A Bargaining Theoretic Approach to Cropsharing Contracts

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(continued on inside back cover)
A Bargaining Theoretic Approach to Cropsharing Contracts

By Clive Bell and Pinhas Zusman*

Sharecropping is a geographically widespread and historically tenacious form of lease. The questions of how the rental share is determined, and how such a form of lease affects resource allocation and the distribution of income pose a challenge to economic theory which has evoked two notable recent attempts to provide an explanation. First, within what may be called the Marshallian tradition, P. K. Bardhan and T. N. Srinivasan (1971) (hereafter B-S) have developed a fairly complete scheme of analysis in which the rental share is regarded by the parties to the contract “as a price-like variable” (1974, p. 1068), i.e., it is parametrically given to all agents. For an exogenously given real wage rate, the supply of and demand for leases determine the rental share. A second approach is that originally formulated by S. N. S. Cheung and subsequently extended and generalized by D. M. G. Newbery (1973a, b), (hereafter C-N). In their system, also claimed to be competitive, both the rental share and the minimum labor input per unit of land are stipulated in all contracts, their values being jointly determined by the landlord's desire to maximize his income subject to the condition that the tenant's income does not fall below his alternative earnings in a perfectly competitive labor market.

True to the spirit of Marshall's analysis (p. 644), B-S show that resource allocation under sharecropping departs from that ruling in a competitive system with fixed rent leases, it being well known that the latter is Pareto optimal. As Newbery (1974) has shown, the fundamental difficulty with B-S's formulation is that there will be excess demand for land for all rental shares less than unity, unless the marginal productivity of land is zero. By contrast, in the C-N scheme of things, the equilibrium which obtains happens to be identical with that ruling in a competitive fixed-rent system and is therefore Pareto efficient. This formulation, however, has two basic and related difficulties of its own. First, there is the dubious enforceability of the minimum labor-intensity provision of the contract. Secondly, if land has to be rationed, both agents are placed in a bargaining situation. Hence, one would expect the rental share which emerges from the bargaining process to reflect the agents' respective bargaining strengths.

Once the overriding importance of the last point is recognized, it is plain that the problem must be reformulated and analyzed within the framework of bargaining theory. This is the salient feature of our approach. Now as Newbery and Joseph Stiglitz have shown, the choice of rental contract is intimately related to the allocation of risk bearing and the provision of incentives in an uncertain world. However, in order to keep the bargaining aspects of the problem in sharp focus, we confine ourselves here to pure cropsharing under certainty.

In addition to these theoretical con-
siderations, the foregoing competitive for-
mulations pay little attention to the fact
that in addition to his labor, the tenant
often possesses land of his own, some capi-
tal, and entrepreneurial and management
skills. In this connection, it is widely
observed that landlord and tenant split the
produce in proportions which are close to,
if not exactly, 50:50. Any analysis which
seeks to explain the latter must take
account of the former. All this suggests
that the assumptions on which both
theories rest must be reconsidered, and
that the analysis of the problem must be
tailored to the characteristics of the agrar-
ian system in question.

In this paper, we discuss first some em-
pirical evidence concerning the existence
and nature of nontradable factors and
market imperfections, which play a key
role in the bargaining process. The evi-
dence relates mainly to northeast Bihar,
but it should survive extension to much of
the lower Indo-Gangetic plain. Secondly,
we employ John Nash's solution to the
bargaining problem, and hence derive the
equilibrium rental share. By way of illus-
tration, we calculate the predicted rental
and factor shares corresponding to some
sets of plausible production parameters.
Finally, we analyze briefly the situation in
which the marginal productivity of land is
zero.

I. Assumptions and Evidence

The most natural question to begin with
concerns the specification of the labor
market. The assumption of a perfectly
certain and elastic supply of labor and
work opportunities at an exogenously de-
termined and parametrically given wage
is not really acceptable at the micro-
economic (village) level. But it will be
retained on account of its simplicity,
which eases the task of analyzing the
effects of relaxing other less defensible
assumptions.

