Market Intervention Policies When Production Is Risky

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(continued on inside back cover)
Market Intervention Policies When Production Is Risky

P. B. R. Hazell and P. L. Scandizzo

The supplies of many agricultural commodities involve important production risks. In analyzing market intervention policies, these risks should enter the analysis as stochastic elements in the slope of the supply function and not just in the intercept term. This specification leads to the result that optimally distorted prices are more efficient for social welfare than competitive market equilibrium prices. Important gains in social welfare may be obtained with risky products through the appropriate use of production quotas and price stabilization schemes designed to optimally distort the market.

Key words: risk, pricing policy, production quotas, stabilization schemes.

Theoretical analyses of market intervention policies are typically based on deterministic models of the market place. When risk is introduced as, say, for the analysis of price stabilization schemes, then the risky components of supply and demand are usually specified to be additive in nature, that is, the intercepts of the supply and/or demand schedules are specified as stochastic.

The supplies of many agricultural commodities, especially crop products, do not really conform to this kind of specification. Rather, production risks are multiplicative in nature, meaning that it is the slope of the supply function which contains the important stochasticities. This specification is suggested for two reasons: first, because it is usually crop area which is price responsive and total production (supply) is then total area times stochastic yield, and second, because the variance of total output increases with total area and is not a constant as assumed in an additive risk model. Multiplicative specifications are frequently used in empirical supply analysis, either in linear programming models (Hazell and Scandizzo) or in the econometric estimation of constant elasticity of substitution and Cobb-Douglas functions.

The purpose of this paper is to explore some of the implications of a multiplicative risk model. It is shown that such a model leads to the rather surprising result that optimally distorted market prices are more efficient for social welfare than the prices determined through a competitive market equilibrium. The implications of this result are explored in terms of desirable market intervention policies and as it affects conventional wisdom on price stabilization schemes and shadow pricing for project analysis.

The Model

In this paper, the supply and demand schedules are assumed linear and only supply is risky.\(^1\) Specifically, the following market structure is assumed:

\[
S_t = \lambda \epsilon_t P^*_t, \\
D_t = a - bP_t, \\
S_t = D_t,
\]

and \(E(\epsilon_t) = \mu, \ V(\epsilon_t) = \sigma^2, \ \text{cov}(\epsilon_t, P^*_t) = 0\) for all \(t\), where \(P^*_t\) is the price anticipated by producers at the time of making production decisions, \(\epsilon_t\) is stochastic yield, and \(a, b, \) and \(\lambda\) are positive constants.

The model has the following features. (a) Anticipated price, \(P^*_t\), is the relevant forecast of \(P_t\) made by producers at the time of committing their inputs for period \(t\). Typically, in agricultural production, there will be a lag between such decisions and the realization of production. As such, \(P^*_t\) incorporates antici-

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\(^1\) There is little rationale for introducing a multiplicative risk term on the demand side, and an additive term does not effect the nature of the results unless correlated with \(\epsilon\).
pations about both $\epsilon_t$ and about demand and total supply. The assumption that $\text{cov}(\epsilon_t, P^*_t) = 0$ rules out the possibility of perfect forecasts (in which case the model would collapse to a simultaneous specification) and implies that no knowledge is available about $\epsilon_t$ other than that the parameters $\mu$ and $\sigma^2$ are known.

(b) No specific assumptions are made about the risk behavior of producers, except that the aggregate supply schedule is upward sloping. However, it is well known that when risk aversion exists, the supply schedule will incorporate a new cost, the compensation to producers for taking risk (Magnusson, p. 65). Typically, this will mean that the supply schedule will have a greater slope coefficient than a risk-neutral supply schedule. (c) Because $\epsilon_t$ is multiplicative, the variance of market supply in each period is

$$V(S_t) = \lambda^2 P^*_t \sigma^2,$$

and this increases with anticipated price. However, the coefficient of variation is

$$V(S_t)^{1/2}/E(S_t) = \sigma/\mu,$$

and this remains constant.

