Development Research Center

Discussion Papers

No. 32

COMMODITY PRICE STABILIZATION
IN IMPERFECTLY COMPETITIVE MARKETS

David M. Newbery
World Bank

December 1981

Note: Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publication to Discussion Papers should be cleared with the authors to protect the tentative character of these papers. The papers express the views of the authors and should not be interpreted to reflect those of the World Bank.


Session 4: Market Structure and Stabilization Policies
1. Introduction

In Newbery and Stiglitz (1981a), we developed a reasonably comprehensive analysis of commodity price stabilization on the assumption that both producers and consumers were price takers. We recognized that trade in many commodities is subject to a variety of distortionary interventions, such as tariffs, quotas, domestic subsidies, acreage restrictions and the like, and analysed the impact of international commodity price stabilization in a distorted world economy (Newbery and Stiglitz, 1981a, Ch. 19, pp 272-83). However, our analysis of distorted markets assumed that the structure of protection and distortion would not be affected by international price stabilization, whilst individual producers and consumers remained price takers within this distorted framework. In short, governments were not assumed to be intervening to exercise international market power, and hence would have no reason to change the pattern of intervention in response to changes in the world market.

Clearly it would be desirable to try and extend the analysis of commodity, price stabilization to deal with imperfectly competitive commodity markets. At least some countries have a sufficiently high share of world trade in specific commodities to give them some market power.
Table 1: Shares of individual countries in world trade of commodities, Averages 1977–79

<table>
<thead>
<tr>
<th>Country</th>
<th>Product</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>&gt;50%</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>Maize</td>
<td>73</td>
</tr>
<tr>
<td>Phillipines</td>
<td>Coconut oil</td>
<td>71</td>
</tr>
<tr>
<td>Bangladesh</td>
<td>Jute</td>
<td>70</td>
</tr>
<tr>
<td>Malaysia</td>
<td>Palm oil</td>
<td>69</td>
</tr>
<tr>
<td>Argentina</td>
<td>Linseed oil</td>
<td>62</td>
</tr>
<tr>
<td>Australia</td>
<td>Wool (greasy)</td>
<td>60</td>
</tr>
<tr>
<td>Yalaysia</td>
<td>Copra</td>
<td>55</td>
</tr>
<tr>
<td>Malaysia</td>
<td>Rubber</td>
<td>51</td>
</tr>
<tr>
<td><strong>25–50%</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>All Cereals</td>
<td>49</td>
</tr>
<tr>
<td>Brazil</td>
<td>Sisal</td>
<td>42</td>
</tr>
<tr>
<td>USA</td>
<td>Wheat</td>
<td>39</td>
</tr>
<tr>
<td>Malaysia</td>
<td>Tin</td>
<td>36</td>
</tr>
<tr>
<td>Morocco</td>
<td>Phosphate Rock</td>
<td>34</td>
</tr>
<tr>
<td>Senegal</td>
<td>Ground nut oil</td>
<td>29</td>
</tr>
<tr>
<td>USA</td>
<td>Cotton</td>
<td>29</td>
</tr>
<tr>
<td>Tanzania</td>
<td>Sisal</td>
<td>28</td>
</tr>
<tr>
<td>India</td>
<td>Tea</td>
<td>27</td>
</tr>
<tr>
<td>Saudi Arabia</td>
<td>Oil</td>
<td>26</td>
</tr>
<tr>
<td>Guinea</td>
<td>Bauxite</td>
<td>25</td>
</tr>
<tr>
<td>Canada</td>
<td>Barley</td>
<td>25</td>
</tr>
<tr>
<td>Cuba</td>
<td>Sugar</td>
<td>25</td>
</tr>
</tbody>
</table>

The popular example today would no doubt be OPEC oil, though it is difficult to determine how far OPEC's market power is exercised by a well coordinated cartel, or subcoalition, or how far it depends on the actions of the largest member, Saudia Arabia. Before 1973, the examples which might have sprung to mind were Brazilian coffee or Indian tea. Table 1 lists countries which had more than 25% of world trade in a given commodity on average over 1977-79. Interestingly, Brazilian coffee does not appear (only 17%), whilst Saudi oil is 26%, near the cut-off point on the list. Of course, trade shares in specific commodities are an imperfect measure of market power, which depends on elasticities of demand and rest of world supply, as well as the extent of domestic production and transport costs, both of which define the site of the relevant market. Amongst the vegetable oils substitution possibilities will clearly limit the exercise of market power, whilst synthetic substitutes likewise constrain the fibre and rubber markets. On the other hand, transport costs may make regional markets more relevant than the whole world market for some commodities whilst a given country's commodity may be sufficiently differentiated from grades supplied by other countries that the country can exercise some market power despite its low overall market share. Nevertheless, despite all these qualifications, it seems reasonable to conclude that some countries (or cartels of countries) have some market power in some commodity markets, and that it is therefore worth enquiring how this market power affects the issue of international price stabilization.

The paper is organised as follows. Section 2 summarises the analysis of the case for international price stabilization in distorted markets.
referred to above in the first paragraph. Most distortions are probably not designed to exercise market power, yet clearly have an impact on the market, and hence on the case for stabilization. The remainder of the paper studies the case in which a producer exercises deliberate market power. In section 3 the various questions this raises are posed, and then answered in the remaining sections. Section 4 computes the optimum stocking rule for a monopolist and compares it with the competitive rule, asking whether he will undertake more or less price stabilization, and on what this depends, and what it implies for overall price stability, given the actions of the other agents. Section 5 investigates the impact of an international buffer stock agency on an imperfectly competitive market.
2. Do Market Imperfections Strengthen the Case For Commodity Price Stabilization?

In some commodities, notably foodstuffs, international trade is subject to a variety of distorting interventions. (See, e.g. Bale and Lutz, 1981; Lutz and Scandizzo, 1980.) These typically emerge when local producers find themselves in competition with imports, and are quantitatively important for sugar, cereals, rice, and, to a lesser extent, meat and dairy products. In most cases, it is difficult to argue that these policies benefit the importing country as a whole, for, with the notable exception of the U.S., the countries imposing the restrictions are small relative to world trade in the commodity. (Arguably, the EEC is the other main exception, though it seems unlikely that the main motive for the Common Agricultural Policy was to improve the EEC's terms of trade, though this may well be one of its consequences.) If this analysis is accepted (for some commodities, and some countries) then the market distortions are not evidence of market power, and it is possible to retain the simplifying assumption that producers and consumers are price takers, though the prices they face will no longer be uniform throughout the market. Much of the conventional analytics of price stabilization theory will still apply, though the distortions introduce additional complications, especially for the welfare analysis. In this section we ask whether the presence of trade distortions increases or reduces the amount of price stabilization which is desirable compared to that desirable in an undistorted market.

To the extent that domestic price support Programmes and sliding-scale tariffs are designed to stabilize domestic prices, distortions may already have reaped many of the potential benefits of price stabilization,
and hence weaken the case for additional buffer stock schemes. However, there are several, admittedly casual, arguments which suggest that distortions may strengthen the case for price stabilization, though little theoretical work has been done to make these arguments precise.

