DISCUSSION PAPER

INERTIA IN EMPLOYMENT

Graham Pyatt

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Development Research Department
Economics and Research Staff
World Bank

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Summary

This paper arises from an attempt to provide a theoretical basis for the observed duality and inertia in labor markets. The standard treatment of short-run equilibrium of the firm is modified by distinguishing the firm's inherited or ex ante labor force from the external pool of potential new recruits. New recruits are assumed to be less efficient than established workers within the firm, and this difference is shown to modify standard results in interesting ways. In particular, it is shown that the actual level of employment will be inert relative to changes in product demand and that, if existing workers are risk averse, then the firm may find itself operating x-inefficiently.
1. Introduction 1/

This paper is concerned with the micro-economic foundations of employment determination at an elementary level and, in particular, with an explanation as to why employment may be insensitive to changes in product market demand. It focuses on the short-run equilibrium of a firm. Assumptions about production technology and product demand are more or less standard, while the assumed objective of the firm is to maximize profits with given capital stock. However, rather than complete the standard model by assuming an exogenous supply of homogeneous labor, two types of labor are distinguished here, viz, the existing workers currently employed by the firm, and others, external to the firm, who might be recruited. The former correspond to the firm's internal labor supply, the latter to its external supply.

A crucial assumption in the analysis allows that internal labor is potentially cheaper than external labor of equal efficiency. Such a differential could arise for a variety of reasons. For example, the firm's activity may require skills which are not readily available externally, so that new recruits would either have to be trained, implying training costs, or simply given time to acquire such skills on the job, with an implied differential in efficiency for the period of learning (by doing). An alternative type of explanation would be that a firm's current level of

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employment exhausted its local labor market, necessitating a premium (removal cost) to attract labor from elsewhere if employment were to expand. Or search costs may be involved in finding new recruits of the same quality as existing workers. In either case, it can be noted, there is no permanent difference between internal and external labor. The distinction is transient. New recruits, following training or relocation, are indistinguishable from established workers. The difference, if any, maintains only within the (short-run) period over which training or learning takes place. However, within this period, there may well be differences, and the potential consequence which matters here is that, per efficiency unit, the supply price of external labor may differ from the reservation wage of existing workers.

The reservation wage of the existing workers is to be defined relative to their current employment. If this reservation wage is \( \tilde{w} \), then the actual wage, \( w \), must exceed \( \tilde{w} \) if existing workers are to prefer retaining their present jobs to seeking some alternative. In this sense, \( \tilde{w} \) measures the attractiveness of alternative employment. Therefore \( \tilde{w} \) is sensitive to the availability of alternative employment as well as to wage rates outside the firm and the level of unemployment benefits under social security programs.

The supply price or opportunity cost to the firm of external labor is denoted \( \bar{w} \). This is also measured in efficiency units so that, while the reservation wage per man may be the same for internal and external labor, any greater efficiency of the former within the firm will imply that \( \bar{w} > \tilde{w} \).

The condition \( \bar{w} > \tilde{w} \) essentially defines the circumstances which are explored in this paper. In the limit, as \( (\bar{w}, \tilde{w}) \to 1 \), the results to be presented will reduce to those of the standard textbook case in which internal
and external labor are indistinguishable. It can be argued that to make the distinction is a useful generalization, since it moves in the direction of greater realism for the model.

The analysis assumes that preferences of existing workers can be expressed collectively, while those of external labor, including any who might be recruited, are of no consequence in the short run, given that their reservation wage requirements are satisfied if the firm in fact offers them jobs. Accordingly, for the short-run determination of wages and employment, it is only the respective preferences of the firm and its existing employees that are of consequence, with the former being measured by profits and the latter by a collective utility function which has as its arguments the wage \( w \) and the proportion of existing workers retained by the firm.

Given these assumptions, the present analysis goes only part-way towards a theory of employment contracts, since a complete specification of bargaining between the firm and its existing workers turns out to be unnecessary to the derivation of some interesting results.

A first step is to explore the potential for mutual self-interest as between the firm and its internal labor force. An agreement on employment and wages under which both the firm and its existing employees are better-off than they would be without each other (i.e., the firm having to recruit an entirely new labour force, and the existing labour force all having to find new jobs), is described as an attractive contract. It is shown in Section 3 below that such contracts will be possible when \( \tilde{w} \) exceeds \( \tilde{w} \), that is, when the supply price of external labor exceeds the reservation wage of internal labour as previously discussed.

Next, beyond the notion of agreements or contracts which are
attractive, is the question of efficiency: an agreement is defined as being efficient if it maximizes the preference function of one party for a given satisfaction level of the other. Efficient agreements are discussed in Section 4 of the paper, which contains the main results.

While actual arrangements in a labor market can perhaps be expected to result in outcomes which are attractive in the sense previously discussed, there can be less confidence in assuming that they will be efficient. However, if indeed they are not efficient, then it follows that (at least) one party could be better off without any loss to the other. Such circumstances would seem to suggest pressures for institutional change in labor markets, so as to encourage efficient agreements to be reached.

The set of efficient contracts is necessarily a subset of those that are attractive. The size, and hence the characteristics of this subset will depend on how the preferences of the firm and its existing workers are specified. Here, throughout, the former are measured by profits. The latter are considered at two levels.

At the first level, the welfare of existing workers is not specified beyond assuming that it is positively related to the number of existing workers retained by the firm and the wage it pays them. On this basis, some interesting results are obtained. Attractiveness and efficiency of a contract imply that existing workers will enjoy goodwill in the sense of having first choice of available jobs. One of three regimes will maintain. The firm may retain all existing workers and, in addition, hire some recruits; or it may simply retain all existing workers; or some of the existing workers may be laid off. But hiring and firing will not co-exist. Also, the size of the existing ex ante labor force will have no effect on the actual level of
employment if the \textit{ex ante} or inherited labor force is below a critical size. Otherwise, with the \textit{ex ante} labor force above the critical level, the existence of this inherited labor force implies a higher level of actual employment than would otherwise maintain.