If the yield term $\epsilon$ is bounded on some positive interval $\epsilon_m \leq \epsilon \leq \epsilon_x$, then the market structure can be portrayed as in figure 1. The expected supply function $E(S) = \lambda \mu P^*$ is linear and passes through the origin. Since $\mu$ is assumed to be the best possible forecast of $\epsilon$, $E(S)$ is the basic behavioral relation on the supply side. In the diagram, if producers anticipate $P^*_t = \rho_t$, then they will plan production for period $t$ so that the expected market output is $S^*_t = \lambda \rho_t$. However, because $\epsilon_t$ is stochastic, the actual supply function can ro-

![Figure 1. Market structure with risky supply](attachment:image.png)
tate in a random way around $E(S)$ to any position contained in the funnel defined by $S|e_{m} = \lambda e_{m} P^{*}$ and $S|e_{x} = \lambda e_{x} P^{*}$. Hence, if expected supply in period $t$ is $S_{t}$, actual supply could take on any value on the line $AB$ in figure 1. Clearly, actual market price is stochastic with $e_{t}$, and the actual price in period $t$ may take on any value between $P^{m}$ and $P^{x}$.

Since the market clears each period, market price is

$$P_{t} = \frac{a}{b} - \frac{\lambda}{b} e_{t} P^{*}.$$  

(4)

Further, since $e_{t}$ is stochastic, then $P_{t}$ must always be stochastic. This means that a competitive market equilibrium exists, it must be defined in terms of convergence properties of the probability density function of $P_{t}$. There are a number of concepts of equilibrium that can be used (Bergendorff, Hazell, and Scandizzo; Turnovsky 1968), but in this paper it is only necessary to consider convergence in the mean and variance of price. (A more complete mathematical treatment of the convergence properties of this market structure is available in Bergendorff, Hazell, and Scandizzo.)

The Equilibrium Mean Price

Taking the expectation of equation (4) over $e_{t}$ and $P^{*}$, expected market price is

$$E(P_{t}) = \frac{a}{b} - \frac{\lambda}{b} \mu E(P^{*}).$$  

(5)

Now if $P^{*}$ is formed in such a way that

$$\lim_{t \to \infty} E(P^{*}) = \lim_{t \to \infty} E(P_{t}),$$

that is, anticipated price is an asymptotically unbiased forecast, then it is clear that $E(P_{t})$ will converge to a limiting value of

$$\lim_{t \to \infty} E(P) = \lim_{t \to \infty} E(P_{t}) = \frac{a}{b + \lambda \mu}.$$  

(6)

As an example, consider the class of cobweb models in which

$$P^{*} = \sum_{t=1}^{m} \gamma_{t-1} P_{t-1} \text{ with } \sum_{t=1}^{m} \gamma_{t} = 1.$$  

Here $E(P_{t}) = \frac{a}{b} - \frac{\lambda}{b} \mu \sum_{t=1}^{m} \gamma_{t} E(P_{t-1})$, which is an $m$th-order difference equation whose solution, if it exists, is indeed equation (6). As Bergendorff, Hazell, and Scandizzo have shown, a sufficient condition for convergence is that $\lambda \mu < b$ which is also the necessary condition for the naive cobweb in which $P^{*} = P_{t-1}$. In essence, this condition simply ensures that $\lim E(P^{*}) = \lim E(P_{t})$.

The price limit $E(P)$ in equation (6) has the property of being a self-fulfilling expectation, that is, if $\lim E(P_{t})$ is the price anticipated by producers, then the expected value of the market clearing price will be $E(P)$. This can easily be shown by substituting $P^{*} = \lim E(P)$ into equation (6) leading to the result

$$E(P_{t}) = \lim E(P_{t}).$$

$E(P)$ is therefore the rational expectation of the model in the Muthian sense. It is also the price corresponding to the intersection of demand and expected supply though, as Hazell and Scandizzo show (p. 239), this result depends critically on the linearity assumptions of the model.