Price stabilization may generate larger benefits in the presence of trade distortions than in their absence for four quite different reasons:

(i) It is often argued that world and domestic prices are more volatile in the presence of trade restrictions than in their absence, and less use is made of the risk-sharing aspects of international trade. If stabilization is desirable under free trade then the greater instability under restricted trade makes it even more desirable. Between 1972 and 1974 the US wholesale grain price more than tripled, while in the EEC it rose by 20 per cent, and in the USSR it apparently remained unchanged. Johnson (1975) argues that the international market could have absorbed the modest world production shortfalls with only modest price increases, but because the shortfalls were only shared by part of the world market the price instability was severe.

At this point, it should be stressed that the link between trade restrictions, market fragmentation, and price volatility is not simple, and many types of trade policy may actually reduce price volatility, as we show below. Another apparent source of market fragmentation is when much of the trade is sold on long-term fixed-price contracts, with a smaller fraction sold on the spot market. The crucial determinant of the volatility of the spot market is then the extent to which consumption decisions are guided by the spot market or the long-term contract price. If consumers are willing to resell on the spot market when the spot price is high, and buy when the price
is low, then the long-term contract is equivalent to hedging on a futures market, with all trade effectively passing through the spot market. In such cases the actual volume of spot trade is a poor guide to the effective volume which determines the price and its volatility. While perhaps not so important for agricultural commodities (except for bilateral trade), such contracts are pervasive for many minerals.

(ii) It has been argued that private speculators are less willing to stockpile and reduce price fluctuations in the presence of market distortions (FAO, 1975; Gray, 1960). The argument is that in such markets speculators have to forecast the actions of governments who have taken responsibility for regulating the markets. There is an important and unresolved question as to how speculators would react to the presence of agents with considerable market power, such as governments fixing tariffs or exchange rates to benefit the domestic economy. A similar issue arises in foreign exchange markets when Central Banks engage in 'managed' or 'dirty' floating. Speculators may believe that this market power will be used to manipulate the market in such a way as to generate profits for the intervening authority at the expense of the speculators (cf. Hart, 1977). If so, the speculators will be less willing to undertake stabilizing speculation, and there will be insufficient stabilization, and additional price stabilization is needed, which would not be required in a competitive market. This second argument is mainly about who will have to do the stabilization, or how much stabilization is needed, rather than about the benefits of the stabilization.

(iii) If the total world consumption of commodities subject to distortion is increased, then there is an additional gain (equal to a reduction in the excess burden of the distortion) equal (roughly) to the increase in
supply multiplied by the difference between consumer and producer price. This benefit is an important component in cost-benefit analysis (see e.g. Boadway, 1974, or Harberger, 1971). The argument is then that price stabilization induces a positive supply response which then generates extra efficiency gains (over and above those realized on competitive markets). Whether in fact price stabilization will lead to increased supply is discussed in Newbery and Stiglitz (1981a, Part V) where it will be seen that the answer is by no means obvious.

(iv) It has been cogently argued (1975) that if price stabilization is achieved by creating buffer stocks, then this will provide an impetus to highly desirable trade liberalization. The argument here is that restricted trade and the resulting price volatility make both the availability and cost of supplies so uncertain to importing countries that they create farm support programmes to ensure supplies, and impose further restrictions to make them viable. If the world price were stabilized and supplies guaranteed, then these restrictive measures would be unnecessary and might be dismantled. Even where the original intervention was inefficient its true cost may remain obscure until the price is stabilized. Once the cost is evident, the source of the inefficiency might be removed.

This last argument may well be the most important, but it requires a theory of the choice of trade policy before it is possible to say how institutional change may affect the structure of trade distortion. Newbery and Stiglitz (1981a, Chapter 24), goes some way towards a theory of the choice of trade policy under risk, and suggests that countries may actually increase protection if world prices are successfully stabilized, but this
ignores the other reasons for trade policy, specifically market power. Buffer stocks by making world demand more elastic may reduce market power and hence lower the optimum tariff. These issues we discuss further below.

On the face of it these arguments seem quite persuasive, but in the absence of any formal model it is difficult to see whether the arguments are generally true, true under restrictive assumptions, or, indeed, false.

Sewbery and Stiglitz (1981a, Ch. 19, pp. 276-82) construct a simple model to analyse the first argument, and the same model could readily be extended to analyse the third argument. The model has linear demand schedules to permit cross-country aggregation and an analytical solution for the world market clearing price. It ignores supply responses (though linear supply schedules could easily be included), distributional issues, attitudes to risk, and concentrates on the aggregate efficiency gains of stabilization in the presence of distortions.

We find that linear trade policies (in which there is a linear relationship between domestic and world prices, as with tariffs) do not affect the benefits of price stabilization significantly, but non-linear policies do. In particular, if countries use quotas rather than tariffs, then even if they do not change with price stabilization, there may be additional benefits over and above the benefits conventionally calculated (assuming no distortions). The reason is that with linear policies the average degree of distortion does not change as the variability of prices change, but with non-linear policies it may be reduced. The latter result is sufficiently interesting to bear repeating the argument contained in Newbery and Stiglitz (1981a, Ch. 19, pp. 280-2).
Suppose that in every country $i$, demand is linear and non-stochastic, while supply experiences additive risk:

\[ D_i = d_i - a_i p_i \]  
\[ S_i = b_i + \tilde{\varepsilon}_i, \quad E\tilde{\varepsilon}_i = 0, \quad E\tilde{\varepsilon}_i^2 = \sigma_i^2 \]  

Here $p_i$ is the domestic price in country $i$ and $\tilde{\varepsilon}_i$ is the randomness in supply, assumed uncorrelated with other countries' randomness, $\tilde{\varepsilon}_j$.

Restrictions on trade are modelled as follows. Domestic excess demand is

\[ Z_i = c_i - a_i p_i - \tilde{\varepsilon}_i, \quad c_i = d_i - b_i, \]  

but this is modified to give a demand for imports

\[ M_i = c_i' - a_i p - a_i \tilde{\varepsilon}_i, \]  

where $p$ is the world price, and $a_i$ measures the degree to which domestic fluctuations are transmitted abroad. Equating (3) and (4) gives the relation between the domestic and world price

\[ p_i = \frac{a_i'}{a_i} p + \frac{1}{a_i} \{ c_i - c_i' - (1 - a_i) \tilde{\varepsilon}_i \} = \left( \frac{a_i'}{a_i} \right) p + \tilde{u}_i. \]  

A pure tariff-cum-export subsidy has $a_i'/a_i = 1 + t_i$, $c_i' = c_i$, $t_i = 1$, but most distortions do not fit easily into this framework since they are typically non-linear. Thus an import quota which always binds has $a_i = 0$, $a_i = 0$, but if it only binds some of the time $p_i$ will be a non-linear function of $p$ and $\tilde{\varepsilon}_i$.