These results suggest that the assumptions set out in Section 2 below imply a labor market which is inert in the sense of being resistant to downward movements in the level of employment. This implication comes out more forcefully when the specification of preferences for existing workers is tightened. This is achieved here by assuming that the collective preferences of existing workers are measured by their expected utility, being defined as an appropriately weighted average of the utility of their reservation wage and that of the wage paid by the firm to those it retains.

Given this stronger statement of preferences it is almost, but not quite, possible to make exact statements about what the level of employment will be, independent of the precise wage or, indeed, of the specific bargaining process. The condition that contracts should be attractive and efficient is now sufficient to go a long way towards determining the level of employment. Specifically, it can be noted that the lowest levels of employment are least likely when the utility function of existing workers is linear in their wage. This is because, in this case, the trade-off between wages and employment which is acceptable to existing workers will be most sensitive to the latter. It will be shown that, in this limiting case, the actual level of employment will be given by the median of three numbers, denoted here by $\bar{n}$, $n^o$ and $\tilde{n}$. Of these, $n^o$ is the actual number of existing employees, while $\bar{n}$ and $\tilde{n}$ are such that $m(\bar{n}) = \bar{w}$ and $m(\tilde{n}) = \tilde{w}$, where $m(n)$ is the marginal revenue product schedule for labor. Necessarily, therefore, with
\( \tilde{\omega} > \hat{\omega} \), it follows that \( \bar{n} < \tilde{n} \).

The result: \( n = \text{median} (\bar{n}, n^0, \tilde{n}) \) describes an inert or 'sticky' labor market. If \( n^0 < \bar{n} \) (implying \( m (n^0) > \tilde{\omega} \)), the firm hires extra labor up to the point that \( m (n) = \tilde{\omega} \). Similarly, if \( m (n^0) < \tilde{\omega} \), the firm will lay-off members of its existing labor force in sufficient number to raise \( m (n) \) to \( \tilde{\omega} \). But for all intermediate cases, with \( \tilde{\omega} \geq m (n^0) \geq \tilde{\omega} \), the firm will neither hire nor fire, and employment will be stationary at the level, \( n^0 \), inherited from the past. History matters, therefore, in this model.

An implication of the above result is that fluctuations in product demand do not necessarily produce fluctuations in employment. In general, cycles in product demand will either have no influence on employment, or they will result in a cyclic response which is both lagged and damped relative to the product demand cycle. This result is a particular aspect of the more general implication of \( n = \text{median} (\bar{n}, n^0, \tilde{n}) \), to the effect that attractiveness and efficiency criteria may well yield short-run labor market equilibria in which there is a failure of the market to adjust levels of employment in response to changes in product markets.

The level of employment is no longer exactly determined in all cases when employees' preferences are such that \( U'(.) < 0 \), i.e. when workers' utility is a non-linear function of the wage. The effect, if any, of non-linearity is for employment to be higher in those cases were \( n \) would otherwise be \( \bar{n} \). Accordingly, introducing risk aversion among existing workers in the form \( U''(.) < 0 \) potentially implies fewer lay-offs. Risk aversion therefore adds to the inertia of employment levels. Just how much it may add is not determined here. But it is shown that it may be enough to imply a level of employment at which the marginal revenue product of labour would be
negative if the firm was to operate at full technological efficiency. Of course, in such a situation, the firm will choose to operate within its technological frontier, simply producing enough output to maximize total revenue. In this way, the present analysis allows the intriguing prospect that x-inefficiency may characterize an attractive and (distributionally) efficient determination of employment and wages at the level of the firm.

This analysis has antecedents in the early literature on employment inertia, notably Oi [1962], and more generally with theories of segmented labour markets as usefully reviewed in Cain [1976]. However, Oi's formulation of the problem is specifically dependent on training costs and regards wages as endogenous. Here, wages for existing workers are clearly endogenous, (although the analysis is not pushed to the point at which they are completely determined), while explicit training costs are not necessary to the argument, which can be sustained on the basis of learning-by-doing alone. Indeed, while the results obtained are thought to be of more general relevance, the initial motivation in undertaking the work reported here was to explore the linkages between duality in wage systems of developing countries and learning-by-doing as an important characteristic of their industrialization processes. While the wage implications of the assumptions made here remain to be fully developed, in the analysis presented, wages for retained employees are in fact bounded, with the reservation wage defining the lower bond. Clearly, then, there is scope within the framework for a dual wage system to maintain.

Finally, the present analysis has some implications for institutional form in the labor market. Collective bargaining by existing workers is taken as given, but there are no restrictive practices, only efficiency differences, to explain \( \bar{w} > \tilde{w} \). However, it can be noted that essentially the same model
would result if there were no efficiency differences between internal and external labor, and instead the former were able, through restrictive practices, to raise the supply price, $\tilde{w}$, of external labor relative to the reservation wage, $\tilde{w}$. Accordingly, these restrictive practices which are reasonably captured by the assumption $\tilde{w} \geq \tilde{w}$ lead to the same model and hence the same conclusions. Moreover, it can now be seen that in situations in which there are efficiency differentials between established workers and new recruits, the formal unionization of labor may be only a cosmetic change, having no effect, of itself, on the level and stability of employment that would otherwise maintain.

While the present paper does not go far into the question of wage determination, these investigations are enough to show that the wage will only coincidentally be equal to the marginal revenue product of labor under efficient agreements. Evidently, therefore, the mutual interest of the firm and its employees in reaching efficient contracts will lead to institutional forms in the labour market in support of behaviour which is different from profit maximization by the firm, given the wage rate, as normally assumed.

