The Manipulation of Futures Markets by a Dominant Producer

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David M. Newbery

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Abstract

This paper studies the impact of a futures market on a cash market in which there is a dominant producer and a competitive fringe of price-taking firms. It is shown that a futures market has two effects: it increases the production of the fringe producers because it enables then to reduce their risk, but it also enables the dominant producer to manipulate futures markets sales to induce the fringe firms to produce less. Thus the dominant producer has an incentive to destroy futures markets, but, failing that, his activities nevertheless increase the efficiency of the futures market (reduce its bias), and reduce producer risk. If fringe producers are (unreasonably) risk averse, and cannot observe the dominant producer's current supply or storage decision, then he has a further incentive to destabilize the cash market.
1. Introduction

The recent development of futures markets in crude oil and oil products raises in an acute form the question of how futures markets operate for commodities whose production is dominated by large producers or cartels. Does the presence of a futures market reduce or increase the market power of the dominant producer? Should consumers encourage the development of such markets or regulate and restrict their operation? Does the market power of the dominant producer in the cash market have any counterpart in the futures market? In particular, does a dominant producer have an incentive to manipulate futures markets? Clearly these and other questions to do with the performance of futures markets for imperfectly competitively produced commodities require us to go beyond the standard perfectly competitive models.

The tradition of treating primary commodities as though they are produced and traded under conditions of perfect competition is surprisingly strong and needs questioning quite apart from the case of oil. In the first place, many governments, in both developed and developing countries, intervene in agricultural markets, so that the natural unit of production is arguably the country, rather than the individual farmer or plantation. These countries often produce a significant share of world production. For minerals, the same companies reappear in many different countries, and the largest companies control a significant fraction of world production. Newbery, (1981) listed 8 commodities for which single countries controlled more than 50 percent of world trade, and a further 13
for which single countries controlled between 25 and 50 percent (averaged over 1977-79). Interestingly, Saudi Arabian oil just qualified in this period with 26 percent (and aight not on 1981-82 data) whilst Brazilian coffee was excluded over this period with only 17 percent.

In the second place, to the extent that primary producers have long been concerned about both the stability and level of commodity prices, there have been made repeated attempts to establish commodity agreements or cartels of varying degrees of formality and durability, and such cartels have been potentially large relative to the market, and well placed to trade on the established futures markets. Of the 21 commodities where country trade shares are above 25 percent, at least 9 have futures markets and are listed in Table 1 below. (In addition, bauxite was one of the commodities, and there is a futures market in aluminium.) Copper, coffee and cocoa, all with active futures markets and moderate producer concentration, did not appear on the original list but are included in Table 1 for completeness.

It therefore seems important to try and model the behaviour of dominant commodity producers confronted with the choice of trading on futures markets. Apart from any other reason, there are powerful incentives, mentioned below, for producers to be discrete about any attempt to manipulate futures markets, and hence manipulation will be difficult unless the motives and strategies of such agents can be clarified. Finally, there is evidence that the coffee cartel engaged in
<table>
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<th>Country</th>
<th>Product</th>
<th>Share (Percent)</th>
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<tr>
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<td>73</td>
</tr>
<tr>
<td>Australia</td>
<td>Wool (greasy)</td>
<td>60</td>
</tr>
<tr>
<td>Malaysia</td>
<td>Rubber</td>
<td>51</td>
</tr>
<tr>
<td>USA</td>
<td>Wheat</td>
<td>39</td>
</tr>
<tr>
<td>Malaysia</td>
<td>Tita</td>
<td>36</td>
</tr>
<tr>
<td>USA</td>
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</tr>
<tr>
<td>Brazil</td>
<td>Coffee</td>
<td>17</td>
</tr>
</tbody>
</table>

extensive market manipulation in the period 1977–79, so that the subject is of more than academic interest. (See the fascinating accounts in Greenstone (1981) and Edmunds (1982)).

The main conclusion of this paper is to show that if all agents have access to the same information, and can predict the production and trading strategy of the dominant producer, the dominant producer has an incentive to manipulate the futures market in order to benefit his position in the cash market, but nevertheless his futures trading activity reduces the bias in that market. Moreover, his market power in the futures market diminishes the more competitive speculators enter the market, and the less averse to risk they are. The second result is that if agents have the same information but cannot observe the production decision of the dominant producer, then he may have an incentive to follow randomized production strategy (or to randomize his storage decision.) However, unless fringe producers are very risk averse, this will be an unattractive strategy.