The Equilibrium Variance of Price

Using equation (4), the variance of market price $V(P_{t})$ defined over $e_{t}$ and $P^{*}$ is

$$V(P_{t}) = \frac{\lambda^{2}}{b^{2}} V(e_{t} P^{*})$$

$$= \frac{\lambda^{2}}{b^{2}} [E(e_{t}^{2} P^{*2}) - E(e_{t}^{2}) E(P^{*2})].$$

Assuming that $\text{cov} (e_{t} P^{*2}) = 0$, then

$$V(P_{t}) = \frac{\lambda^{2}}{b^{2}} [E(P^{*2}) \mu_{2} - E(P^{*})^{2} \mu^{2}],$$

where $\mu_{2}$ denotes the second moment of $e$ around 0. Adding and subtracting $\mu_{2} E(P^{*})^{2}$ and using $\sigma^{2} = \mu_{2} - \mu^{2}$ gives

$$V(P_{t}) = \frac{\lambda^{2}}{b^{2}} [\mu_{2} V(P^{*}) + \sigma^{2} E(P^{*})^{2}].$$  

(7)

Taking limits and again assuming $\lim E(P^{*}) = \lim E(P)$,

$$\lim V(P_{t}) = \lim V(P_{t})$$

$$= \frac{\lambda^{2}}{b^{2}} \left[ \mu_{2} \lim V(P^{*}) + \frac{a^{2} \sigma^{2}}{(b + \lambda \mu)^{2}} \right].$$  

(8)

The limiting variance of market price $\lim V(P)$ is therefore seen to be a linear function of the limiting variance of anticipated price $\lim V(P^{*})$ and has its smallest value when $\lim V(P^{*}) = 0$. However, since it has been assumed that $\lim E(P^{*}) = \lim E(P_{t})$, then the minimum value for

$$\lim V(P_{t}) = E_{\ast} \{ E[P_{t}^{*}] \} - E_{\ast} \{ E[P_{t}] \}.$$  

(9)
lim $V(P)$ must occur exactly when producers anticipate the mean price lim $E(P)$ in each and every period. This supports Muth’s contention that the self-fulfilling expectation is the most rational.

More generally, however, equation (8) says that the equilibrium variance of market price will be smaller the more accurate the anticipated price each period. Thus, in the class of cobweb models discussed earlier, Bergendorff, Hazell, and Scandizzo have shown that the naive cobweb has the largest lim $V(P^*)$ and hence the largest lim $V(P)$ and that the weighted cobwebs have progressively smaller lim $V(P^*)$ and lim $V(P)$ as $P^*$, tends to lim $E(P)$.

A Welfare Analysis

The equilibrium mean price lim $E(P)$ in equation (6) was derived under the quite reasonable assumptions that the market converges and that on average, producers settle on the self-fulfilling expectation as their anticipated price. It is now appropriate to consider whether this price is also a welfare-maximizing price.

Following Massell and Turnovsky (1974), social welfare will be measured using the expected values of the consumers’ and producers’ surplus. Consumers’ surplus is the expected value of the area under the demand curve and above actual market price $P_t$. Algebraically,

$$W_t = \int_{P_t}^{a-b} (a-bP)dP,$$

which after some algebra evaluates at $W_t = \lambda \varepsilon_t P^*_t 2/2b$. Taking expectations over $\varepsilon_t$ and $P^*_t$, and adding and subtracting $\lambda_2 \mu_2 2b$, gives