2. Basic Assumptions

The analysis is built on a set of five assumptions which are described in this section, together with notation and some preliminary results.

2.1 Production Technology

The output level of the firm is denoted $q$. Implicitly, capital is fixed and it is assumed that raw material inputs are strictly complimentary. Output therefore depends only on the level of employment, $n$, and on techno-
logical efficiency. The specification of this relationship is:

Assumption 1: Output, \( q \), is bounded via employment, \( a \), as
\[
q < \hat{q}(n) \text{ for all } n > 0, \text{ where } \hat{q}(n) \text{ is such that }
\]
(i) for \( n = 0 \), \( \hat{q}(n) = 0 \); and
(ii) for all \( n > 0 \), \( \hat{q}'(n) > 0 > \hat{q}''(n) \).

The function \( \hat{q}(n) \) therefore describes the technological limits on output. It is assumed to be twice differentiable and characterized by diminishing returns with respect to the variable input, \( n \).

2.2 Product Demand

It is convenient to define product demand in terms of a revenue function, \( v(q) \), which is net of raw material costs. Hence \( v(q) \) is value added, and specified as:

Assumption 2: Net output or value added of the firm is denoted by \( v(q) \) such that
\[
(i) \text{ for } q = 0, \ v(q) = o; \text{ and }
(ii) \text{ for } q > 0, \ v'(q) < 0 \text{ and } v'(q) < 0
\]
depending on \( q < q^* \).

The assumption \( v''(q) < 0 \) implies simply that marginal revenue diminishes as \( q \) increases. Beyond this, the formulation assumes that there exists a level of output, \( q^* \), at which marginal (net) revenue is zero. No loss of generality is involved since \( q^* \) may be arbitrarily large.

Assumptions 1 and 2 can be combined to yield the following statement about the marginal revenue product of labor:

Result 1: If \( \hat{v}(n) = v [\hat{q}'(n)] \) and \( m(n) = \hat{v}'(n) \) then,
from assumptions 1 and 2 it follows that:

(i) for \( n = 0 \), \( \hat{v}(n) = 0 \) and \( m(n) = \hat{v}(n)/n \); and

(ii) for \( n > 0 \), \( m'(n) = \hat{v''}(n) < 0 \) while \( m(n) > 0 \),

depending on \( n < n^* \), where \( n^* \) is such that

\[ q(n^*) = q^* \text{, i.e. } m(n^*) = 0. \]

Proof is standard and therefore omitted. The result states that the marginal revenue product of labor diminishes, and that it has value zero when \( n = n^* \).

It should be noted that the marginal (net) revenue product of labor is defined only when the firm is fully exploiting its technological capability, i.e., when \( q = \hat{q}(n) \).

2.3 Preferences of Current Employees and Labor Supply

The firm's labor input, \( n \), is a measure of employment in efficiency units, taking the efficiency of an existing worker as numeraire. If this total input is made up of \( e \) units supplied by existing workers and \( r \) units from new recruits, then \( n = e + r \); and the total wage bill is now \( we + \hat{w}r \), where \( w \) is the wage paid to those of the existing workers who are retained by the firm, and \( \hat{w} \) is similarly the wage per efficiency unit of new recruits. Hence, if new recruits are half as efficient as existing workers, then \( \hat{w} \) is twice the wage per man actually paid to new recruits. If there are \( n^0 \) existing workers, then \( e \), the number of those retained, must be such that \( e \leq n^0 \). Similarly \( r \) efficiency units of new labor are hired, and \( r \geq 0 \); and \( n^0 - e \) of the existing units are laid off, where \( n^0 - e \geq 0 \).

The notation introduced above contributes to the definition of labor supply as follows:
Assumption 3:

(a) External labor is in perfectly elastic supply at a wage per efficiency unit of \( \bar{w} \); and

(i) The collective preferences of the \( n^0 \) existing workers are given by

\[
U(e, w) \text{ such that } \frac{\partial U}{\partial e}, \frac{\partial U}{\partial w} > 0; \quad \text{or}
\]

(ii) \( U(o) = \left\{ e U(w) + (n^0 - e) U(\bar{w}) \right\}/n^0 \)

such that \( U'(\cdot) > 0 \geq U''(\cdot) \).

Part (a) of this assumption is straightforward. Part (b) calls for some explanation. It has two versions, either of which implies that existing workers, as a group, have a collective preference function. Under (i) this is characterized as being an increasing function of \( e \) (which is the number of existing workers offered continuing employment by the firm), and \( w \) (which is the wage they are paid). The alternative provided by (ii) defines \( U(o) \) as a weighted average of \( U(w) \) and \( U(\bar{w}) \). Since \( U'(w) > 0 \geq U''(w) \) by assumption, \( U(\cdot) \) can be interpreted as a utility function. Hence \( U(w) \) is the level of utility to be associated with being offered continuing employment. If \( U(\bar{w}) \) is similarly defined as the level of utility contingent on being laid-off, and if the \( e \) jobs offered to the \( n^0 \) members of the existing labor force are allocated at random, then it follows that \( U(o) \) is the expected utility of an existing member of the labor force, and \( o \) can therefore be interpreted as the mean equivalent wage for such a person. Similarly, \( \bar{w} \) can be thought of as the mean equivalent wage contingent on being laid-off and, as such, will be a measure of the alternative prospects for either a new job or being dependent on social security. Only if \( w \) is greater than \( \bar{w} \)
will the firm's ex ante or existing labor force prefer continuing employment with the firm. Accordingly, \( \tilde{w} \) may be referred to as the reservation wage, relative to current employment, of existing workers. As such \( \tilde{w} \) will be related to \( \tilde{w} \). In the simplest scenario, internal and external workers will have the same reservation wage per man, so that the ratio \( \tilde{w}/\tilde{w} \) measures the relatively greater efficiency, within the firm, of its existing workers, by virtue of their firm-specific skill or some other comparative advantage.