The paper is organized as follows. The next section discusses the role of information and the importance of specifying what the different market participants know when making their decisions. Section 3 sets out the model of the futures market and demonstrates that, assuming the dominant producer is less risk averse than the fringe producers, he would benefit from the collapse of the futures market. Section 4 discusses the incentives for manipulating the futures market when agents have full information, first as a large speculator, then as a large producer. This demonstrates the main result that he benefits from
manipulation as a large producer, no matter what the source of risk. The final section discusses the extent to which the dominant producer can benefit from varying his decisions from year to year when his competitors cannot observe the current decision, but can only predict the average supply.

2. The Role of Information in the Manipulation of Futures Markets

The simplest model in which to examine the incentives for futures market manipulation is one of a non-storable agricultural final consumption commodity produced in an annual cycle. At the time of planting the future weather (and hence output) are uncertain, as, consequently, is the post harvest market clearing price, p. Since the commodity is not stored, nor further processed, the only agents concerned are the producers, consumers, and speculators. I shall assume that consumers have no incentive to hedge, and so the only participants in the futures market will be the producers and speculators. 1/ The futures market opens at the start of the crop season, and contract expires after the harvest is in and the spot market clearing price has been established. Thus farmers make their production decisions at the same time as choosing their futures trades, and knowing the futures price, pf.

1/ If the commodity is further processed, then processors will also hedge, and experience different risks than producers, as will stockholders for storable commodities. Modelling this behaviour is important for some questions — notably whether futures markets are biased — but are not obviously relevant for market manipulation. See Stein (1979) and Anderson and Danthine (1982) for models with processors and stockholders.
Since we are interested in modelling manipulation by a dominant producer, I shall assume that there is one dominant producer (to be thought of as the marketing board of a large exporting country, or its counterpart for a cartel) facing a fringe of competitive small producers — the farmers in the rest of the world. The dominant producer is well organised enough to arrange adequate income insurance for its own farmers (via domestic price supports, direct transfers, etc.) and large enough to secure adequate intertemporal income smoothing by borrowing and lending, so that it acts as a risk neutral agent. (See Newbery and Stiglitz, 1981, 14.3, pp. 201–204). The fringe farmers are risk averse.

The first and crucial issue to address in discussing market manipulation is what information the various participants have, and what use they make of it in choosing their strategies. It is convenient to distinguish three cases, with very different implications for market manipulability. At one extreme, the dominant producer may hold rational expectations about demand and supply conditions whilst the fringe producers may follow some naive forecasting or decision rule. This implies that the dominant producer knows what this decision rule is and can compute the fringe decisions given information available at the start of the crop year, and, given his own decisions, can predict the joint distribution of his output and the market clearing price (or a sufficient statistic of the distribution — see Newbery and Stiglitz, Ch. 10, 11.) Clearly, the dominant producer is well placed to manipulate futures markets and to exploit his superior information and forecasting ability. Hart (1977) has studied this form of manipulation, which relies on being
able to systematically mislead the other agents and hence cause them to make losses. It appears that the alleged coffee market manipulation described by Greenstone (1981) and Ednunds (1982) relied on this strategy, since in 1977 the cartel bought July futures heavily, and also bought physical coffee to prevent it reaching the market, to squeeze the market. Had speculators possessed the same information as the cartel (in particular, the cartel's plans) then this maneuver would not have succeeded.

At the other extreme, all agents may have access to the same information when making their decisions, and hold rational expectations, in the sense that they know the model which describes the (stochastic) determination of market equilibrium. This would be the case if agents knew the objectives (utility functions), production functions, nature of demand, and the joint probability distribution describing the stochastic elements, and were able to compute the equilibrium choices of all agents. There is thus complete symmetry of information.

The third possibility lies between the two extremes, though closer to the full rational expectations equilibrium just described. Agents have full-information about production and utility functions, and the nature of risk, but cannot observe production decisions. In an unchanging world in long run equilibrium agents could make the same production decisions each year, and these would then be predictable. Fringe agents, being individually insignificant, would have no incentive not to make such predictable decisions, but the dominant producer may benefit from randomizing his production about soce (ultimately)
predictable level in order to introduce asymmetries into the information system. He would then be better placed to predict futures market clearing prices, since he would know his own production decision, and the fringe would face less predictable prices, or greater risk.