Finally the relationship between agricul-
tural worker and tenant status is probably
asymmetric. It is always open to the
tenant to work for others as well as (or
instead of) on the land he has leased in;
but the landless laborer cannot just elect
to become a tenant if that status requires
capital and husbandry skills to which he
can get access only with great difficulty,
if at all. In this connection, it should be
noted that in both India as a whole and
Bihar, household operational holdings
which are partly owned and partly leased
greatly outnumber those which are wholly
leased in, and that the former account also
for the lion's share of all land leased in.1
Thus, tenants are drawn mainly not from
the mass of landless laborers, but from the
ranks of the small peasantry possessing
land of their own as well as skills and
capital (or access to capital), all of which
are traded (if, indeed, they are tradable at
tall) in imperfect markets.

Moreover, the micro data analyzed by
Krishna Bharadwaj for several Indian
States and by Bell for northeast Bihar
reveal that both categories of tenant
households make very extensive use of
hired labor for cultural labor operations
and harvesting. Thus the costs of super-
vising cultivation itself (though not the
enforcement of a minimum input bundle,
of course) fall on the tenant. Moreover, as
the work is done by casual laborers, who
must be contacted in sufficient numbers at
the right time, there are transactions costs
of labor hire which are passed on to the
tenant by the landlord leasing out his
land. Factors such as choice of crop, time-
liness of operations, and good husbandry,
where the scope for them is large, indicate
that purely entrepreneurial skills are also
involved (see C. H. H. Rao). All these
point to a vital managerial element in the
tenant's role for which he must be
sufficiently rewarded.

1 See, for example, the Indian National Sample Survey
(nos. 36, 113, 144) and S. K. Chakraverty, p. 99.
The second major query surrounding the stylized competitive models is the absence of nonlabor inputs other than land. Here the principal difficulty stems from the nature of the market for animal draught power, especially on the supply side. In India, there is no historical evidence of human traction (or digging) for land preparation (except for small-scale vegetable cultivation), and there is a broad social disdain for such methods (see Prufull Sanghvi). Now, if the markets for both this input and credit were perfect, Cheung's contract could be extended to stipulate the minimum draught input per acre as well as the rental share and minimum labor intensity. However, the field data suggest that: (a) hired draught is only a very small fraction of total draught inputs, in marked contrast to labor; and (b) owning at least one pair of bullocks or buffalo is a sine qua non for obtaining a lease, and that this distinguishes tenants owning no land of their own from the general class of landless laborers.

These findings are backed up by the comments of tenants during interviews: although there existed a daily rate for the hire of a bullock team season by season, it was extremely difficult in practice to hire. No cultivator could afford to rely on the hire market, nor, indirectly, could any landlord. Hence the likelihood that tenants would own some land, for land and draught livestock ownership are very highly correlated. Nor are such observations confined to Bihar. In his study of allocative efficiency in an Eastern Uttar Pradesh village, David Hopper was unable to compare the calculated marginal product of bullock labor with its market price because "there was not an active local market for bullock rentals to provide a comparison" (p. 623).

The upshot of this examination of the assumptions underlying the competitive models is that the conclusions drawn from them may be highly misleading. In itself, the generalization of those analyses to many kinds of labor and nonlabor inputs presents no difficulties, provided all markets are perfect. But once it is perceived that tenants are distinguished from ordinary landless laborers by their command over factors whose services are traded (if directly tradable at all) in markets which are inherently and highly imperfect, then major revisions become necessary. First, the equilibrium condition that under perfect certainty the tenant's income must be equal to his alternative earnings as a wage laborer has to be dropped. Secondly, it cannot be replaced by a generalized equivalent to include the alternative earnings of factors other than "worker labor" owned by tenants because the markets for such factors are not even remotely perfect. Thirdly, while it may be safe to assume that the market for unskilled labor is perfect, any formulation of the problem must respect the fact that the crux of the land lease decision lies in the nontradability of managerial skill and draught power.

These imperfections are intimately connected with the issue of whether or not the landlord has the ability to enforce contracts specifying a set of minimum input intensities on sharecropped land. In addition to the a priori arguments advanced by B-S that such contracts are not enforceable, there is empirical evidence to that effect for northeast Bihar. The detailed testing of the competing hypotheses advanced by the two schools has been undertaken elsewhere (Bell), with the Marshallians coming off the better. Although there is no space here to go into much detail, the fact that most tenants also own land allows a search for the influ-

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2 In this connection, it is worth noting Richard H. Day's conclusion that mechanization contributed in a major way to the decline of sharecropping in the U.S. South by providing an alternative to tenant-owned draught power.