It can be noted for future reference that:

Result 2: Assumption 3 b(ii) is a special case of 3b(i)

which requires \( w > \tilde{w} \).

2.4. The Firm's Objective

Given production technology and product demand, the firm is assumed to seek maximum profits. Thus

Assumption 4: The objective of the firm is to maximize short-run profits, denoted by \( \pi \), where

\[
\pi = v(q) - w(e - \tilde{w}r) - w(e - \tilde{w}r)^2
\]

\( e \) = employment of existing workers,

\( w \) = wage paid to existing workers,

\( r \) = employment in efficiency units of new recruits, and

\( \tilde{w} \) = wage paid (per efficiency unit) to new recruits.

This leads directly to the following standard result:

Result 3: If the firm is restricted to its external supply of labor, (i.e. \( e = 0 \)), which is perfectly elastic at wage rate \( \tilde{w} > 0 \), then the firm will set output \( q \)
and employment \( n = \bar{n} \) such that:

(i) if \( \bar{w} > \hat{m}(o) \), then \( b = o = q \); and

(ii) if \( \bar{w} < \hat{m}(o) \), then \( q = \hat{q}(\bar{n}) \) where \( \bar{n} \) is such that \( m(\bar{n}) = \bar{w} \) and \( o < \bar{n} < n^* \).

If \( \pi(\bar{w}) \) denotes the profits earned in consequence of the maximizing behavior specified by Result 3, then the relationship between \( \pi(\bar{w}) \) and \( \bar{w} \) will be as shown in Figure 1. When \( \bar{w} \) is zero, the firm will maximize value added by having \( m(n) = o \) and hence \( n = n^* \) and \( q = q^* \). Otherwise, for \( \bar{w} > o \), the convexity of the relationship between \( \pi(\bar{w}) \) and \( \bar{w} \) follows directly from the diminishing marginal revenue product for labor established in Result 1(ii).

**Figure 1**

The relationship between profits, \( \pi(\bar{w}) \) and the external supply price of labor, \( \bar{w} \), when the firm is precluded from utilizing internal labor, \( (e = o) \).
3. **Attractive Agreements**

3.1 **Definition**

The assumptions introduced in the previous section involve five endogenous variables, \( e, r, n, w \) and \( q \). Without necessarily implying that any or all of these variables are the subject of collective bargaining, a determination of them can be referred to as a contract or agreement between the firm and its existing employees. Such an agreement can be described as attractive if it implies that the firm makes more profit than it would by simply relying on the external labor market; and also that current employees are better off than if they were to do likewise. These considerations are formalized in the following definition:

**Definition:** An agreement \((e, r, n, w, q)\) is attractive if and only if both

(a) \( r = v(q) - w - \tilde{w} \times r > \pi(\tilde{w}) > 0 \); and

(b) \( w > \tilde{w} \)

for non-negative values of \( e, r, \) and \( q \) such that \( 0 < e < n^0 \), \( q < q(n) \) and \( e + r = n \).

3.2 **Existence**

Given the definition of an attractive agreement and the assumptions previously set out, the conditions under which such agreements can exist may be explored. It is useful to do so in terms of some further notation, which is introduced in Result 4:
Result 4: Let \( v^*(n) = \max(\hat{v}(n), \tilde{v}(n^*)) \) and
\[ y(n) = \frac{\{v^*(n) - \pi(\bar{w})\}}{n} \]
then
\[
\min(n, n^{**})
\]
(i) (a) \( v^*(n) = \int \_0^\infty m(v)dv > \hat{v}(n) > v(q) \) and
(b) \( dv^*(n)/dn = \max(m(n), o) \); and
(ii) (a) \( ny(n) = \tilde{v} \bar{n} + \int \_\bar{n}^\infty m(v)dv \)
so that
\[ \min(n, n^{**}) \]
(b) \( y(\bar{n}) = \min(\tilde{w}, m(o)) > y(n) \);
(c) \( y(n) \rightarrow o \) as \( n \rightarrow \infty \); and
(d) \( \max(0, m(n)) > y(n) \) and \( dy(n)/dn > o \)
according as \( n > \bar{n} \).

The function \( v^*(n) \) introduced in Result 4 is simply \( \hat{v}(n) \) when \( n < n^* \), and otherwise has the constant value \( \hat{v}(n^*) \). Hence, unlike \( \hat{v}(n) \), \( v^*(n) \) does not diminish as a function of \( n \) when \( n \) exceeds \( n^* \). Result 4(i)(a) and (b) formally express these properties of \( v^*(n) \) and some implications. Their proof is obvious and therefore omitted.

Result 4(ii)(a) follows directly from the definitions of \( y(n), v^*(n) \) (as in Result 4(i)(a)), and \( \pi(\bar{w}) \). Result 4(ii)(b) follows from it. If \( \bar{w} > m(o) \), then \( \bar{n} = o \) and the integral measuring \( y(n) \) takes the limiting value \( m(o) \). Alternatively, if \( \bar{w} < m(o) \), then \( \bar{n} > o \). Now \( m(\bar{n}) = \bar{w} \) by definition of \( \bar{n} \), (Result 3(ii)), while \( m(v) < m(\bar{n}) \) for all \( v > \bar{n} \), (Result 1(ii)). Hence \( \bar{v}(\bar{n}) = \bar{w} > y(n) \) for \( n > \bar{n} \). Similarly, \( m(v) > m(\bar{n}) \) for all \( v < \bar{n} \), so \( \bar{w} = y(\bar{n}) > y(n) \) for all \( n < \bar{n} \). To establish 4(ii)(c) note that, as \( n \rightarrow \infty \), \( \min(n, n^*) + n^* \) so that
ny(n) + constant: hence \( y(n) + o \). Finally, it is easily shown that
\[ n^2 y'(n) = v_m(v) \int_{n}^{\min(n, n^*)} \min(n, n^*) m(v) \, dv. \]
The second part of Result 4(ii)(d) follows directly from this, given that \( m'(v) \leq o \). Moreover, since
\[ ny'(n) = \max(o, m(n)) - y(n), \]
the first part of Result 4(ii)(d) can be obtained from the second.