Thus a quota set at a fixed level $q_i$ implies the following import demand equation (compare equations (3) and (4):

\[ M_i = \begin{cases} 
    c_i - a_i p - \tilde{\varepsilon}_i, & \text{if } \tilde{\varepsilon}_i \geq k_i(p) \\
    q_i - \tilde{\varepsilon}_i, & \text{if } \tilde{\varepsilon}_i < k_i(p) 
\end{cases} \]
where \[ k_1(p) = c_1 \cdot q_1 - a_1 p \] (7)
is the level of domestic supply risk below which the import quota binds. The domestic price is then
\[
P_i = \begin{cases} 
p, & \theta_i > k_1(p) \\
1 \frac{a_1(c_1 - \theta_i - \bar{q}_i)}{\bar{q}_i}, & \theta_i < k_1(p) 
\end{cases}
\] (8)
which is a non-linear function of \( p \).

It is obviously difficult to solve for the international price when a large number of countries have stochastic non-linear excess demand functions, but we can gain some qualitative insight by examining the effect of a quota on a single small country. A country is small in this context if \( \text{Cov}(\bar{q}, \theta_i) \) is sufficiently small to be ignored, so that the actions of the country are assumed not to affect the world price.

Moreover, since \( \theta_i \) is uncorrelated with \( \theta_j, \theta_i \) will therefore be uncorrelated with the world price \( \bar{p} \). In the absence of international price stabilization, \( k_1(\bar{p}) \) is a random variable, and the probability of the quota restricting trade is
\[ \pi_1(\bar{p}) = \text{Pr}\{\theta_i > k_1(\bar{p})\} = G(\theta_i(\bar{p})) \]
where \( G(\theta_i) \) is the distribution function of \( \theta_i \), while if the world price is perfectly stabilized at \( \bar{p} \), then the probability becomes
\[ \pi_1'(\bar{p}) = \text{Pr}\{\theta_i < k_1(\bar{p})\} \]
If the probability density function of \( \theta_i \) is \( g(\theta_i) = dG/d\theta_i \), symmetric about its mean of zero, and unimodal, as shown in Fig. 1, then \( \pi_1(\bar{p}) \) is the shaded area.
We wish to know whether the proportion of the time that the quota binds increases with price stabilization; i.e., if

$$\pi_i(\bar{p}) > \pi_i^*(\bar{p}),$$

which will be the case if $$\pi_i$$ is a concave function of $$p$$. Since $$p$$ is a linear function of $$k_i$$ from equation (7), this is equivalent to

$$\frac{d^2\pi_i}{dp^2} = a_i^2 \frac{d^2\pi_i}{dk_i^2} = a_i^2 \frac{dg_i(k_i)}{dk_i} < 0$$

or $$g$$ must be increasing over a range of values near $$k_i(\bar{p})$$. This is equivalent to requiring $$k_i(\bar{p}) < 0$$, as in Fig. 1, or the quota must bind less than half the time. If $$k_i(\bar{p}) > 0$$, and the quota binds more than half the time, then price stabilization will reduce the proportion of the time it binds.

Obviously, the more often the quota binds, the greater will be the disparity between the domestic and world price, and hence the greater will be the degree of distortion and the total cost of distortion (ignoring, as mentioned before, the distribution of the cost and the costs of risk aversion).

Summarizing, we can say that price stabilization which does not
change the mean world price leads to a reduction in the degree of distortion caused by a quota, provided that the quota binds less than half the time after stabilization. It, however, the quota then binds more than half the time, price stabilization will have increased the degree of distortion. In quite plausible circumstances, then, price stabilization will generate some additional efficiency gains by reducing the average level of distortion in the world economy.

Moreover, the change in average distortion has other effects as well. The average domestic price is an increasing function of the degree of distortion as measured by the probability $π_1$, because the quota raises the domestic price relative to the no-quota case. For a small country whose supply risk is independent of world risk, price stabilization has no effect on average profits in the absence of a quota (assuming as before no change in the mean world price) but with a quota average profits fall, since the average domestic price will fall with stabilization. (We continue to assume that the quota binds less than half the time after stabilization.) This fall in average profits may lead producers to reduce supply. Since the quota encourages inefficiently high domestic supply, this fall in supply will improve world efficiency, and generate a world welfare gain equal, roughly, to the fall in supply times the average excess of domestic over world price. This factor will augment the efficiency gains due to the fall in average distortion. If we continue to trace through the effects of the quota, we note that the average demand for imports rises with stabilization, since the quota restricts imports less often, compared with the no-quota situation. This will tend to raise the world price, and to offset the fall in supply. Given assumptions about the distribution of $π_1$, p, and the production function it would be possible to identify each of these effects and find their overall size and direction.
3. **Price Stabilization in a Monopolized Commodity Market**

In a competitive market economy with a complete set of futures and insurance markets the market equilibrium is Pareto efficient and market intervention by, for example, 2x1 International Buffer Stock Agency, to further stabilise commodity prices, would be inefficient. In such a market, private agents would provide the efficient level of storage activity and price stabilization. Now, whilst it can frequently be argued that commodity markets are competitive, it is clear that the market structure is not complete, lacking insurance markets and most futures markets. In such cases Newbery and Stiglitz (1982) show that in general a competitive market equilibrium is not even constrained Pareto efficient, so that in principle a Government could make everyone better off by market intervention (e.g. setting taxes, or subsidising storage activities).

However, they also show that if agents are risk-neutral and hold rational expectations, then the competitive equilibrium will be efficient. The reason is obvious. If agents are risk-neutral, and hold common (objective) beliefs about the economy, then they would not wish to trade on risk markets even if they existed. Similarly, if they hold common (objective) beliefs about futures prices, they would not wish to trade on futures markets, and in both cases such markets would be redundant. Their absence therefore makes no difference to the market equilibrium, which will remain Pareto efficient.

It will greatly simplify the analysis if we continue to make these assumptions of risk-neutrality and rational expectations, though a full analysis would clearly relax these assumptions. (See, in particular, Newbery
and Stiglitz, 1981a, Chapters 13, 14.1, 15 and 30; and Newbery and Stiglitz 1981b). In making these assumptions we are following tradition (e.g. Gustafson, 1958; Samuelson 1971), though most writers appear unaware of the strength of these often implicit assumptions. Under certain additional assumptions (stable linear demand, stationary stochastic supply, and some restrictions on various parameters) it is possible to solve analytically the competitive (and thus efficient) stockpiling rule (see Newbery and Stiglitz 1981a, Ch. 30; 1981b). At this stage the main point to notice is that the rule is very different from the favored intervention rule proposed for International Buffer Stock Schemes, which would attempt to keep prices within a specified band width. The efficient rule would store a fraction (somewhat greater than one half) of the excess of harvest plus last year's carryover above some critical amount. (See below.) This rule remains a good approximation (and a natural starting point for interactive numerical solution) when the simplifying assumptions do not hold. (See Newbery and Stiglitz, 1981a, Ch. 30; Gustafson, 1958).