3 Four central hypotheses are set up and tested by Cheung (pp. 55-61), who uses evidence from prerevolutionary China, Taiwan, Korea, and Japan.
TABLE 1—MEAN INPUTS AND OUTPUTS PER ACRE ON TENANT HOLDINGS

<table>
<thead>
<tr>
<th>Cropping Intensity</th>
<th>Intermediate Inputs*</th>
<th>Hired Labor*</th>
<th>Yield*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sharecropped plots</td>
<td>1.40</td>
<td>33.0</td>
<td>35.6</td>
</tr>
<tr>
<td>Owned plots</td>
<td>1.76</td>
<td>64.3</td>
<td>46.7</td>
</tr>
</tbody>
</table>

Source: Bell.

* Rupees per net acre sown.

ence exerted by nontradables and other imperfections on the allocation of resources. For each such tenant, his access to inputs (traded or nontraded) and his aversion to risk are common elements in his decisions concerning the allocation of resources between owned and sharecropped plots. These sources of variation across farmers can be eliminated in the following way: for each farmer, form a vector whose elements are the differences between output and inputs per acre on owned and sharecropped land, respectively. This procedure amounts to a "pairing" of observations, a classic and powerful experimental method which has been neglected in the sharecropping literature. Table 1 summarizes mean input and output levels by the tenure status of land cultivated for a sample of 31 farmers, cropping intensity being a proxy for land services (and hence, indirectly, for draught services). Given the multivariate nature of the situation, an appropriate way of testing the significance of the vector of differences on owned and sharecropped land, namely [0.36, 31.3, 11.1, 183.6], is to employ Hotelling's $T^2$ statistic. The computed value of $T^2$ is 31.37, which is significant at the 0.1 percent level.

These findings suggest that the tenants were highly successful in "diverting" both nontradable and tradable resources to the land which they owned, contrary to the competitive theory offered by C-N. While landlords may have had a mind to enforce contracts specifying the mean vector of input intensities and yields for owned land, the task was apparently beyond them in practice.

II. Bargaining Over the Rental Share

The foregoing discussion suggests a number of fruitful departures from competitive assumptions. We shall confine our attention to just two of them: the inclusion of nontradable factors and the rationing of leases by landlords. These necessitate the use of a quite different analytical approach, namely, a bargaining game formulation. To keep matters simple, we assume that tenants are landless, but endowed with managerial and husbandry skills, and draught power. The introduction of the tenant's own land would add the complications of an allocation problem to those of finding a solution to the bargaining game.

Suppose just one landlord and $n$ $(<\infty)$ identical tenants reside in each locality and all land is homogeneous. The landlord, who does not cultivate any land himself,4 possesses $\alpha$ units of land, while each tenant household is endowed with one unit of labor. The labor market is perfect so that the marginal utility of leisure equals the wage rate $(w)$ and tenants maximize income net of the imputed cost of labor. Contracts specifying minimum intensities are unenforceable so the bargaining process will determine the rental share $(r)$ as the sole provision of the contract. With the existence of nontradable factors, it is reasonable to assume that each tenant's output is a strictly concave, twice continuously differentiable function of his land holding $(h)$ and unskilled labor inputs $(l)$.5

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4 This assumption is made for simplicity's sake. The value of the landlord's leisure is then independent of the number and nature of the contracts he concludes, and therefore will not feature in the analysis.