The function \( y(n) \) is illustrated in Figure 2 for the alternative cases \(\bar{w} > m(o) \) and \(\bar{w} < m(o) \). The graph of \( m(n) \) versus \( n \) is also shown in the figure, and this defines a level of employment, \( \tilde{n} \), such that \( m(\tilde{n}) = \bar{w} \). This will be referred to subsequently. Here it can be noted that the importance of the function \( y(n) \) derives largely from the following result:

**Result 5:** A necessary condition for an agreement \((e, r, n, w, q)\) to be attractive is that \( y(n) > w > \tilde{w} \).

The condition \( w > \tilde{w} \) is evident from the definition of an attractive agreement provided above. The additional requirement \( y(n) > w \) can be established as follows. By definition, \( ny(n) = v^*(n) - \pi(\bar{w}) \), while \( v^*(n) \geq v(q) \) from Result 4(i)(a). But profits, \( \pi \), are given by \( \pi = v(q) - we - \bar{w}r \). Hence, by substitution, \( e(y(n) - w) + r(y(n) - \tilde{w}) \geq \pi - \pi(\bar{w}) \).

Now \( \pi > \pi(\bar{w}) \) is necessary, by definition, for an attractive agreement, as is \( e > 0 \) and \( r > 0 \). Since \( y(n) < \bar{w} \) from Result 4(ii)(b), it follows that \( y(n) > w \) is a necessary condition for an agreement to be attractive as stated in Result 5. This condition imposes an upper bound on the wage, \( w \), to be paid to existing workers, and this bound is a function of \( n \), the aggregate level of employment as shown in Figure 2.
Case (a): $\bar{w} \geq m(o)$

Case (b): $\bar{w} < m(o)$

Figure 2
Restrictions on the Set of Attractive Contracts
Following on from Result 5, the following theorem can now be proved.

**Theorem 1:** Attractive agreements exist if and only if both

\[ \min(\bar{v}, m(o)) > \bar{w} \]  
and \[ n^o > o. \]

Since \( e \), the number of existing employees retained, must be such that \( e \leq n^o \), it follows that \( o < n^o \) is necessary for \( e > o \), i.e. the initial number of employees must be positive. Next, from Result 4(ii)(b),

\[ \min(\bar{w}, m(o)) > y(n), \]  
while \( y(n) > w > \bar{w} \) is necessary for an attractive agreement from Result 5. Hence \( \min(\bar{w}, m/o) > \bar{w} \) is a necessary condition for an attractive agreement, as stated in the theorem, and the two sides of this inequality provide, respectively, upper and lower bounds for \( y(n) \) and \( w \). Figure 2 is drawn on the assumption that the condition \( \min(\bar{w}, m(o)) > \bar{w} \) is satisfied.

To show that these same conditions are sufficient, let \( e = \min(n, n^o) \) by assumption. If \( o < n^o \), then there exists \( n \) such that \( o < n < n^o \) and, for such \( n \), \( e = n > 0 = r \), which is consistent with an attractive contract. In addition, let \( q = q^*(n) \) by assumption, where \( q^*(n) \) is simply \( q(n) \) provided \( n < n^* \); and \( q^*(n) = q^* \) if \( n > n^* \). Then

\[ v(q) = v^*(n) = ny(n) + \pi(\bar{w}) \]  
where \( v^*(n) \) and \( y(n) \) are defined in Result 4. Hence profits, \( \pi \), are given by \( v(q) + w + \bar{w}r = \pi(\bar{w}) + n(\pi(n) - w) \). Accordingly, \( \pi > \pi(\bar{w}) \) and \( w > \bar{w} \), as required by an efficient contract, if \( y(n) > w > \bar{w} \), i.e. if the necessary condition required by Result 5 is satisfied. Since \( \min(w, m(o)) > y(n) \), from Result 4 (ii) (b), it follows that if \( \min(\bar{w}, m(o)) > w \), then \( y(n) > w > \bar{w} \) is possible. Hence the conditions specified in the theorem are sufficient for attractive contracts to exist. Indeed, the argument shows that all points \((n, w)\) within the areas ABC
of Figure 2 are consistent with attractive agreements when \( q = q^*(n) \), 
\( e = \min(n, n^0) \) and given that \( r = n - e \).

The following corollary of the theorem can be established by reference to Figure 1:

**Corollary 1:** Given \( n^0 > 0 \), a necessary and sufficient condition for the existence of attractive agreements is that 
\[ r(\tilde{w}) > r(\tilde{\tilde{w}}) \]

The corollary states simply that, if a firm has an existing labor force, then both the firm and these employees can potentially reach an attractive agreement if, pursuing profit maximization and using only external labor, the firm would prefer to be faced by perfectly elastic labor supply at a wage of \( \tilde{w} \) rather than \( \tilde{\tilde{w}} \). It can be noted (from Result 3) that, in the former case the firm would employ \( \tilde{n} \) efficiency units of labour, where \( \tilde{n} \) is defined by 
\[ \tilde{n}(\nu) = w. \]

### 3.3 Some Implications:

Theorem 1 establishes that, under certain conditions, there is scope for both the firm and its current employees (the *ex ante* labor force) to gain from an appropriate selection of values for the variables \( e, r, n, w \) and \( q \).