$$E(P^*)_2$$

(9) \hspace{1cm} E(W) = \frac{\lambda_2 \mu_2}{2b} [V(P^*) + E(P^*)_2].$$

To evaluate the producers’ surplus (or realized profit), it is necessary to recognize that production costs incurred in period $t$ depend on anticipated price $P^*$, and not on actual price $P_t$. This is because the model is specified with lagged production and less than perfect price forecasts. Diagrammatically, the surplus can be shown as in figure 2. Suppose producers again anticipate $P^*_t = \rho_t$ for period $t$, so that expected market supply is $S^*_t$. Production costs are then the value of the triangle $OBS^*_t$ and these become fixed for period $t$ regardless of the outcomes for $\varepsilon_t$ and $P_t$. Producers’ revenue depends directly on actual yields and prices. Suppose actual supply is $S_t$ in figure 2. Then actual market price is $P_t$ and producers’ revenue is the area of the rectangle $OP_tCS_t$. The producers’ surplus for period $t$ is therefore $OP_tCS_t - OBS^*_t$.

Algebraically, the producers’ surplus is calculated as follows:

$$\Pi_t = P_tS_t - \int_0^{S^*_t} \lambda \mu P^*_t \frac{S}{\lambda \mu} dS,$$

that is, total actual revenue minus total costs as measured by the area under the expected supply function from 0 to $S^*_t$. After some algebra,

$$\Pi_t = \frac{a}{b} \lambda \varepsilon_t P^*_t - \left( \frac{\lambda_2 \mu_2}{b} \varepsilon_t^2 + \lambda_2 \mu_2 \right) P^*_t^2.$$

Hence, taking the expectation over $\varepsilon_t$ and $P^*_t$, and adding and subtracting $\frac{\lambda_2 \mu_2}{2b} E(P^*)_2$,

(10) \hspace{1cm} E(\Pi) = \frac{a}{b} \lambda \mu E(P^*)

$$- \left( \frac{\lambda_2 \mu_2}{b} + \frac{1}{2} \lambda \mu \right) V(P^*)$$

$$- \left( \frac{\lambda_2 \mu_2}{b} + \frac{1}{2} \lambda \mu \right) E(P^*)_2.$$

The social welfare measure $E(SW)$ used here is the sum of the expected values of the consumers’ and producers’ surplus. Adding equations (9) and (10) together,

(11) \hspace{1cm} E(SW) = \frac{a}{b} \lambda \mu E(P^*)

$$- \frac{1}{2} \left( \frac{\lambda_2 \mu_2}{b} + \lambda \mu \right) V(P^*)$$

$$- \frac{1}{2} \left( \frac{\lambda_2 \mu_2}{b} + \lambda \mu \right) E(P^*)_2.$$

The expected value of social welfare is therefore a function of the mean and variance of the price anticipated by producers at the time of making their decisions. For any fixed $V(P^*)$, the maximum of this function occurs when

$$\frac{\partial E(SW)}{\partial E(P^*)} = \frac{a}{b} \lambda \mu - \left( \frac{\lambda_2 \mu_2}{b} + \lambda \mu \right) E(P^*) = 0,$$

that is, when the expected value of producers’ anticipated price is

(12) \hspace{1cm} E(P^*) = \frac{a \mu}{b \mu + \lambda \mu_2}.

Since equation (11) is a decreasing function
in $V(P^*)$, the global maximum occurs when $V(P^*) = 0$, that is, when producers anticipate $a\mu(b\mu + \lambda\mu_2)$ in each and every period.

Assuming equation (12) is satisfied, then by substituting equation (12) into equation (5) and taking expectations, the expected market clearing price would be:

$$E(P_t) = \frac{a(b\mu + \lambda\sigma^2)}{b(b\mu + \lambda\mu_2)}.$$  

(13)

Equation (13) is not the market equilibrium price $\lim E(P)$ derived earlier in equation (6) nor is equation (12) a self-fulfilling expectation; yet it is this combination of anticipated and expected prices which maximizes the selected measure of social welfare. This is a somewhat surprising result, for it implies that there is an optimal distortion for the market.

Taking the ratio of equations (12) and (13) and simplifying,

$$\frac{E(P^*)}{E(P_t)} = \frac{b\mu}{b\mu + \lambda\sigma^2} = \frac{|\xi_d|}{|\xi_d| + R^2},$$  

(14)

where $|\xi_d| = \frac{b}{\lambda\mu}$ is the absolute value of the elasticity of demand evaluated at the competitive market equilibrium, and $R$ is the coefficient of variation for $\epsilon$, that is, $R = \sigma/\mu$.