5 For simplicity, fully traded intermediate inputs such as seed, fertilizer, etc. are lumped together with unskilled labor. Table 1 indicates that this procedure is also empirically defensible.
Consider the landlord's dealings with a single tenant in the situation in which contracts with all other tenants have been closed at a rental rate \( \hat{r} \). If these two agents agree on a contract specifying a rental share \( r \), their respective incomes (including the value of the tenant's leisure) will be:

\[
Y_1 = (1 - r)F(h, l_i) - w_1 + \omega \\
Y_2 = (n - 1)F[(\alpha - q)/(n - 1), l] + rF(h, l_i)
\]

where \( l_i \) is the total labor input (i.e., family plus hired hands) of the tenant in question, and \( h \leq q \), \( (\alpha - q) \) being the area of land leased out to the other tenants and, because they are identical, equally divided among them. Now suppose that the marginal product of land is positive \( (F_2 > 0) \) so that there is excess demand for tenancies \( (r < 1) \), and land is fully utilized \( (q = h) \) and rationed. Maximization by the single tenant operating \( h \) units of land, and by all other tenants, implies also that

\[
(1 - r)F_2(h, l_i) = (1 - \hat{r})F_2[(\alpha - h)/(n - 1), l_i] = \omega
\]

respectively. If, however, the two agents fail to agree, the tenant will fall back on wage labor and the landlord will distribute the \( h \) units of land thus freed among the other \( (n-1) \) tenants, who change their labor inputs accordingly. In this situation, the tenant's and landlord's incomes become, respectively:

\[
y_1 = \omega \\
y_2 = (n - 1)\hat{r}F(\alpha/(n - 1), l')
\]

where the labor input on each of the \( (n-1) \) tenancies is now given by \( (1 - \hat{r})F_2(\alpha/(n - 1), l') = \omega \). The disagreement payoffs \( (y_1 \) and \( y_2 \)) thus represent the bargaining power of the tenant and the landlord, respectively. The solution of the bargaining problem must reflect the respective bargaining positions of both parties. In this connection, it should be noted that in the event of a disagreement, the landlord's ability to distribute any tenant's parcels among the other claimants makes his position close to what it would be if he were to engage in cultivation himself. Indeed, at the margin, he would be indifferent between such a redistribution and a little extra self-cultivation.

We now employ John Nash's method to arrive at a solution to this bargaining game. Define

\[
N \equiv (Y_1 - y_1)(Y_2 - y_2)
\]

Then any value of \( r \) which maximizes \( N \) is a solution. However, since labor utilization by tenants is not part of the contract, tenants' behavior, as given by equation (2), constitutes an external constraint on the bargaining game. That is, the influence that the rental share exerts on labor utilization is recognized by both parties and affects the outcome of the game. Nash's solution also requires that \( Y_1 - y_1 > 0 \) for \( i = 1, 2 \). That is, neither the tenant nor the landlord will enter into nonremunerative contracts. Substituting (1) and (3) into (4), we get the following first-order condition for \( N \) to be a maximum:

\[
\frac{dN}{dr} = - F(h, l_i)\{rF(h, l_i) + \hat{r}(n - 1)\} \\
\cdot \left[F(h, l) - F(\alpha/(n - 1), l')\right] \\
+ \left[F(h, l_i) + rF_2\frac{dl_i}{dr}\right] \\
\cdot \left[(1 - r)F(h, l_i) - w_l\right] = 0
\]

where, from (2)

\[
\frac{dl_i}{dr} \leq \frac{F_2}{(1 - r)F_{22}} \leq 0
\]

In Nash's solution of a two-player cooperative bargaining game, the agreement payoffs to the parties depend on their "disagreement payoffs" and the feasible set of all payoffs. The solution is consistent with the following four axioms: (i) "group rationality"; (ii) von Neumann invariance of the utility indicators; (iii) "independence of irrelevant alternatives"; and (iv) symmetry of players. Alternatively, Nash's solution may be viewed as the outcome of some plausible bargaining processes (see R.D. Luce and H. Raiffa).
Thus, any \( r^0 \) satisfying (5) is a function of \( r \). That is, the rental share negotiated in a particular contract depends on the share stipulated in the other \((n-1)\) contracts. The equilibrium configuration of contracts is such that if any contract expires, all other contracts being given, renegotiation will yield the very same contract. Recalling that by assumption all tenants are identical, then symmetry considerations imply \( r^0 = r \). Hence, by (2), \( l_1 = l \) and \( h = \alpha/n \) (implying, as one would expect, that all tenants would cultivate parcels of equal area). Thus, from (5), we have

\[
(6) \quad r^0 = \frac{1 - \beta_2}{\{(n+1)-(n-1)\} F'(\cdot) - \beta_2[1+\epsilon_2(1-\beta_2)]}
\]

where \( \beta_2 \) is the point elasticity of output with respect to labor; \( F'(\cdot) = F(\alpha/n, l) \) is the output on each farm in the event of agreement; \( F'(\cdot) = F(\alpha/(n-1), l') \) is the output on each of the other \((n-1)\) tenancies if there is a disagreement; and \( \epsilon_2 = F_2/l_2F_{22} \) (the inverse of the elasticity of the marginal productivity of labor). Thus equation (6) provides a general determination of \( r^0 \) once \( \alpha, n, w, \) and the technology are specified.