Given this benchmark of the competitive stocking rule for the efficient level of price stabilization, a number of interesting questions can now be asked of an imperfectly competitive commodity market. Again, in the interests of simplicity, we shall confine attention to a monopolist facing a large number of competitive consumers. First, on the assumption that the producer does the storage and price arbitrage, how does the amount of storage and hence price stabilization compare under monopoly with the efficient level? Do monopolists perform more or less' price stabilization? The answer to this question immediately raises related questions - under what circumstances do monopolists
do less stabilization, and which agents, consumers or producers, will carry the stocks? Again, the answer will affect the next question, which is, if consumers have a comparative advantage in carrying the stocks, how does the change in their demands (for consumption and storage) affect the market power of the monopolist?

The next set of questions concern the consequences of establishing an International Buffer Stock to stabilize prices. First ask what effect this would have on the market if the Buffer Stock followed conventional rules, and did not attempt to use its potential market power. Do different buffer stock rules (e.g. band width rules or competitive rules) differ in their robustness to monopoly manipulation?

Finally, what might happen if the Buffer Stock Agency were able to use its market power to countervail the monopolist? (The more plausible scenario, in which the Agency acts as a cartellizing influence on the primary producers is subsumed in the analysis which treats the monopolist as stockholder.)

4. The Optimal stock rule for a monopolist

The simplest model which allows an analytical solution in the competitive case (see Newbery and Stiglitz 1981b) contains the following elements:

(i) There is a stock \( S_{t-1} \) carried forward from the previous year.

(ii) To this is added a random harvest \( h_t \), so that at date \( t \) the amount available for consumption, \( C_t \), and for storage, \( S_t \), is the total supply, \( x_t \):

\[
x_t = h_t + S_{t-1} = C_t + S_t.
\]
We assume that there are no losses in storage (though these are easy to handle – see Samuelson, 1971), and that planned production does not change each year (e.g., for a tree crop like coffee). If weather and other random factors are serially uncorrelated, this implies that the harvest is also 2 serially uncorrelated, stationary random variable. We choose units so that the average harvest is one unit:

$$Eh_t = 1, \text{Var } h_t = \sigma^2, C_v(h_t, h_{t'}) = 0, t \neq t'. \quad (10)$$

Finally, we assume that demand is stationary and non-stochastic, so that the marker clearing price $p_t$, depends only on current consumption, $c_t$:

$$p_t = P(C_t) \quad (11)$$

When it comes to solving explicitly for the storage rule we shall assume that the demand schedule is linear, with elasticity $\varepsilon$ at the pre-stabilization mean price, $\bar{p}$. If, on average, consumption equals harvest, so that stocks neither continually increase nor decrease, the (inverse) demand schedule will be

$$p(C) = \bar{p}(1 - \frac{1}{\varepsilon}(C-1)) \quad (12)$$

Social welfare can be measured in money units, since we assume risk neutrality and ignore income distribution, as

$$U(C) = \bar{p}(1 + \frac{1}{\varepsilon})C - \frac{1}{2\varepsilon}C^2 \quad (13)$$

so that

$$p(C) = \frac{dU}{dC} \bar{p}$$
The competitive storage rule can be found by maximizing expected social welfare (the approach taken by Gustafson, 1958, and Samuelson, 1971) or derived directly from the competitive arbitrage conditions. It is easy to demonstrate the equivalence of these two approaches (see Newbery and Stiglitz, 1981a) so we shall follow the second, more transparent approach.

If the annual storage costs excluding interest is \( k \) per unit at the margin, then a speculator who buys after the harvest at price \( p_t \) and sells after the next harvest at price \( p_{t+1} \) will have made a marginal profit (in money terms at date \( t+1 \)) of

\[
p_{t+1} - (p_t + k)(1+r)
\]

per unit, where \( r \) is the rate of interest. If speculators are risk neutral then they will store nothing if

\[
E \ p_{t+1} < (p_t + k)(1+r)
\]

but will otherwise continue to store until they have driven up the current price and driven down the expected future price to the point where

\[
(p_t + k)(1+r) = E \ p_{t+1}
\]

These two cases can be combined in the fundamental competitive arbitrage equation

\[
p_t + k \geq \delta \ E \ p_{t+1} \quad \text{complimentarily}
\]

where \( \delta = 1/(1+r) \) is the discount factor. Our first objective is to investigate the form of the competitive storage rule, and this appears to
be difficult because the expected price next year depends on planned carryovers next year, which in turn depends on the planned carryovers in the following year. To find the storage rule now we need to know the storage rules to be followed hereafter. To solve the problem we need to start with a terminal date at which the carryforward is specified, and hence known, and work backwards. This is the standard method of solving stochastic programming problem, but with a long time horizon is computationally demanding. If, however, the time horizon is allowed to lengthen and if the expected present discounted value of terminal stocks, $E^T S_T P_T$, tends to zero, then the influence of the future on the present also tends to zero, and we can instead of looking for a particular solution (which depends on $S_T$), look for a stationary stock rule which is independent of terminal stocks, and hence independent of time. Since price only depends on consumption from (11), and since from (9)

$$C_t = x_t - S_t$$

it follows that we are looking for a stock rule which is a function only of total supply $x_t$:

$$S_t = f(x_t) \geq 0$$

Contrast the form of this rule with the bandwidth rule, where if the demand is

$$C = D(p)$$

and the price band has upper and lower intervention points $p^u$, $p^l$, then the stock rule is
The bandwidth rule has storage depending on both supply and ha\footnote{est over the central range, and thus is not a competitive arbitrage rule. More importantly, it follows that it is not an efficient method of stabilising consumption in a competitive market, which, given our assumptions about risk neutrality, is the basic reason for stabilising prices.

To return to the problem of finding the competitive stock rule, we seek a function $f(x)$ which solves equation (14), which, given (11), (15) and (16), can be written

$$p(x_t - f(x_t)) + k \geq \beta \mathbb{E} p(\tilde{h}_{t+1} + f(x_t) - f(\tilde{h}_{t+1} + f(x_t)))$$

complimentarily (18)

The problem in solving this arises from the right hand side, in the term $f(h + f(x))$. One key feature of the rule is, however, immediate.

