate of interest. In particular, see U. S. Congress (1968).

14. See Marglin (1963a, 1963b, and 1967a) and the UNIDO Guidelines.

15. See Friedman (1957), Ando and Modigliani (1963), and 'Lucch and Morishima (1972).


17. For discussion of the relationship between the rate of savings and income distribution, see Hahn (1951), Kalecki (1939), Robinson (1956), or Hahn and Matthews (1964).
18. See Sen (1960), Marglin (1967a, 1967b), and Lefeber (1968), for theoretical discussion of the relationship of savings and choice of technique in a dualistic economy. This topic is discussed at more length in Section IV.


20. The rate of return on any capital good is the sum of the capital gains and "own" interest rate or rentals. Defining the gross rate of return as \( r_i \), and the price of the capital good as \( p_i \), and the marginal product (or rental value) as \( MP_i \), we know that

\[
r_i = \frac{p_i}{P_i} + \frac{MP_i}{P_i}
\]

It can be demonstrated (see Dorfman, Samuelson, and Solow (1958)) that the rate of return \( r_i \) is equal for all capital goods. Therefore, as long as capital gains are not negligible, the overall rate of return cannot be measured by the marginal value product alone.

21. Alternative values of \( i \) can be used to define cross-over points which limit the range of the social parameters \( w_i \) and \( 1 \). See Chapters 12, 13, and 18 of the UNIDO Guidelines.
22. At one time, the conventional wisdom was to evaluate projects using a shadow wage equal to the marginal product of unskilled labor in traditional agriculture, a value often taken to be zero or near zero. For examples of this view, see Chenery (1953) or Tinbergen (1958). Galenson and Leibenstein (1955) and later Sen (1960) pointed out that this policy would lower the economic growth rate by limiting the amount of domestic savings. This would be true if workers entirely consumed their increased wages and the government could not generate additional tax revenues. For more recent discussions, see Marglin (1967b), Sen (1972), and Newberry (1972).

23. This is the approach taken in the UNIDO Guidelines; see Chapters 6, 14, and the case studies. Little and Mirrlees choose to value everything in terms of investment and, therefore, they correct the consumption items by the price of consumption relative to investment. However, the two approaches are conceptually equivalent.

24. The Guidelines do note the possibility of time dependent $s$ and $q$ in an appendix to Chapter 14. Formulas are developed to handle this, but the authors suggest that data limitations may be too severe to make the corrections worthwhile.

25. The Little-Mirrlees formula for the shadow price of capital is developed on page 167 of the Manual. The value of the capital stock in year $T$ is taken to be $\frac{1 + sq^T}{1 + 1}$. The reader should note that there is an error in the formula for the shadow price. The formula given in footnote 1 of page 167 for the summation of the value of incremental consumption is incorrect and should read
\[(c - m)n \left[ \left( \frac{1 + r}{1 + s} \right)^{r} - 1 \right] \frac{1 + 1}{r - 1} \cdot \]

26. The formula given in the Guidelines for the shadow price of investment is \( \frac{(1-s)q}{i-sq} \). This puts some restriction on the value which the social discount rate, \( i \), can assume. Namely, it must be greater than the growth rate \( sq \) if the shadow price is to be less than infinity. A complete discussion can be found in Chapter 14.

27. See Chapter 13 and the case studies in the Manual.

28. Although the social discount rate may also be changing in response to varying growth in per capita consumption and variations in the elasticity of the marginal utility of consumption, we continue to ignore this possibility as a necessary simplifying assumption.

29. The incremental capital-output ratio can be interpreted as the marginal social product of capital; i.e., the marginal product assuming optimal employment of all other resources, including labor. For a discussion of this interpretation, see Marglin (1967b).

30. Note that \( sq \) can also be interpreted as the economic rate of growth.
31. The use of the alternative marginal product, especially in the private sector, has been recommended by Harberger (1965, 1967) and others. They say that using the social discount rate of consumption will lead to inefficient use of resources. As long as we correct the net benefit stream using the shadow price of capital, this criticism is not true. See Chapter 14 of the Guidelines for a comparison of alternative methods of discounting.

32. It may be impossible to accept all projects which have positive present value if there are overall investment budget constraints. In this case, the most profitable set of projects consistent with the capital investment budget should be chosen. This method might give some indication to the budget authority as to how it initial allocations to various ministries and sectors should be revised. Of course this procedure is, strictly speaking, only valid if the choice of any set of projects does not seriously affect the weights and prices which were utilized for calculating the present values. Otherwise, partial equilibrium techniques are inappropriate shortcuts.
REFERENCES


a potentially tradeable item at the world price if the government will be unable to resist pressures to domestically produce the item. On the