Clearly this ratio is less than 1, so that $E(P^*) < E(P_t)$. Further, taking the ratio $E(P^*)/\lim E(P)$ and simplifying,

$$\frac{E(P^*)}{\lim E(P)} = \frac{b\mu + \lambda\mu_2}{b\mu + \lambda\mu_2} = \frac{|\xi_d| + 1}{1 + |\xi_d| + R^2},$$  

(15)

Note that once $P_t$ is fixed over time, the market automatically attains a new equilibrium so that there is no need to consider the asymptotic properties of $E(P_t)$.
which is also less than 1 because $R^2 > 0$. Hence, $E(P^*) < \lim E(P)$.

Taking these two results together, the optimal distortion involves a cutback in production compared to the competitive equilibrium. Producers should plan on the basis of a lower expected price than the competitive equilibrium price $\lim E(P)$, and as a result they will realize a higher market price on average.

The derivation of equation (12) guarantees that this strategy leads to the maximum gains to consumers and producers as a whole, and hence $E(SW)$ is larger when producers anticipate equation (12) rather than $\lim E(P)$. The way in which this gain is allocated among consumers and producers can be seen by using equations (9) and (10): For the same $V(P^*)$, consumers gain as $E(P^*)$ increases while producers lose. Since $E(P^*)$ in equation (11) is smaller than $\lim E(P)$, the distortion benefits producers at the expense of consumers.

The Nature of the Distortion

The existence of an optimal distortion price requires some explanation. In a very different context, Brainard has also shown how a multiplicative risk model can lead to unconventional results. Basically, it can be attributed to two factors in the model specification: first, because production costs are dependent on anticipated price $P^*$, and not on actual market price $P^*_e$. This means that there is no fixed relationship between revenue and costs and that for some $P^*_e$, the yield $\epsilon$ outcome may, in conjunction with the inelasticity of demand, 5 conspire to cause revenue to fall below costs to the extent that there is a net welfare loss to society. In itself, this cost is not sufficient to distort the market. 6 However, because of a second feature of the model, the multiplicative risk term, the variance of market supply $S^*$ increases quadratically as producers move up the expected supply function, so that the possibility of costs exceeding revenue also increases. Clearly, the distortion in the market arises from the trade-off between the surpluses from higher outputs and the net welfare loss associated with wasted resources.

Let $F$ denote the ratio of expected market-clearing price assuming an optimal distortion to the expected market-clearing price given a competitive equilibrium, that is, $F = E(P^*)/\lim E(P)$, where $E(P^*)$ is defined in equation (13). Then $T = (F-1)100\%$ measures the percentage market price distortion, having a value of 0 when the optimally distorted price $E(P_d)$ is the same as the equilibrium price $\lim E(P)$. Using equations (14) and (15),

$$T = \frac{R^2}{\xi_d} \cdot \frac{100\%}{\xi_d + \frac{R^2}{\xi_d} + 1},$$

$T$ is therefore seen to be a function of the demand elasticity evaluated at the competitive market equilibrium and the coefficient of variation of $\epsilon$. For fixed $R$ the distortion is larger the more inelastic the demand, but the distortion disappears at the limit as the demand elasticity is increased towards infinity. The distortion also increases with $R$, so that the more risky the production the greater the optimal market price distortion. In a deterministic market, $R = 0$ and the optimal distortion is 0.

In order to provide some feel for the magnitudes of the optimal distortions involved, $T$ has been calculated for various values of $\xi_d$ and $R$ in table 1. It is often claimed that the aggregate demand for agricultural commodities is inelastic. If this is so (say for non-traded goods), then for only moderate degree of production risk ($R = 0.5$), the optimally distorted market price $E(P^*_d)$ should be at least 11% higher than the competitive market equilibrium price $\lim E(P)$. For higher risk situations, the optimal distortion ratio becomes quite large indeed for inelastic demands.