Result 5 provides some conditions which such appropriate values must satisfy, including the condition \( \tilde{w} > w > \tilde{\tilde{w}} \). Now, if the difference between \( \tilde{w} \) and \( \tilde{\tilde{w}} \) derives entirely from efficiency differences between new recruits and established workers, as has previously been suggested as a possibility, then if \( \lambda \) measures the relative inefficiency of new recruits it follows that 
\[ \lambda \tilde{w} = \tilde{\tilde{w}}. \]

Hence the condition \( \tilde{w} > w > \tilde{\tilde{w}} \) can be written as \( \tilde{w}/\lambda > w > \tilde{\tilde{w}} \) which implies that established workers are to be paid more than their reservation...
wage but less than their greater efficiency relative to external labor would justify. A system of payment based simply on efficiency differences (a piece-rate system) is ruled out, therefore, not necessarily by the firm, but by the potential mutual concern of the firm and its employees to secure an attractive agreement. Similarly, if the firm was to pay the same wage per man to both new recruits and established workers, then this wage would have to exceed \( \bar{\omega} \) to retain the latter, and would be unnecessarily extravagant with respect to the former. The condition \( \bar{\omega} > \omega > \bar{\omega} \) therefore implies giving some recognition, but less than full recognition, to the greater efficiency of established workers. The greater efficiency of established workers generates a rent. The condition \( \bar{\omega} > \omega > \bar{\omega} \) implies that this is to be shared in some way between these workers and the firm, by paying a premium or differential to established workers, but one that is less than a full reflection of their efficiency difference.

4. Efficient Agreements

4.1 Definition:

From this point on, it will be assumed that the conditions required by Theorem 1 are satisfied so that attractive agreements exist and it is therefore up to the labor market, however constituted, to make a determination of the particular agreement to be adopted. Depending on how the labor market works, a range of outcomes is possible, and these way or may not be efficient according to the following definition:

**Definition:** An agreement \((e, r, n, \omega, q)\) is efficient if it is attractive and, in addition, there is no other attractive agreement which existing workers prefer while the firm is
worse off; or which the firm prefers while the existing workers are not worse off.

The set of attractive agreements which are efficient constitute the contract set. Actual contracts can be expected to correspond to points in this set. If they do not, then it is likely that the market mechanisms/institutions which brought them about will be under some pressure to change.

4.2 Weak Conditions for an Efficient Contract

Assumption 3(b)(i) provides a weak formulation of the preferences of the ex ante labor force. Accordingly, the set of contracts which are efficient relative to assumption 3(b)(i) is only weakly restricted. It is defined by Theorem 2:

Theorem 2: Under assumptions 1 to 4, a contract \((e, r, n, w, q)\)
is attractive and efficient only if:

(i) \(q = q^*(n)\);

(ii) \(e = \min (n, n^0)\) so that

\[r = n - e = \max (n - n^0, 0)\]; and

(iii) if \(n^0 < \bar{n}\), then \(n = \bar{n}\); and

if \(n^c > \bar{n}\), then either

(a) \(n = n^0\) and \(m (n^0) > w\); or

(b) \(\bar{n} < n < n^0\) and \(w > m (n)\)

To prove the theorem, it can be noted that the collective preferences of existing workers are given by assumption 3(b)(i) as \(U(e, w)\), while \(v = v(q) - w\) measures the preferences of the firm. Since the former depend only on \(e\) and \(w\), efficiency requires that the latter must be a maximum with respect to \(q, n\) and \(r\), subject to given \(w\) and
\( e = n - r \). Given that \( w < \bar{w} \) is necessary from Result 5, this implies (i) and (ii, of the theorem, and hence that \( \pi = v^* (n) + (\bar{w} - w) \min (n, n^0) - \bar{w} n \), and \( U = U (\min (n, n^0), w) \). It now follows that if \( n > n^0 \), \( U \) is independent of \( n \) and \( \pi \) increases as \( n \) increases if \( m(n) > \bar{w} \). Similarly, if \( n < n^0 \), \( U \) increases with \( n \), (since \( n = e \) and \( \partial U / \partial e > 0 \) by assumption), while \( \pi \) increases with \( n \) if \( m(n) > w \). Accordingly, if \( \bar{n} > n^0 \) then, for given \( w \), \( U \) increases as \( n \) increases to \( n^0 \), and is constant thereafter; while \( \pi \) increases as \( n \) increases to \( n^0 \), (provided that \( w < \bar{w} \)), and continues increasing with \( n \) until \( n = \bar{n} \), declining thereafter. Since \( w < \bar{w} \) is necessary for an attractive contract it follows that, when \( n^0 < \bar{n} \), to be both efficient and attractive, a contract must be such that \( n = \bar{n} \), thus establishing the first part of (iii) of the theorem. It remains to consider the case \( n^0 > \bar{n} \). In this case, as \( n \) rises to \( \bar{n} \), with \( w \) given, \( U \) increases, and continues to rise until \( n = n^0 \), being constant subsequently. Profits \( \pi \) also increase as \( n \) rises initially to \( \bar{n} \), (provided \( w < \bar{w} \)), and profits must decline if \( n \) increases beyond \( n^0 \) because if \( n > n^0 \) when \( n^0 > \bar{n} \), then \( m(n) < m(n^0) < m(\bar{n}) = \bar{w} \). Between these two possibilities, with \( \bar{n} < n < n^0 \), the relationship between profits and employment is uncertain. Such values of \( n \) may therefore correspond to attractive and efficient contracts. Accordingly such contracts may be characterized by \( n = n^0 \) with \( m(n) > w \), or by \( \bar{n} < n < n^0 \) with \( w > m(n) \). These possible characteristics correspond to the remaining possibilities under (iii) of Theorem 1.