This paper will ignore the first kind of manipulation, since its proper study would require a model of information conveyed by the market. At this stage it does not seem particularly useful to build further models of sophisticated agents playing against naive agents who follow well defined and systematically incorrect forecasting rules, though it would be very interesting to explore the extent to which a dominant producer can exploit informational advantages over intelligent but less well informed agents. (See, for example, Kyle, 1981). Instead, I shall concentrate on the symmetric information rational expectations case, and discuss the last kind of asymmetry briefly. The reason for this is simple — if agents can manipulate markets even when other agents know exactly what they are planning to do, then this is a robust result, since they will presumably have even more incentive to manipulate if they have additional informational advantages to exploit.

3. Incentives for the Destruction of Futures Markets

In the model just described, a risk neutral dominant producer facing risk averse competitors is harmed by the establishment of a futures market, and hence has an incentive to undermine or destroy the market. The reason is that futures markets reduce risk and hence induce a positive supply response from the fringe which depresses the market clearing price
and hence reduces the dominant producer's profits. Since he gains no insurance benefits from the futures market, he is unambiguously worse off with the market than without.

To establish this result, we need to model the production decisions of the farmers and the futures trading decisions of farmers and speculators. Following well-established tradition, assume that agents have constant absolute risk aversion and that prices and quantities are jointly normally distributed. This allows us to use the mean-variance analysis of the standard capital asset pricing model, and gives rise to linear trading rules which can be aggregated and solved in closed form. These admittedly strong assumptions can be defended as second order approximations to a more complex reality, and should not prove critical in the analysis which follows.

Suppose that output, q, after the harvest depends multiplicatively on the weather (θ) and the level of inputs, x:

\[ q = \theta f(x), \quad \theta = 1, \quad \text{Var} \theta = \sigma^2, \]  

(1)

In the absence of a futures market, the farmer's income is

\[ y = \tilde{p}q - vx \]  

(2)

where the input price is v, and the output price is \( \tilde{p} \), a random variable at the time of planting. If \( 
\) is the coefficient of risk aversion, and
the farmer chooses to maximize expected utility, then his decision problem is equivalent to maximizing

\[ W = E y - \frac{1}{2} \lambda \text{Var } y \]  

(e.g. see Newbery and Stiglitz, 1981, p. 85). Thus the farmer's choice of \( x \) satisfies

\[ \hat{p} f'(x) = w \]  

where

\[ \hat{p} = E p \theta - \lambda f \text{Var } p \theta \]  

is the action certainty equivalent price (i.e., that price which, in the absence of uncertainty, would lead the farmer to choose the same action, or level of input, \( x \).

If there is a futures marker, the farmer can, in addition, choose his level of futures sales, \( z \), in which case his income will be

\[ y = \hat{p} \hat{q} + z(p^f - \hat{p}) - wx \]  

and his objective is to maximize
\[ W = E \ p \ q - w x + z(p^f - \bar{p}) - \frac{1}{2} A \{ \text{Var} \ p q - 2 z \ \text{Cov} \ (p, p q) + z^2 \ \text{Var} \ p \} \]  \hspace{1cm} (7)

where \( \bar{p} = E p \) is the expected post-harvest price. If \( z \) can be positive or negative (i.e., forward purchases are also possible) then the expected utility maximizing choice of \( z \) is

\[ z = \frac{\text{Cov} \ (p, p q)}{\text{Var} \ p} - \frac{\bar{p} - p^f}{A \ \text{Var} \ p}. \]  \hspace{1cm} (8)

Speculators, on the other hand, have no risky production or other sources of risky income, and sell \( z^s \) forward (or, more accurately, buy \(-z^s \) forward) to maximize

\[ W^s = z^s (p^f - \bar{p}) - \frac{1}{2} A^s z^s 2 \ \text{Var} \ p \]

\[ z^s = -\frac{\bar{p} - p^f}{A^s \ \text{Var} \ p}. \]  \hspace{1cm} (9)