Given the size of these distortions, it is pertinent to inquire into the magnitudes of the associated welfare gains. Using equation (11), these can be expressed in terms of percentage gains as

$$Z = \frac{|G|}{100\%},$$

where

$$G = \frac{E^*(SW) - \lim E(SW)}{\lim E(SW)}$$

and $\lim E(SW)$ denote, respectively, the expected social welfare assuming producers anticipate $E(P^*)$ as defined in equation (12) and the equilibrium price $\lim E(P)$ as defined in equation (6). For simplicity, suppose producers anticipate these prices in each period so that $V(P^*) = 0$; then, after some algebra, 7

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5 Remember linear demand curves become increasingly inelastic as the quantity increases.

6 In an additive risk model, for example, with the same kind of lagged specification, a price distortion does not arise.

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1 Substituting equation (12) for $E(P^*)$ in equation (11), setting $V(P^*) = 0$ and simplifying,

$$E^*(SW) = \frac{a\mu^\lambda}{2b(\mu + \lambda)}.$$
Table 1. Percentage Price Distortions (T) and Percentage Welfare Gains (Z) for Variations in Market Parameters

<table>
<thead>
<tr>
<th>ξa</th>
<th>R = 0.25</th>
<th>R = 0.5</th>
<th>R = 1.0</th>
<th>R = 2.0</th>
<th>R = 3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Z</td>
<td>T</td>
<td>Z</td>
<td>T</td>
<td>Z</td>
</tr>
<tr>
<td>-0.50</td>
<td>8.0</td>
<td>0.2</td>
<td>28.6</td>
<td>2.8</td>
<td>80.0</td>
</tr>
<tr>
<td>-0.75</td>
<td>4.6</td>
<td>0.1</td>
<td>16.7</td>
<td>2.1</td>
<td>48.5</td>
</tr>
<tr>
<td>-1.00</td>
<td>3.0</td>
<td>0.1</td>
<td>11.1</td>
<td>1.6</td>
<td>33.3</td>
</tr>
<tr>
<td>-2.00</td>
<td>1.0</td>
<td>0.0</td>
<td>3.8</td>
<td>0.7</td>
<td>12.5</td>
</tr>
<tr>
<td>-5.00</td>
<td>0.2</td>
<td>0.0</td>
<td>0.8</td>
<td>0.2</td>
<td>2.9</td>
</tr>
<tr>
<td>-10.00</td>
<td>0.1</td>
<td>0.0</td>
<td>0.2</td>
<td>0.1</td>
<td>0.8</td>
</tr>
<tr>
<td>-50.00</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\[ G = \frac{(|ξ_d| + 1)^2}{(|ξ_d| + 1 + R^4)(|ξ_d| + 1 - R^4)} - 1. \]

Values of Z are reported in Table 1 for the corresponding values of T. When R = 0.5 and demand is inelastic, the optimal distortion leads to a welfare gain of at least 1.6%. This corresponds to the price distortion of 11%. Generally, the welfare gains from the distortion are quite small for low R and might be ignored. However, for R-values of 1.0 or larger, the welfare gains are disturbingly large. For example, if R = 1.0 and the elasticity of demand is -0.75, expected social welfare can be increased by 48.5% by optimally distorting a competitive market.

It might be objected that the magnitudes of these distortions and welfare gains result from the simplifying linearity assumptions of the model. However, the linearity assumptions are least important when demand is inelastic, and it is exactly in these cases that the significant results are obtained.

Market Intervention Policies

The existence of an optimal price distortion suggests a rather new rationale for market intervention policies in competitive markets. They should be used to reduce the expected market supply from \(a\lambda\mu(b + \lambda\mu)\), the level realized in a competitive equilibrium, to \(a\lambda\mu^2(b\mu + \lambda\mu)\), the output corresponding to the optimal distortion price in equation (13). This would raise the average market price from the competitive equilibrium level but in so doing would actually raise the level of aggregate social welfare as measured by the sum of expected values of the consumers' and producers' surplus.