The optimum storage rule is non-linear. This follows because stocks must be non-negative, so $f(x) = 0$ below some critical value of $x$, for which

$$p(x_0) + k = \beta \mathbb{E} p(\tilde{h} - f(\tilde{h})).$$

(19)
In the Appendix we show how to solve equation (18) for the special case of a two point distribution:

\[ h = \begin{cases} 
1 + u & \text{Prob } \rho \\
1 - yu & \text{Prob } 1 - \rho
\end{cases} \gamma = \frac{\rho}{1-\rho} \]  

(20)

so that

\[ \bar{h} = 1, \text{Var } h = \gamma u^2 = \sigma^2. \]

In this case the competitive stocking rule is linear beyond \( x_0 \):

\[ f(x) = a(x - x_0) \quad x_0 \leq x \leq x_m \]  

(21)

where

\[ a = \frac{1 + \beta - \sqrt{(1 + \beta)^2 - 4 \beta \alpha}}{2 \beta \alpha} \]

\[ x_o = \frac{a - \alpha \beta}{1 - \alpha \beta}, \quad x_m = \frac{1 - \alpha \beta}{1 - \alpha} \]  

(22)

and

\[ a = 1 + \varepsilon (1 - \beta + k/\bar{p}), \]

provided the parameters lie in a certain range:

\[ 1 - \gamma u + \alpha (x_m - x_o) < x_o < 1 ru. \]  

(23)

Given specific values of \( \beta, \varepsilon, k/\bar{p}, u, \) and \( \rho \), \( a \) and \( x_o \) can be found and equation (22) checked to ensure the validity of (21).

So far we have derived the competitive rule, but fortunately the monopoly stocking rule now follows immediately, for the monopolist is interested in arbitraging marginal revenue, rather than price. Since for a linear demand schedule the marginal revenue is also linear, the monopolists' problem is isomorphic to the competitive problem. Instead of the arbitrage rule of equation (14) we have
where $m_t$ is the marginal revenue, and in this case, whilst

$$p(c) = \bar{p} \left( 1 + \frac{1}{\bar{e} - e} \right)$$

$$m(c) = \bar{p} \left( 1 + \frac{1}{\bar{e} - e} \right)$$

Instead of the storage rule having the form (see Appendix)

$$f(x) = \frac{1}{1 + \beta} \left[ x - a + \beta Ef \{ \bar{h} + f(x) \} \right], \ x \geq x_0$$

it takes the form

$$f(x) = \frac{1}{1 + \beta} \left[ x - \frac{1}{2} (a + \beta) + \beta Ef \{ \bar{h} + f(x) \} \right], \ x \geq x'_0$$

where $x'_0$ now satisfies (22), replacing $a$ by $1 \ (a + \beta)$.

It is interesting to note that the slope of the stocking rule is unchanged, but the amount of stocking is unambiguously increased, since $\beta < \frac{1}{a} < a$. Fig. 2 illustrates.

Fig. 2 Monopoly and Competitive rules compared
Numerical Example

\[ \rho = 0.75 \quad \text{(Bad harvests once every four years)} \]

\[ u = 0.20 \quad \text{(Harvests 1.2 or 0.6, coefficient of variation (CV) of output 35%)} \]

\[ \varepsilon = 2.5 \quad \text{(elasticity, unstabilized CV of price = 13.9%)} \]

\[ \beta = 0.95 \quad \text{(5% interest rate)} \]

\[ k/p = 3\% \quad \text{(storage costs)} \]

then \[ a = 1.2 \]

\[ \alpha = 0.6835 \]

Competitive storage rule: \[ f^C(x) = 0.6835(x - 1.2), \quad x \geq 1.2 \]
(Thus, since harvests never exceed 1.2, there is never any storage in the competitive case, and prices would not be stabilized any further.)

Monopoly storage rule: \[ f^m(x) = 0.6835(x - 0.9563), \quad x \geq 0.9563 \]

(Maximum supply \( x_m = 1.7263 \), maximum stockpile 0.5263)

This stocking rule implies average stocks of 25.6% of average harvest, which is substantial. Moreover, it reduces the CV of consumption from 35% to 22%, and lowers the CV of prices from 14% to 9%.

The effect of stockpiling in this example is to substantially improve monopoly profits and reduce average consumer surplus relative to the no stabilisation monopoly equilibrium. The extra loss of consumer surplus averages about of (riskless) consumer expenditure on the crop.

The results of this section can now be summarized in a Proposition 1. A monopolist facing a stable linear demand schedule will undertake more
price stabilization through storage activities than a competitive market producing the same average supply, and will thereby be able to exploit the consumer more effectively.

This immediately raises the next question.

4.1 When do monopolists stabilize more than the competitive market?

Consider a situation in which, with a given level of current supply, it is just not worth a competitive market storing, so that

\[ p(Q_0) + k = \varepsilon E_p(Q_1) \]  

(25)

where period 0 is today, and period 1 is the next period. In such cases it will pay a monopolist to store if

\[ \frac{dR(Q_0)}{dQ_0} - k < \frac{BE}{dQ_1} \frac{dR(Q_1)}{dQ_1} \]  

(26)

where \( R(Q) \) is revenue from the sale of \( Q \). This can be rewritten as

\[ p(Q_0)(1 - \frac{1}{\varepsilon_0}) + k < \varepsilon E_p(Q_1)(1 - \frac{1}{\varepsilon_1}) \]

where \( \varepsilon_i = \varepsilon(Q_i) \) is the elasticity of demand when sales are \( Q_i \). Substituting from (25), the monopolist will store more if

\[ \frac{\varepsilon_0}{\varepsilon} < 1 - \frac{(1 + r)k}{p} \]  

(27)

where \( r \) is the rate of interest and \( \bar{\varepsilon} \) is an average price elasticity:

\[ \bar{\varepsilon} = \frac{E_p}{E_p/\varepsilon} \]

The accumulated proportional storage cost, \( (1+r)k/p \), takes values between 1.3% and 5.1% for the six core commodities considered by Newbery and
Stiglitz (1981a, Table 20.7, p 295). Equation (27) demonstrates immediately that if the demand schedule has constant elasticity the monopolist will do less stabilization than a competitive market, but this result is very sensitive to the shape of the demand schedule. A slight fall in price elasticity at lower prices (by, e.g., 5–10%) will induce the monopolist to undertake more stabilization. With a linear demand schedule (for which the elasticity falls rapidly) this tendency can be quite pronounced, as the previous section demonstrated.

Of course, there may be quite different reasons why a monopolist might prefer price stability. He might fear that temporarily high prices would induce new entrants into the industry and reduce his future profits, and hence might limit price (or, more to the point, maintain the equivalent of excess capacity in the form of buffer stocks to deter entry. See Spence, 1977). Ignoring such strategic issues, though, we can summarise these results as follows.

Proposition 2. A monopolist facing a stable constant elasticity of demand schedule or a schedule whose price elasticity falls as price rises will undertake less price stabilization than a competitive market (producing the same average supply). Whether or not the monopolist does more or less stabilization depends sensitively on the shape of the demand schedule.

If monopolists undertake less stabilization than the competitive market, then we have to consider the possibility that other agents (consumers, or, more probably, independent stock holders) will perform further arbitrage, which will in turn alter the gross demand schedule (i.e. including demands for additions to storage) facing the monopolist. This may lead the monopolist to change his behavior. Thus it is important to ask:
4. Who performs the storage activities in an imperfectly competitive market?