This has the same form as the second term in equation (8), which thus allows us to interpret that as the speculative component, whilst the first term is the hedging component. In our simple model, the speculators can only be persuaded to take a long position (in which they provide the
forward purchases which balance the farmers' forward sale) if $p^f$ is below the expected future price. The normal backwardation provides the risk premium which covers the cost of transferring risk from farmers to speculators. However, it is worth pointing out that even in this simple model in which there are no processors or stockholders whose hedging needs might complement those of the farmer's, it is still possible for $p^f$ to equal or exceed the expected future price. To see this we need to look at equilibrium in the futures market, in which net futures sales must be zero. In the absence of any other agents this implies that the sum of sales of farmers and speculators must be zero, or, if all agents share common beliefs, and face the same risk, 9:

$$0 = \sum (z^f + z) = \frac{\text{Cov}(p,p\theta)}{\text{Var} p} \sum q - \frac{\bar{p} - p^f}{\text{Var} p} \sum \left( \frac{1}{A} + \frac{1}{A^s} \right).$$

(10)

Let $\alpha$ measure the effective degree of risk aversion of the market as a whole:

$$\frac{1}{\alpha} = \sum \left( \frac{1}{A} + \frac{1}{A^s} \right).$$

(11)

Thus if there are $n$ farmers and $m$ speculators, all equally risk averse, $\alpha = A/(n + m)$, whilst if any agent is risk neutral, then $\alpha = 0$ and the market acts risk neutrally. If average total production is then the bias in the futures market, or the extent of normal backwardation, is

$$\bar{p} - p^f = \alpha \bar{q} \text{Cov}(p,p\theta).$$

(12)
and the equilibrium futures sale by farmers $i$ is, from (12) and (8)

$$z^i = \beta^i - 1 \frac{\text{Cov}(p, p_\theta)}{\text{Var} p}$$

(13)

where, for farmer $i$

$$\beta^i \equiv 1 - \frac{\bar{a}}{A^i} , \quad 0 < \beta^i < 1 ,$$

is a measure of the extent to which the farmer is more risk averse than average ($A^i/a$) and more exposed to risk than average ($\bar{a}/\bar{q}$). If all farmers are identical and there are no speculators, $\beta = 0$, whilst if there is one risk neutral speculator, so $a = 0$, then $\beta = 1$. In between, if there are $n$ farmers and $m$ speculators, all equally risk averse, then, for an average farmer

$$\beta = 1 - \frac{n}{n + m} = \frac{m}{n + m} .$$

(14)

The term $\beta$ can also be thought of as the extent to which the farmer is able to share the risk with other agents in the economy. The larger is the number of speculators, the more the farmer is able to transfer risk to then, and the more heavily he is willing to be involved in the futures markets. The less risk averse speculators are, the smaller will be $\alpha$, and the more willing they will be to accept the farmers' risk, and again, the more the farmer will be willing to trade these risks in the future markets.
If, as assumed, \( p \) and \( \theta \) are jointly normally distributed, Appendix A shows that equation (13) can be written

\[
\frac{z}{\sigma} = \beta^\prime (1 + r\sigma_a / \sigma_p) \tag{15}
\]

where \( a, a_p \) are respectively, the coefficients of variation of output and price, and \( r \) is the correlation coefficient between output and price. Clearly, this expression can be negative, in which case from equation (12) \( p_f \) will exceed \( p \) and the futures market will exhibit contango instead of backwardation.

When the farmer can hedge on a futures market, his action

certainty equivalent price, \( p_f \), is, from (7)

\[
p_f = E_p \theta - A_f \text{VAR}_p \theta - z \text{Cov} (p \theta, p) \tag{16}
\]

where \( z \) is the optimal futures trade, satisfying (8) or (13). Suppose now that a futures market is introduced into an otherwise unstabilized market, and that it has no effect on the beliefs or information about future prices. If it had no effect on the post market price distribution, then from (5) and (15)

\[
p_f - p = A z \text{Cov} (p \theta, p) - A_f \beta \text{Cov}^2 (p \theta, p) / \text{VAR}_p \tag{17}
\]

substituting from (13). In this case the futures market unambiguously increases the action certainty equivalent price, which, other things
equal, will induce a supply response, and thus lower the average spot price.