The first part of Theorem 1, that \( q = q^* (n) \), states simply that if \( n \) exceeds \( n^* \), then output is held back to level \( q^* \) to avoid the reduction in revenue, \( v(q) \), otherwise entailed in producing more than \( q^* \). By
imputation, if \( n > n^* \), then \( q < q(n) \) and the firm is producing within its technological limits. In this sense, Theorem 1 does not exclude the possibility that \( x \)-inefficiency can co-exist with distributive or contractual efficiency.

Under (ii) of Theorem 1, \( e = \min(n, n^0) \), which implies that existing workers get first choice of available jobs. Contractual efficiency excludes the possibility that the firm will simultaneously hire new labour and lay-off new recruits.

If \( n^0 < \bar{n} \), the existing labour force cannot provide the scale of labour input that the firm would choose if it had no internal labour supply. In other words, once all existing workers have been retained at some wage less than \( \bar{w} \), it remains profitable for the firm to recruit extra labour, at a cost per efficiency unit of \( \bar{w} \), up to the point at which \( n = \bar{n} \). This is the only situation, according to Theorem 1, in which the firm will hire labour, and it corresponds closely to the standard neo-classical result as set out in Result 3. It can be noted that the twin criteria of attractiveness and efficiency are shown by the theorem to be sufficient to uniquely determine the level of employment at \( n = \bar{n} \) when \( \bar{n} > n^0 \), irrespective of the precise level of wages paid to the existing labour force.

When \( n^0 \neq \bar{n} \), the employment level is not so precisely determined, and it may depend to some extent on the wage. Specifically, for \( n^0 > \bar{n} \), Theorem 2 implies that either (a) \( n = n^0 \), with the wage at some level in the interval \( y(n^0) < w < \bar{w} \); or (b) \( n \) and \( w \) are such that the point \((n, w)\) lies within that part of the area ADC in Figure 2 which also satisfies \( n < n^0 \).

These results can be expressed alternatively as in the following corollary of Theorem 2:
Corollary 2: If an agreement is both attractive and efficient
then either \( n = \max (n^0, \bar{n}) \) and \( y(n) > w > \bar{w} \), or
\( \bar{n} < n < n^0 \) and \( y(n) > w > \max (\bar{w}, m(n)) \).

Evidently, the conditions restrict the set of possible outcomes, especially as they relate to employment, and it is only incidentally that the wage, \( w \), will be equal to labour's marginal revenue product, \( m(n) \).

Finally it can be noted that while Figure 2 shows \( y(n^*) > \bar{w} \), this is not necessary. Now \( y(n^*) = \{ \hat{v}(n^*) + \pi(\bar{w}) \} / n^* \), so that the previous inequality can be written as \( \hat{v}(n^*) - \bar{w} n^* > \pi(\bar{w}) \). Hence:

Result 6: Theorem 2 does not exclude the possibility that an efficient contract can imply \( x \)-inefficiency if
\[
\pi(\bar{w}) - \pi(\bar{w}) > \{ \hat{v}(\bar{n}) - \hat{v}(n^*) \} - \bar{w} \{ \bar{n} - n^* \} > 0.
\]

This is evidently stronger than the condition \( \pi(\bar{w}) - \pi(\bar{w}) > 0 \) provided by Corollary 1 for the existence of attractive contracts.

4.3 Stronger Conditions for an Efficient Contract

The results in Theorem 2 are obtained on the basis of a definition of efficiency which involves the weak specification of preferences for existing workers as in Assumption 3(b)(i). Strengthening this as in 3(b)(ii) implies further restrictions for contractual efficiency, and hence a more restricted set of efficient contracts:

Theorem 3: If the preferences of existing workers can be expressed simply in terms of their mean equivalent income, \( \bar{w} \), then the conditions of Theorem 2 for an attractive contract to be efficient are strengthened in the case \( n^0 > \bar{n} \) to the extent that if \( n < n^0 \),
then \( \max (o, m(n)) = w - \frac{[(U(w) - U(\tilde{w})]/U'(w)} \) and \( n > \bar{n} \).

To prove this result, it can be noted that Theorem 1 determines \( q \) and \( e \) as \( q = q^*(n) \) and \( e = \min (n, n^0) \), while either \( n = \max (n^0, n) \) or \( n < n^0 \). Hence either \( u \) is uniquely determined or

\[ U(w) = \left\{ n U(w) + (n^0 - n) U(\tilde{w}) \right\}/n^0 \] and \( v = v^*(n) = v^n \). Developing this latter case, profits are constant if \( ndw = \{ \max (o, m(n)) - w \} \) \( d \). Given this restriction, it can readily be shown that

\[ n^0dU/dn = \left\{ U(\tilde{w}) - U(w) \right\} + \{ \max (o, m(n)) - w \} U'(w), \] implying that \( \max (o, m(n)) = w - \frac{U(w) - U(\tilde{w})}{U'(w)} \) is a necessary condition for an efficient agreement as stated in Theorem 3.