In order to illuminate the influence exerted by these factors on the rental share, we shall now derive solutions for technologies in which the elasticities of substitution between land and unskilled labor are zero, unity, and infinity, respectively.\(^7\)

(A) **Zero elasticity of substitution technology:** \( F = \min \{l', l\} \), \( 0 < \gamma, 0 \leq 1 \). As tenants are maximizers, \( h = l \). Without loss of generality, let \( \alpha = n \) so that only family labor is used. By writing down the agreement and disagreement payoffs and maximizing \( N \), it may be shown that in a universal cooperative solution, \( h = \alpha, h^0 = l^0 = 1 \), and

\[
(7) \quad r^0 = (1-w)/(2+(n-1)[1-(1-1/n)^{-r}])
\]

which, for \( n \) sufficiently large, becomes

\[
(7') \quad r^0 = (1-w)/(2-n)
\]

Now for the lease contracts to be remunerative, \( w < 1 \) (the level of output on each tenancy). Thus (1-w) is the "surplus" to be divided between the tenant's nontradables and the landlord's land. If there are numerous tenants and constant returns to scale in land and unskilled labor alone \( (\nu = 1) \), \( r^0 = 1-w \), which is the C-N solution. The competitive share of nontradables is \((1-\nu)\), so that the competitive rental share is \((\nu-w)\), which is less than \( r^0 \) from (7') for \( \nu < 1 \), the difference reflecting the landlord's bargaining power.

An important implication of a fixed coefficient technology is that there is no incentive element in the contract, as labor inputs per acre are fixed. The landlord has no enforcement problem, and \( r \) acts purely as a distributive instrument. Under these production conditions, it follows immediately that

\[
dl_1/dr \bigg|_{w, h} = 0
\]

Hence, from (3) and (5), \( I^1 - y^1 = I^2 - y^2 \), i.e., the agents' gains over their respective disagreement payoffs are equal.\(^8\)

(B) **Cobb-Douglas technology:** \( F = Ah^h/l^l \), \( A \) constant, \( 0 < \beta_1 + \beta_2 \leq 1 \). Here, consider the limiting case when \( n(n-1) \) —the extra area received by each of the other \((n-1)\) tenants if the \( n \)th fails to reach a contract—is small.\(^9\) Then, by expanding \( F' \) as a Taylor series about \( F(\cdot) \), and using the condition that the other

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\(^7\) It turns out that the derivation using a general CES technology does not yield simple interpretable expressions.

\(^8\) For this production technology, the utility possibility frontier \( l'(y', l') \) for given \( \alpha, n, \) and \( w \) is linear.

\(^9\) For example, in a village possessing 1000 acres and 100 tenant households, \( a/(n-1)w < 0.1 \) acres.
tenants alter their labor inputs on receiving an extra \( \alpha' n(n-1) \) units of land so as to maintain \( F_2 - \omega/(1-r^0) \). Some manipulation gives

\[(8) \quad r^0 = (1 - \beta_2)/(2 - \beta_1/(1 - \beta_2))\]

which is independent of factor endowments and the wage rate. As \( L \) decreases with \( r \) for given \( \omega \) and \( k \), the rental share must now play an allocative as well as a distributive role.

It is instructive to compute \( r^0 \) from (8) for various pairs of \( \beta_1 \) and \( \beta_2 \). If there are constant returns to scale in \( h \) and \( l \) \( (\beta_1 + \beta_2 = \nu = 1) \), (8) reduces to \( r^0 = \beta_1 \) which is the C-N solution. However, resource allocation is not Pareto optimal, for

\[ F_2 = \omega/(1-r^0) > \omega \]

when contracts stipulating a minimum labor intensity cannot be enforced and the elasticity of substitution between \( h \) and \( l \) is nonzero. Thus, while land receives its competitive cost share, labor inputs per unit of land are lower than would be under fixed rents, and production is not efficient.