An obvious and effective way to achieve this goal would be through a system of production or marketing quotas. Perhaps these should be regarded as a social norm for agricultural markets in which production is risky and demand is price inelastic!

The optimal distortion can also be implemented, at least partially, through appropriate use of price stabilization schemes and, in developing countries, through the shadow prices used in project appraisal.

Price Stabilization Schemes

Equation (11) shows that for fixed \(E(P^*)\), the realized social welfare \(E(SW)\) increases as the variance of anticipated price \(V(P^*)\) is reduced. However, from equation (7) it is known that the variance of actual market price \(V(P)\) is a linear and increasing function of \(V(P^*)\) so that any price stabilization scheme which leaves \(E(P^*)\) unchanged must lead to a gain in total welfare. It is also clear from equation (9) that consumers lose through price stabilization while equation (10) indicates that producers gain. These results are consistent with results obtained by Waugh, Oi, Massell, and Turnovsky (1974) for additive risk models.

The optimal stabilization scheme to maximize \(E(SW)\) is obviously one in which
V(P_t) = V(P^*) = 0, and producers are paid a fixed price of P^* = aμ/(bμ + λμ). Since then E(P_t) > P^*, there is a "natural" margin on average to cover storage and administration costs and there may even be a profit for the stabilizing agency. Such a scheme, while socially desirable may not benefit producers, however, unless they also share in the profits of the stabilizing agency. Of course, unless there was a quota system constraining output to correspond with the anticipated price P^*, any producers' share of profits would have to be transferred in a suitable manner, say through a dividend scheme, so as not to increase their anticipated price P^*.

It is interesting to consider the maximum profit that the stabilizing agency can extract without leaving producers any worse off than before stabilization.\(^8\) Put another way, what fixed price (call it P') must producers be paid each period if their average realized profits are to remain the same as they would be in a competitive market equilibrium? To answer this question it is convenient to assume that production is constrained through a quota scheme to correspond to the anticipated price P^* = aμ/(bμ + λμ) and that the stabilizing agency therefore realizes an average market price of E(P) = a(bμ + λμ) as derived in equation (13). To simplify matters further, let it also be assumed that in the equilibrium and unstabilized state, producers anticipated the mean price lim E(P) in each and every period, so that the variance of anticipated price is 0.

Now since producers receive the fixed price P' but act as if they anticipate P^*, their realized average surplus is

\[
E(\Pi) = E(S,P') - \frac{1}{2} E(S|P^*)P^* = \lambda \mu P^*P' - \frac{1}{2} \lambda \mu P^{*2}.
\]

The problem is to find P' which equates the above average surplus with that realized under the competitive situation in which producers anticipate lim E(P). This surplus is, using equation (10),

\[
E(\Pi) = \frac{a}{b} \lambda \mu \lim E(P) - \frac{\lambda}{2b} (2\lambda \mu + b\mu) \lim E(P)^2.
\]

Hence, equating the two surpluses and solving for P' gives

\[
P' = a \left( \frac{\lim E(P)}{P^*} \right) - \left( \frac{2\lambda \mu + b\mu}{2b\mu} \right) \left( \frac{\lim E(P)^2}{P^*} \right) + \frac{1}{2} P^*.
\]

It is not too difficult to derive the conditions under which P' must be larger than P^*. This is when P' = P^* > 0 or equivalently when

\[
\frac{a}{b} \left( \frac{\lim E(P)}{P^*} \right) - \left( \frac{2\lambda \mu + b\mu}{2b\mu} \right) \left( \frac{\lim E(P)^2}{P^*} \right) - \frac{1}{2} P^* > 0.
\]