A large number of factors are relevant even in the simpler competitive case, and some of them are reviewed in Newbery and Stiglitz (1981a, Ch. 14, pp. 195-6). If agents are not risk neutral, then stocks provide a convenience yield in the form of insurance, and the agent whose-net cost of storage (carrying costs less convenience yield) are lowest will have a comparative advantage in storage. Thus, if the aggregate demand facing producers is elastic, and prices are negatively correlated with output (e.g. if supply shocks generate price instability) then producers derive a positive convenience yield for stocks. If, on the other hand, demand fluctuations are the source of the price instability, consumers will enjoy a positive convenience yield unless prices vary with their incomes. Intermediate producers (i.e. agricultural processors) enjoy a positive convenience yield since their profits will be negatively correlated with input prices.

On the other hand, transport and handling costs will convey a cost advantage on producers, since downstream users will have to incur additional interest costs on the marketing margin. However, economies of scale in storage, access to cheaper finance, hedging facilities, better market information, and risk pooling advantages may offset these cost disadvantages and confer the comparative advantage on middle men. The following section assumes that all these various factors cancel out, so that producers, consumers and middle men face equal net storage costs.

Consider the case in which the consumption demand schedule is stable and has constant elasticity of demand. The argument of the previous section then shows that, if only producers stockpile, the monopolist would perform
less stabilizing action than a competitive market. Consequently, there would be enough remaining price variability to justify further price stabilization by competitive agents, assuming they face no cost disadvantage. What would happen if such agents intervened and collectively achieved a competitive degree of price stabilization?

The analysis of Newbery and Stiglitz (1981a, Ch. 30) and Gustafson (1958) shows that the qualitative form of the stocking rule remains unchanged as the shape of the demand schedule and the form of the probability density of harvests varies. The level of supply below which there is no stocking \( (x_0) \) does depend on two features, but can be found relatively easily. The stocking rule will typically be a convex function of supply, whose general shape is readily approximated by a piecewise linear form. (Newbery and Stiglitz, 1981a, pp. 435–438.) If competitive consumers undertake the storage activity, then the producer will face a displaced gross demand schedule (for consumption plus storage additions) as shown in Fig. 3.

![Fig. 3 Effective demand schedule facing monopolist](image)
The form of the demand schedule is approximately

\[ C = q + \tilde{S} = p^{-\varepsilon} \quad q \leq x_o - \tilde{S} \]

\[ C = ax, + (1 - a)(q + \tilde{S}) = p^{-\varepsilon} \quad q \geq x_o - \tilde{S} \]  \hspace{1cm} (23)

where \( \tilde{S} \) is the (random) carryover from last year, and \( x_o \), defined by (21), represent the linear approximation to the competitive stocking rule. The net effect of the consumer stockpiling is to make the elasticity of demand increase as the supply increases and price falls, and hence to further discourage the monopolist from undertaking any storage. Thus, providing there is no cost disadvantage facing the consumers, they will presumably displace the monopolist and all storage will be undertaken by consumers.

Summarising this argument we have, depending on who does the stabilization:

Proposition 3. If consumers face the same or lower net storage costs (including interest charges) than producers, then prices should be either more stable, or at worst, no less stable, than in competitive markets handling the same average production.

If, on the other hand, consumers face cost disadvantages, these prices may remain less stable under monopoly.

5. The Effect of International Buffer Stocks on Imperfectly Competitive Markets

So far we have restricted attention to either the producer or the competitive consumers, either of which might carry stocks and hence stabilize prices. If, however, an International Stabilization Authority is set up, then it will presumably be given rules for operating its intervention.
activities, and these rules may affect the market facing the monopolist. It is interesting to consider three cases. The first would enquire into the likely consequences of introducing the favored Band With rule into the imperfectly competitive market. The second case would examine the consequences of introducing the buffer stock rule which would be optimal in a competitive market, but which would not necessarily be best for an imperfectly competitive market. Finally, the most difficult but most interesting question is to ask that rule would be optimal in an imperfectly competitive market.

5.1 Bandwidth Rule

This rule is specified in equation (17) and generates an effective demand facing the producer as shown in Fig. 4 below (assuming sufficient stocks are on hand, i.e., at least $h_1$).

![Fig. 4. Effective demand under the Bandwidth Rule](image-url)
The same figure shows the marginal revenue facing the monopolist, and it is this schedule which is relevant for his production and storage decisions. First, note that if the monopolist retains any short run supply discretion once the stochastic uncertainty has been resolved, then the bandwidth rule can actually destabilise prices. For suppose that in bad years the supply schedule is $S_1$, whilst in good years it is $S_2$. Without stabilization the price would be either $p_1$, or $p_2$, but with stabilization the monopolist would choose to supply $h_1$, or $h_2$ (significantly more variable) at prices $p^u$, $p^l$.

It is quite likely in this case that the buffer agency will overaccumulate stocks since the floor price $p^l$ may generate a very attractive marginal revenue for the monopolist.

If the producer has negligible short run supply discretion (i.e. the case considered in the previous section) then the appropriate question to ask of the international buffer scheme is whether it is sustainable, or whether it will induce speculative attacks which render it non-viable.

The issue of vulnerability to speculative attack has been addressed by Salant (1979) and Newbery and Stiglitz (1981, p 413-4). In a competitive market in which the upper and lower price bands are set at $\tilde{p}(1+b)$, $\tilde{p}(1-b)$, it will pay producers or speculators to speculate an a price rise whenever the price falls to the floor price unless

$$\beta(1-b) + k > \beta E \tilde{p} = \bar{\beta}$$

or

$$b < 1 - \beta + \beta \bar{p} = r + c, \quad c \equiv k/\bar{p}$$

If the bandwidth is set further apart than this, then private agents will speculate against the stockpile. Moreover, even if the buffer agency sets
the bandwidth appropriately, then if its own stocks are insufficient to sustain the ceiling price, the right hand side of (29) will be increased, since the expected next period price will be biassed up by the probability of a shortfall too large to be net by the agency's buffer stock, resulting in a price above \( \bar{p}(1 + b) \) and hence raising \( \bar{E}p \) above \( \bar{p} \).

In a non-competitive market the agency is subject-to additional speculative pressures from the monopolist, for it is clear in Fig. 4 that the expected marginal revenue next period is substantially raised by the intervention prices. Even if it would not pay a competitive speculator to buy now in the expectation of future profit, it might well pay a monopolist. As an example, consider the linear demand schedule of equation (12), and suppose the distribution of harvest \( h \) is symmetric about \( \bar{h} = 1 \). Let

\[
\text{Prob} \left\{ h \geq 1 + \epsilon b \right\} = \frac{1}{2} (1 - \Pi)
\]

so that

\[
\text{Prob} \left\{ 1 - \epsilon b < h < 1 + \epsilon b \right\} = \Pi
\]

then, with no producer carry forward expected marginal revenue becomes

\[
Em = \bar{p} \left\{ \frac{1}{2} (1 - \Pi)(1 - b + 1 + b) + \Pi (1 - \epsilon) \right\}
\]

\[
= \bar{p} \left( \frac{1}{2} - \Pi/\epsilon \right)
\]

instead of \( \bar{p} (1 - \epsilon) \). The monopolist will thus store if his current harvest, \( h \), satisfies

\[
1 - \epsilon b < \bar{h} < 1 + \epsilon b
\]