More generally, from Table 2, we see first that \( r^0 \) exceeds the output elasticity of land by a good margin if \( F \) is highly concave. This is noteworthy because although the landlord is a potential monopolist, all land is under cultivation. For the assumptions on the technology ensure excess demand for land by tenants \( (\beta_1 \text{ and } \beta_2 \text{ are constant, and } F_1 > 0 \text{ everywhere}) \). Also, when \( F \) is strictly concave, a maximizing landlord will not keep land out of cultivation, since \( r^0 \) is insensitive to changes in \( \alpha \) when \( \alpha' n(n-1) \) is sufficiently small, and total output will fall if there is a failure to reach agreement with one or more tenants. Secondly, there is a marked tendency for \( r^0 \) to take values nearer to the magic 0.5 than \( \beta_1 \) for virtually all but extreme \( (\beta_1, \beta_2) \) pairs. Thirdly, for plausible values of \( \beta_1 \) and \( \beta_2 \), \( r^0 \) clusters closely around 0.5. Fourth, for example, reported values of \( \beta_1 \) in the range 0.2 0.4 and of \( \beta_2 \) in the range 0.02 0.2 (p. 615).

A related issue is the structure of factor shares corresponding to the ruling value of \( r^0 \). The share of (field) labor, whether supplied by the tenant's own household or by the landless is given immediately by equation (2):

\[ s_{w} = \omega F(\cdot) = (1 - r^0) l F_2(\cdot)/F(\cdot) = (1 - r^0) \beta_2 \]

Thus, following the Marshallian rule for the use of the variable factor not only depresses employment, but also reduces labor's share in output. The residual output, of course, accruies to the nontradable factor. Denoting the factor shares of land, field labor, and nontradables by \( s_L \), \( s_{w} \), and \( s_n \), respectively, the most plausible values of \( \beta_1 \) and \( \beta_2 \) yield the sets of factor shares shown in Table 2. Taking the competitive structures of factor shares as a benchmark, it is seen that: (a) the landlord exploits the tenant; (b) both the tenant (as manager) and the landlord exploit labor, though only in the fifth case is the landlord's exploitation of the tenant more than offset by the tenant's exploitation of labor.

(C) Land and labor perfect substitutes: \( F = (h+l)^\nu \), where \( \nu \leq 1 \). Here, \( F_1 > 0 \) everywhere, so there is excess demand for land if \( r < 1 \). Also, from (2), \( \nu(1-r)(h+l)^{\nu-1} \omega \). Thus, if land is redistributed in the event of a disagreement, the labor input employed by each of the other \( (n-1) \) tenants will fall so as to keep \( (h+l)^{\nu-1} \) constant. Hence production on the other tenancies will not change following a redistribution.
of land, and the landlord's bargaining position is the weaker thereby. In this case, the Nash function takes the simple form

$$N = rF(\cdot)\left[1 - (1 - r)F(\cdot) - wL\right]$$

from which we obtain, at length,

$$r^o = \frac{(1 - \beta_2)(1 - \nu)}{(2 - \nu - \beta_2)}$$

where $0 \leq \beta_2 \leq \nu$ and is also a function of $\omega$.

To round off the analysis, it is instructive to compare the rental shares ruling in the above cases as $w$ varies, but with given factor endowments ($\alpha, \eta$) and returns to scale ($\rho$). In the zero elasticity of substitution case (7'), it is immediate that $r^o$ is linearly decreasing in $w$, with $r^o = 1/(2 - \nu)$ ($\leq 1$ for $\nu \leq 1$) at $w = 0$, and $r^o = 0$ at $w = 1$ ($w \leq 1$ if lease contracts are remunerative to tenants). Now in a Cobb-Douglas technology, $r^o$ is constant and less than unity for $\nu < 1$. It is easily shown by comparing (7') and (8) that there exists some $0 < w^* < 1$ such that the rental share ruling in a Cobb-Douglas technology is less than that in a Leontief technology if $w < w^*$, and conversely if $w > w^*$.