This can be simplified to

\[
\frac{2a\mu}{\lim E(P)} - \frac{(2\lambda \mu + b\mu)}{\lim E(P)^2} > \mu \frac{P^*}{\lim E(P)^2}
\]

and, using lim E(P) = a/(b + λμ) on the left-hand side,

\[
b\mu - 2\lambda \mu > \mu \frac{P^*}{\lim E(P)^2}.
\]

Rearranging terms and using |ε| = b/λμ, this reduces to

\[
(17) \quad \frac{1}{2} |\epsilon| \left[ 1 - \left( \frac{P^*}{\lim E(P)^2} \right) \right] > R^2.
\]

As previously shown, P^* < lim E(P) so that the left-hand side of equation (17) is positive. Hence, for any |ε| ≤ 1, R would have to be considerably smaller than 1/2 if producers are not to require a price P' greater than P^* to maintain their surplus.\(^9\)

**Shadow Prices for Project Analysis**

Shadow prices for agricultural products are typically used in project analysis to remove market price distortions arising from overvalued foreign exchange rates. As such, they serve to increase the value of the product (improving the desired terms of trade between the agricultural and nonagricultural sectors) and encourage agricultural investments.

The results in this paper suggest a rather

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\(^8\) The fact that P' may be less than lim E(P) let alone less than P^* is a little surprising, since it might be expected that only if P' > lim E(P) could (P') be maintained with the lower expected supply. The result reflects the substantial gains to be had from price stabilization when demand is linear. This is because with linear demand and unstabilized prices, large (small) outputs tend to lead to price determination on the inelastic (elastic) part of the demand schedule, and in both cases the revenue effect is unfavorable for producers, leading to a low producers' surplus on average.
pervasive role for shadow prices. Products should be valued at the optimal anticipated price \( a \mu/b(\mu + \lambda \mu) \) in equation (12), which is less than the expected price at market equilibrium. This means that projects for risky commodities should be penalized. Again, the rationale for this lies in the social cost of wasted resources associated with periods in which production costs exceed revenue and these costs increase with expected production.

Shadow pricing in this way would not only penalize risky products but might seem to actually worsen the desired terms of trade for agriculture. However, in this case the shadow price used for guiding investment decisions is not really the desired realized price. This is \( a(b\mu + \lambda \sigma^2)/b(\mu + \lambda \mu) \) as derived in equation (13), which would be higher than the market equilibrium price. Because of this, the desired terms of trade to be realized for agriculture might still improve.

Conclusions

This paper demonstrates that when risk enters the market supply schedule in a multiplicative way, then the expected price and output determined in a competitive market equilibrium may not necessarily be the best in terms of social welfare. Rather, there exists an optimal market distortion involving a higher average price and lower expected supply. This optimal price distortion can only be quite large when demand is inelastic (more than 10% with moderate production risks), but important welfare gains may be had from using market intervention policies to introduce the desired distortion. Such policies might take the direct form of a quota system or, less directly, include appropriate use of price stabilization schemes and shadow prices for project analysis.

These results have been obtained using a number of simplifying assumptions. In particular, it has been assumed that supply and demand schedules are linear, that supply has unity elasticity throughout, and that the sum of the expected values of the consumers' and producers' surplus is a suitable measure of social welfare. Changing these assumptions does of course lead to some modifications in the results. For example, introducing non-linearities can, though not necessarily, reduce the size of the optimal distortion, while introducing an intercept term in the demand schedule increases or reduces the size of the optimal distortion depending on whether the supply elasticity at equilibrium is less than or greater than unity. (The derivation of these and other extensions are available from the authors upon request.) Perhaps the most important limitation of the analysis is the measure of social welfare used. The consumer surplus does not, for example, consider income effects arising from changes in the price of the marketed commodity. In a developed country context where food is but a small part of consumers' expenditure, this shortcoming might be ignored but extreme caution is necessary in interpreting the results for developing countries if the marketed commodity is an important wage good. These problems require more sophisticated analysis before final policy conclusions can be drawn, but the problem of optimal conclusions is likely to remain.

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