The relationship \( \max (o, m(n)) = w - \frac{U(w) - U(\tilde{w})}{U'(w)} \) is graphed in Figure 3. For \( n > n^* \), \( \max (o, m(n)) \) is zero so that \( w \) has some constant value such that \( w \left( w(w) = U(w) - U(\tilde{w}) \right) \). Since \( U'(w) > o > U''(w) \) by assumption, the expression \( w - \frac{U(w) - U(\tilde{w})}{U'(w)} \) has the sign of \( (\tilde{w} - w) \). Since the expression has value \( \tilde{w} \) when \( w = \tilde{w} \), it must therefore be zero for some value of \( w \) in excess of \( \tilde{w} \) as shown in the figure. For values of \( n \) less than \( n^* \), \( \max (o, m(n)) \) decreases as \( n \) increases. Hence for \( \bar{n} < n < n^* \), Figure 3 shows a monotonic increasing relationship between \( n \) and \( w \) such that if \( n = \bar{n} \) then \( w = \tilde{w} \). For \( n > \bar{n} \), \( \max (o, m(n)) > m(\bar{n}) \); and \( U''(w) < 0 \) implies that \( w - \frac{U(w) - U(\tilde{w})}{U'(w)} < \tilde{w} \). Since \( m(n) = \tilde{w} \), it follows that the relationship between \( w \) and \( n \) is not defined for \( n < \bar{n} \).
Figure 3

\[ \max[\hat{m}, \bar{m}(n)] = w - \frac{[U(w) - U(\hat{w})]}{U'(w)} \]
To explore further the implications of Theorem 3, it can be noted that if existing workers are not risk averse, so that $U''(\cdot) = 0$, then

$$w - \frac{[U(w) - U(\bar{w})]}{U'(w)} = \bar{w} \text{ for all } \bar{w}.$$ 

Hence Theorem 3 implies that $n = \bar{n}$ if $n < n^0$. The following corollary of Theorem 3 is therefore established:

**Corollary 3:** If the preferences of existing workers depend only on their expected wage *ex ante*, i.e. on

$$(e/n^0) w + \{1-(e/n^0)\} \bar{w},$$

then a necessary condition for an efficient contract is that $n = \text{median } (\bar{n}, n^0, \bar{n})$.

This simple result has some interesting implications. First, it implies that there can be no indeterminacy in the level of employment for an efficient contract unless there is an element of risk aversion on the part of workers, i.e. $U''(\cdot) < 0$. Similarly, a distributionally efficient contract cannot be $\kappa$-inefficient unless existing workers are risk averse. And finally, there is a degree of inertia in employment relative to fluctuations in product demand which is independent of risk aversion on the part of workers. Each of these last points requires some elaboration.

From Theorems 2 and 3, when $n^0 > \bar{n}$, an attractive and efficient contract must imply $\bar{n} < n < n^0$ and $m(n) = w - \frac{[U(w) - U(\bar{w})]}{U'(w)}$. In addition, from Result 5, $y(n) > \bar{w}$. Since $y(n) = w$ implies a monotonic decreasing relationship between $n$ and $w$, (see Figure 2), while

$m(n) = w - \frac{[U(w) - U(\bar{w})]}{U'(w)}$ implies a monotonic non-decreasing relationship (see Figure 3), these two equations can be satisfied simultaneously only at a unique point, which can be denoted $(\bar{n}, \bar{w})$, where $\bar{n} > \bar{n}$ and $\bar{w} > \bar{w}$.

From the diagrams, it is relatively straightforward to see that $(\bar{n}, \bar{w})$ must
exist, i.e., that the two functional relationships between \( n \) and \( w \) must have a point of intersection in the region \( n > \tilde{n}, w > \tilde{w} \).

The point \((\tilde{n}, \tilde{w})\) is of some interest because it can be shown that, for an efficient contract, \( n < \min (n^0, \tilde{n}) \) i.e., that \( \tilde{n} \) is an upper bound on the level of employment, irrespective of how large the initial endowment of labour, \( n^0 \), might be. This follows from the fact that if \( n > \tilde{n} \), then \((n, w)\) must be such that both \( m(n) = w - [U(w) - U(\tilde{w})]/U'(w) \) and \( y(n) > w \).

If \( \tilde{n} \) is an upper bound on \( n \), it now follows that \( \tilde{n} > n^* \) is a necessary condition for a distributively efficient contract to be compatible with \( x \)-inefficiency. In other words:

**Corollary 4:** A necessary condition for a distributively efficient contract to imply technological \( x \)-inefficiency is that

\[
y(n^*) > w^* \text{ when } w^* \text{ is such that } \int \text{dlog}(U(w) - U(\tilde{w}))/\text{dlog}w = 1 \text{ for } w = w^*
\]

Finally, Figure 4 develops the implication of Corollary 3 for the sensitivity of employment to fluctuations in product demand. In the figure, product demand is assumed to follow some cycle over a series of short runs, which is reflected in cycles for \( \tilde{n} \) and \( \tilde{n} \) via movements in the marginal revenue product schedule \( m(n) \). However, cycles in \( \tilde{n} \) and \( \tilde{n} \) do not necessarily imply any change in the employment level \( n \): if \( n^0 \) is initially between the levels \( A \) and \( B \) in Figure 4(a), then the initial value of \( n = n^0 \) will maintain throughout. Alternatively, if \( n^0 \) is initially at point \( C \), say, then the rule \( n = \text{median} (\tilde{n}, n^0, \tilde{n}) \) implies that \( n \) will follow the path \( C D E F G \), at which point it reaches level \( B \),
and will remain there subsequently. An alternative scenario is provided by Figure 4(b). Here again, there is some uncertainty about the initial path for \( n \). Starting at \( C \), for example, \( n \) will follow the path \( C \rightarrow D \rightarrow E \rightarrow F \rightarrow G \). At \( G \), \( n^0 = \bar{n} \), and \( n \) will remain at \( n^0 \) even though \( \bar{n} \) falls away, until point \( H \) is reached. At \( H \), \( n^0 = \bar{n} \) and \( \bar{n} \) falls, bringing \( n \) down to point \( I \). Employment now remains constant at this level until \( J \), at which point it increases again, etc.

The implication of Figure 4 is that employment will be inert relative to product demand. After an initial settling down, cycles in product demand may produce a damped response in employment or no response at all. And if there is a response, then this will appear as a lagged response, with the product demand cycle leading the cycle in employment.
Figure 4

Comparative Static Effects of Cyclical Movements in Product Demand on the Level of Employment
References