The perfect substitutes case is more complicated. Here, note first that by (2), $\beta_2(\omega) = \nu(1 + l)$, where $l^0 = 1 (\alpha = \eta)$. By definition, $0 < \beta_2(\omega) \leq \nu$. Also $\beta_2(\omega) \to 0$ as $l \to 0$ and $\beta_2(\omega) \to \nu$, its upper bound, as $l$ becomes large. As $\beta_2(\omega) \to 0$, $r^o \to (1 - \nu)/(2 - \nu)$, and as $\beta_2(\omega) \to \nu, r^o \to (1 - \nu)/2. By differenentiating $r^o$ in (9) with respect to $\beta_2, we see that dr^o/d\beta_2 < 0$ for $\nu < 1 so that $r^o$ decreases monotonically as $\beta_2$ increases from 0 to $\nu$. Now as $r^o \to (1 - \nu)/2$ for large $l$, it follows from the tenants' maximizing behavior $[\rho(1 - \nu)/(1 + \cdot)\nu = \omega]$ that this is the limiting value of $r^o$ as $w \to 0$. Moreover, because $r^o = (1 - \nu)/2$ is the maximum of all $r^o$ on $\beta_2 \in [0, \nu]$, and as the Cobb-Douglas value in (8) exceeds this value, the rental share in technology $C$ is less than that in $B$ for all $w > 0$. A further deduction from tenants' decisions regarding labor utilization is that because $r^o = (1 - \nu)/(2 - \nu)$ for $\beta_2(\omega) = 0$ (which implies $l = 0$), then no labor will be used if $w \geq \nu(1 - (1 - \nu)/(2 - \nu) = \nu/(2 - \nu) < 1$. In this range of $w$, $r^o = (1 - \nu)/(2 - \nu)$. Also, we note from (9) that if $\nu = 1$, then $r^o = 0$ for all $w \geq 0$, a result also obtained by Newbery (1974, p. 1063). The intuitive interpretation of the behavior of the rental shares in different technologies is that for a given wage rate, the higher the elasticity of substitution, the greater the incentives (lower $r$) to use labor the landlord will provide to the tenant. An increase in the wage rate improves the bargaining strength of the tenant and thus tends to lower the rental share. However, the associated fall (if any) in the latter depends in a complicated way on the elasticity of substitution and the wage rate.

### III. Some Land Uncultivated: Causes and Consequences

To complete the analysis, we now consider the cases which may lead to some land remaining unutilized. At first glance, this may come about in two ways: either all tenants are able to choose a parcel size such that $F_1 = 0$ (assuming an appropriate production function); or the parcel size is
also determined in the bargaining process. Let us deal with each of the cases in turn.

Treading a now familiar path, we have in the former case, if a contract is reached, conditions described by equation (1) above. If there is a failure to reach agreement, then we have

\[ y^1 = w, \quad y^2 = (n - 1)F(h, l) \]

In this case,

\[ N = [(1 - r)F(\cdot) - w]_r[F(\cdot)] \]

where \( F(\cdot) = F(h, l) \). Note that \( N \) is now independent of the contracts reached with the other tenants. Recall that labor inputs applied by the tenant depend on the rental share: equation (2)—and that tenant maximization implies \( F(h, l) = 0 \). Differentiating \( N \) partially with respect to \( r \), we get

\[ \frac{\partial N}{\partial r} = -rF(\cdot)^2 + \left[ F(\cdot) + rF', \ldots \right] \cdot [1 - r]F(\cdot) - w]_r \]

By differentiating totally the tenant’s maximizing conditions, viz., \( F_1 = 0 \) and (2), and hence solving for \( d_1 \), we obtain

\[ \frac{d_1}{dr} = \frac{1}{(1 - r)\Delta} \]

where \( \Delta = F_1F_{22} - F_{21} > 0 \) by the strict concavity of \( F \). Hence, for \( N \) to be a maximum simultaneously for all contracts, we have, from (12),

\[ r^0 = \frac{(1 - \beta_2)}{2 - \beta_2[1 + (1 - \beta_2)F_{2F31}/\Delta]} \]

Now, for some \( r^0 > 0 \), but arbitrarily small, \( [(1 - r^0)F(\cdot) - w]_r] > 0 \) if land is at all cultivated. Also from (12),

\[ \frac{\partial N}{\partial r} \bigg|_{r=0} = F(\cdot)[(1 - r)F(\cdot) - w]_r] > 0 \]

for the same reason. Then, since \( 0 \leq r^0 \leq 1 \), the maximum value of \( N \), and thus the solution to the bargaining game, exists and is associated with a positive rental share. This despite the fact that the marginal productivity of land is zero.