(i.e. there is no buffer stock intervention), and

\[
1 + \frac{1}{\epsilon} - \frac{2h}{\epsilon} + k/\bar{p} < \frac{\beta Em}{\bar{p}} = \beta (1 - \epsilon)
\]
\[ 1 + \varepsilon b > h > \frac{1}{2} \left( 1 + \varepsilon \Pi + \varepsilon (r + c) \right) \]

which is a very weak condition indeed (Note \( \Pi \) depends on \( b \)). Thus if \( \varepsilon = 0.95 \), \( c = 32 \), \( h \) is roughly normal with CV of 242, \( \varepsilon = 3 \), then a competitive band width rule would require \( b \leq 8\% \), but if \( b \) is set at 8\% (so that \( \Pi = 0.67 \)), then the nonopolist will store on his own behalf whenever \( 0.94 < h < 1.24 \), i.e. 35\% of the time.

It is clear that the same problem will arise even when the consumer demand schedule is non-linear. Thus if an international buffer agency is set up and instructed to maintain prices within a bandwidth, then any monopolist will probably (depending on the various parameters) be able to speculate against the agency, buying whenever the price falls near, but not quite as far as, the floor price in order to sell the following period (or periods).

5.2 The competitive rule

Suppose, however, that the buffer agency is instructed to follow the competitive storage rule (i.e., the rule relating storage to supply which would prevail in a competitive market). Such a rule would have two obvious rationalas - it would break even if there were constant storage costs (or make a profit with increasing marginal costs), and it would be efficient given the already mentioned assumptions of risk neutrality etc. It would have the additional advantage when confronting a monopolist subject to stochastic supply shocks of being non-manipulable. There are two possible cases, depending whether the monopolist would undertake more or less stabilization than a competitive producer. If he would undertake more (e.g. the demand schedule is linear) then the buffer agency will never intervene,
since the remaining price variability would be inadequate to justify competitive intervention. If he would undertake less, (e.g. if he faced a constant elasticity demand schedule) then the agency would have a comparative advantage in storage, as discussed above in section 4, and would undertake all storage, with the monopolist performing no storage.

There remains one curiosum, which is a consequence of the kink induced in the effective demand schedule at the point at which the agency first adds to storage ($x_0 - S$ in Fig. 3). Newbery (1978) demonstrated that it would in general pay a monopolist facing a kinked demand schedule (one which becomes more elastic just below the kink) to introduce randomness into an otherwise non-random market. The reason is that the monopolist can exploit the discontinuity in the marginal revenue facing him. (Contrast the kinked demand theory of oligopoly, in which the demand schedule becomes less elastic below the kink, which provides an explanation for price stability.)

6. Countervailing International Price Stabilization

If some commodity markets are recognized to be monopolised or cartellised, then instead of following a passive role, more suited to a competitive market, the international stabilization agency might be set up to try and mitigate the monopolistic distortions. At this stage it should be pointed out that the political impetus behind the call for an Integrated Programme for Commodities (by UNCTAD in Nairobi, May 1976) was more interested in countervailing the imperfections on the buying side by creating producer cartels, rather than defending the consumers against monopolistic producers, and it would be interesting to extend the analysis
to consider monopsony importers facing competitive producers, but that will have to await another occasion. Some of the insights obtained by studying the more tractable monopoly producer case may, however, be useful.

This case has already been the subject of a most interesting study by Nichols and Zeckhauser (1977). They considered the case of a large consuming nation (such as the U.S.), containing many competitive final consumers, facing a cartel (obviously OPEC provided the notion for addressing the question, but they drew attention to cartels in bananas, bauxite, coffee, copper, iron ore, mercury, phosphate, and tin). They demonstrated that under certain circumstances it would pay the consuming nation to build up a strategic stockpile, whose presence would suppress prices in future periods, even when the supply conditions of the producing cartel were non-random and stationary, so that on competitive price arbitrage grounds there would be no case for stockpiling.

The reference model had the following features. The cartel maximizes the present discounted revenues net of production costs by setting the supply price each period. The consuming nation maximizes the present value of net consumers' surplus (consumers' surplus less storage and interest charges on the stockpile), by choosing its stockpile level each period. Consumers decide on current consumption given the current price. In the simplest model, the world lasts two periods, in the first of which the consumer government buys stocks for resale next period. The game is one of full information in which the supplier plays first (announcing a price) and the consumer government plays second (choosing a stockpile level). The game is solved recursively, and can be solved numerically for any finite number
of periods. With zero production and storage costs the effect of the stock-
pile was to make future demand more elastic and hence reduce the deadweight
loss of the monopoly. Consumers benefit from this, whilst with linear demand
and two periods, producers also gain because of the higher first period
demand and price. In short, both parties gain from the countervailing
market power which reduces deadweight inefficiency losses, though as the
number of time periods increase, so the consumers gain relatively to
producers.

These results are quite striking, but need careful interpretation. First, the benefits are measured by comparing an interventionist stockpiling
strategy by the sole consuming country with no intervention at all. The
fact that the country is the sole consumer means that the market structure
is one of potential bilateral monopoly, rather than pure monopoly, end
it might have been more logical to compare stockpiling with other counter-
vailing actions such as import tariffs.

Second, the cartel's supply strategies are rather special, and
correspond more closely to OPEC's strategies than those of an agricultural
commodity cartel (though there are additional complicating factors in the
case of exhaustible resources which substantially modify the conclusions,
as the authors recognise). Only if short run supply is highly elastic
will it be feasible for a cartel to announce a supply price and allow
consumers then to dictate demand and hence supply. If this elasticity of
supply is to be achieved by producer stockpiling then the same
intertemporal issues arise as with exhaustible resources, and the analysis
must be substantially modified. It is clear that this assumption affects
the relative market powers of the two parties substantially. If, for example, the producer always has the first say, and specified a supply price, then the importer can never usefully impose an import tariff, since this, by assumption, will not affect the supply price. Clearly, if the importer plays first, and announces a tariff, then the roles are reversed and the producer faces a different demand curve against which he optimises, and the resulting market equilibrium is quite different. This is dramatically illustrated in the following numerical example, which uses the same model as Nichols and Zeckhauser (1977, pp 70–74).

Tariffs vs. Stockpiles in the Nichols–Zeckhauser model

We employ the same notation and format as the two period model. Consumption, C, and price, p, are related by

\[ C = K - ap, \]

the discount factor is 6, and there are no production or storage costs. With no consumer action, the price is set at the monopoly level \( p_m = K/2a \), and consumers enjoy a present value of consumer surplus \( V^m = K^2 (1 + \beta)/8a \).