It is important to relate the analysis of this case to the discussion of the competitive formulations in the literature. Recall that, as Newbery has shown, \( F_1 = 0 \) is the only condition under which a B-S equilibrium will exist. Given fixed factors in limited supply (in this case, nontradables) together with an abundance of land, it is highly plausible that the marginal product of land will fall to zero if the parcel size is made sufficiently large. This may well be the case in some parts of Latin America. What is striking is that our analysis predicts a positive rental share in this situation. That the rental share is positive stems directly from the landlord’s power to prevent a would-be tenant from cultivating land if the landlord so chooses.

The second case is one in which both the size of the parcel and the rental share are determined by bargaining. To solve this new bargaining problem we maximize \( N \) in (11) with respect to \( r \) and \( h \), subject to the land availability constraint \( h + (n - 1)h \leq \alpha \), and taking into account the fact that tenants are maximizing on their own account: \( (1 - r)F = w \) and \( (1 - r)F_1 \geq 0 \). The conditions for a Nash solution (maximum \( N \)) are obtained by setting

\[ \frac{\partial N}{\partial r} = 0 \]

\[ \frac{\partial N}{\partial h} \geq 0 \]

complementarily \( h + (n - 1)h \leq \alpha \)

Suppose the land availability constraint does not bind and \( (1 - r)F_1 > 0 \). We have, from (15),

\[ (1 - r)F + F_1 \leq \frac{d_1}{dh} \cdot [(1 - r)F(h, l) - w]_r] = 0 \]

Tenants’ employment decisions imply \( d_1/dh = -F_{21}/F_{22} > 0 \) for well-behaved pro-
duction functions. Also, $(1-r)F(h, l) - \omega_l > 0$ if production is at all undertaken. But then condition (16) is not satisfied, for all terms on the left-hand side are positive. Hence, $F_i = 0$ if the land availability constraint is not binding. By symmetry, the landlord must reach the same contract with all tenants, so that $h = h^0 = h$ and $r = r^0 = r$, with $F_i [h^0(r^0), l^0(r^0)] = 0$. We are then back to the first case. Alternatively, suppose that the land availability constraint binds; but then we are in our original formulation of the problem, in Section II.

We have thus established that if any land lies uncultivated, then the marginal productivity of land is zero and the rental share is positive. This result contrasts both with the competitive solution, in which the rental share is zero if $F_i = 0$, and with the monopoly solution, in which both the rental share and $F_i$ are positive in the face of land lying uncultivated.

IV. Concluding Remarks

In this paper, we have used bargaining theory to derive determinate solutions for the rental share. We have adopted this approach because both the nature of the contract and the features of the agrarian system necessitate it, while competitive formulations must ignore such considerations. In practice, of course, the bargains which are struck need not conform exactly to those predicted by Nash’s solution. But if Nash’s solution is empirically valid, and given that the bargaining process is repeated year after year in circumstances of unchanging technology and social institutions, then there will be a long-term tendency for contracts to center on some average value. Moreover, if in most situations the solution is not very different from one-half, agents will save themselves the bother of detailed arithmetic and settle for that magic number, which has also the advantage of sounding equitable. Thus that particular rental share is elevated to the status of a social norm.

It goes without saying that some of the drastic simplifications in our formulation may vitiate the analysis. In particular, when a would-be tenant can deal with more than one landlord his bargaining position is stronger than in our formulation. But given the spatial dimensions of agricultural production and the resulting costs of cultivating widely scattered parcels, our neglect of the alternative opportunities stemming from the existence of other landlords may not prove to be too serious. To introduce such considerations into the analysis would entail very considerable complications, which we have chosen not to pursue at this stage.

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