If the sequence of events is (1) cartel selects \( p_1 \), (2) consumers select \( C_1(p_1) \) and their government chooses stock, S, (3) cartel selects \( p_2 \) (4) consumers select \( C_2(p_2) \) and import \( C_2 - S \), then Nichols and Zeckhauser compute the optimum \( p_1, p_2 \) and S. With \( \beta = 0.9524 \) (rate of interest = 5%) they show that the optimal stockpiling strategy raises the net present value of consumer surplus by 7.7% and the cartel’s revenue rises 12%.

If, in contrast, the sequence is (1) cartel selects \( p_1 \), (2) consumers choose \( C_1(p_1) \), (3) the government sets an ad valorem tariff \( \tau \) for the second period (4) the cartel selects \( p_2 \), and (5) consumers demand \( C_2(p_2(1 + \tau)) \),
then it is easy to show that the optimum tariff is $\tau = 100\%$, in which case the present value of consumers' surplus is $K^2(1 + 2\beta)/8\alpha$, or an increase of 49% over the no intervention case. The cartel loses 24.4% of its present discounted revenue, though it is interesting to note that there is no change in deadweight loss, merely a redistribution from producers to consumers.

The main conclusion to draw from this fascinating study of the countervailing power of importer's stockpiling strategy is that the benefits of any such strategy depend very much on the initial and final market equilibria. With bilateral monopoly different solution concepts (i.e. different restrictions on allowable strategies, including their timing) have very different implications for the distribution of benefits and the magnitude of deadweight losses. Thus allowing one or other players to increase h's range of allowable strategies to include stockpiling (with its attendant implications for the location and elasticity of supply and demand schedules within any period) can be expected to change the equilibrium. However, without knowing what constrains the original choice of strategies it is difficult to say who would benefit from stockpiling (i.e. who would undertake it), and also it is difficult to say what form optimal intervention would take.
Appendix Characterising the Optimum Stock Rule

The following results are defended in Newbery and Stiglitz (1981b).

(i) The competitive stock function \( f(x) \) is continuous and nonotonically increasing.

(ii) Its derivative \( f'(x) \) is less from unity.

(iii) In a stationary world with bounded harvests stocks are also bounded. If the maximum possible harvest is \( h_m \), then there is a unique number \( x_m \) such that

\[
x_m = h_m + f(x_m)
\]

and for all \( x > x_m \) \( (> h_m) \)

\[
f(x) < x - h_m
\]

This result is perhaps best appreciated by Figure A1.

![Figure A1 Storage Rule](image-url)
If by some unforeseen event supply ever rises above some level $x_n$, say to $x^*$, the stock level must steadily decrease, for even a sequence of bumper harvests $h_n$ will lead to a successive decrease in supply and hence stocks, as shown by the arrowed line in Figure A1.

For the two point distribution considered in Section 3, and for the linear demand schedule of equation (12), the stocking rule can be solved as follows. Equation (18) can be rewritten as follows.

\[
\bar{p}(1 - \beta) \left(1 + \frac{1}{\bar{x}}\right) + k \geq \frac{D}{\bar{p}} \left\{ x - f(x) - BE \left[ h + f(x) - f\{h + f(x)\} \right] \right\}
\]

where $x_0$ is defined by equation (19). Equation A2 can be rearranged to give

\[
\begin{align*}
\{ f(x) \} &= \left\{ \begin{array}{l}
\frac{1}{1 + \beta} \left[ x - a + \beta \frac{f(h + f(x))}{p} \right], \quad x \geq x_0, \\
0, \quad x < x_0.
\end{array} \right.
\end{align*}
\]

where $a$ is a constant:

\[
a = 1 + \epsilon(1 - \beta + k/p) = 1 + \epsilon(c + r)
\]

and $c$ is the total annual storage cost excluding interest, as a fraction of the average price $\bar{p}$.

Provided the harvest takes a discrete distribution, then it is possible to show that for certain parameter values the solution to (A2) is a piecewise linear rule, in which the optimum competitive stock rule is linear beyond $x_0$. 
\[ f(x) = \alpha(x - x_0), \quad x_0 \leq x \leq x_m \]  

where \( x_m \) is the maximum potential supply defined by equation (A1).

Substitute (A5) into (A3):

\[ (1 + \beta)f(x) = x - a + \beta E f(h + \alpha(x - x_0)), \quad x_0 \leq x \leq x_m \]

The problem lies in the behaviour of the term \( h + \alpha(x - x_0) \), which may be greater or less than \( x_0 \), the point at which \( f(x) \) is non-linear.

Define

\[ \text{Prob}(h \geq x_0 - \alpha(x - x_0)) = \pi(x) \]  

\[ E[h|h \geq x_0 - \alpha(x - x_0)] = H(x) \]  

then

\[ (1 + \beta)f(x)' = x - a + \alpha \beta \pi(x)(H(x) + \alpha(x - x_0) - x_0), \quad x_0 \leq x \leq x_m. \]

The stock rule can only be linear over \([x_0, x_m]\) if \( \pi(x) \) is independent of \( x \) over this range; and \( H(x) \) is constant or linear in \( x \). This is equivalent to requiring

\[ \text{Prob}(h | (1 + \alpha)x_0 - \alpha x_m \leq h \leq x_0) = 0. \]  

(A9)
For example, if the harvest has a two point distribution

\[
h = \begin{cases} 
  1 + u & \text{Prob } \rho \\
  1 - \gamma u & \text{Prob } 1 - \rho 
\end{cases}
\]

so that

\[
E h = 1, \quad \text{Var } h = \gamma u^2 = \sigma^2.
\]

To ensure a linear stock rule we require that if the current harvest is low \((h = 1 - \gamma u)\), then no matter how large was the carryover, current storage must be zero, and if the current harvest is high \((h = 1 + u)\), then even with zero carryover, some stocking must occur. In that case

\[
\pi(x) = \rho, \quad H(x) = 1 + u.
\]

Equation (A8) can be solved for \(x_0\):

\[
0 = x_0 - a + \alpha \beta \rho \left(1 + u - x_0\right)
\]

so

\[
x_0 = \frac{a - \alpha \beta \rho (1 + u)}{1 - \alpha \beta \rho}
\]
and α:

\[(1 + \beta)\alpha = 1 + \alpha^2 \beta \rho \]

\[\alpha = \frac{1 + \beta - \sqrt{(1 + \beta)^2 - 4\beta \rho}}{2\beta \rho} \quad (A12)\]

The maximum stock is found from equation (A1):

\[x_m = \frac{1 + u - \alpha x_0}{1 - \alpha} \quad (A13)\]

\[S_m = \alpha(x_m - x_0) = \frac{\alpha}{1 - \alpha} (1 + u - x_0) \quad (A14)\]

The condition of positive carryovers if and only if the current harvest is good is equivalent to

\[1 - \gamma u + S_m < x_0 < 1 + u \quad (A15)\]

which is identical to equation (A9). Given specific values of \(\beta\), \(c\), \(\gamma\), \(u\), and \(\rho\), \(\alpha\) and \(x_m\) can be found and checked to see if they satisfy the constraints of equation (A15).
References