The dominant producer derives no insurance benefit from the futures market, since he is risk neutral, but is clearly harmed by the increase in fringe supply, and hence will be hostile to the introduction of futures markets, and will have an incentive to destroy them. He would, in particular, have an incentive to play on the populist sentiments of regulators by encouraging them to believe that futures markets can be manipulated to the disadvantage of producers, if he thought this might provoke moves to restrict or close future markets. He would have an incentive to convince other potential participants in the market that the market was manipulated, or rigged, and hence the game unfair and not worth playing. There is some evidence that futures markets known to be dominated by government marketing agencies are unpopular, and consequently thin (Gray, 1960). Since the dominant producer certainly has an incentive to mislead other agents, it should not be difficult to so convince these agents, and there is the consolation that if an attempted squeeze or other manipulation fails in its primary purpose, it may succeed by discrediting the market.

The remedy for this kind of behaviour is to provide as much public information as possible about the actions of the dominant producer, in order to broaden the market and increase the number of traders. From (17), the fringe supply response increases with $\beta$, i.e. from (14), with the number of speculators, $n$. 
4. **Incentives for Manipulating Futures Markets with Full Information**

If futures markets are, however, successfully introduced and maintained, they will affect the trading environment of the dominant producer, and he will have an incentive to manipulate the futures market even if all participants know what strategy he is pursuing. At this point, it is perhaps worth defining manipulation. The Commodities Trading Commission defined it thus in 1977:

"... conduct intentionally engaged in resulting in an artificial price that does not reflect the basic forces of supply and demand.

A finding of manipulation in violation of the (Commodities Exchange) Act requires a finding that the party engaged in conduct with the Intention of affecting the market price of a commodity (a? determined by the forces of supply and demand) and as a result of such conduct or course of action an artificial price was created. (at 21, 477, CFTC Dkt No.75-4, Feb. 18, 1977 [1975-77 Transfer Binder] CCH Comm. Fut. L. Rep. 120, 271, cited by Greenstone, 1981, p. 11.)

This seems acceptable if we interpret "artificial price" as a price differing from the competitive price. Clearly, a large producer will know that his actions will affect the price — the question is whether he takes advantage of this power. The competitive equilibrium in the futures market with a risk neutral agent is one in which \[ p^f = p \] and the market is unbiased. Any trading strategy which fails to yield this outcome is, in our full information model, evidence of manipulation.

A dominant producer has two different motives for manipulating the futures market. First, as large agent, he may be able to exercise market power in the futures market quite independently of his activities on a producer. This he does by influencing the spread between the futures
price \( p^F \) and the expected cash price \( \bar{p} \), both directly by trading on the futures market and hence affecting \( p^F \), and indirectly, by changing \( p^F \) he can change the action certainty equivalent price, \( \bar{p}^F \), and hence induce a supply response which affects the cash price, \( \bar{p} \).

The second way in which the dominant producer can advantageously manipulate the futures market is to change the futures price, and hence, via a change in the action certainty equivalent price, induce a supply response by the fringe which increases the dominant producer's profits. If, for example, he can lower the futures price this will reduce fringe supply and increase his sales price. However, this is not costless, as he may lose on his futures trading activities, and the extent to which this strategy is profitable requires a careful cost–benefit analysis.

The next two sections consider each of these motives separately in special cases, chosen to yield a quantifiable estimate of the importance of such manipulation. The first simplifying assumption is that both demand and ex-ante supply schedules are linear. If the production function of a representative farmer is

\[
f(x) = \frac{1}{n}x + \sqrt{(2n^2/n+k)}
\]

(18)

where \( k \) is some constant chosen to yield a sensible form for \( f(x) \) for low levels of output, then, assuming the input price of \( x \) is unity, planned output will be

\[
f(x) = \frac{1}{n} \{1 - \eta + \eta \bar{p}\}
\]

(19)
and aggregate average supply, \( \bar{Q} \), will be

\[
\bar{Q} = 1 - \eta + \eta \hat{p}
\]

(20)

where \( \hat{p} \) is again the action certainty equivalent price. If the demand schedule is

\[
p = 1 + \frac{1}{\varepsilon} - \frac{\bar{Q}}{\varepsilon} + \tilde{u}, \quad \tilde{u} = 0
\]

(21)

then, in the absence of any uncertainty, market equilibrium in this competitive market would be \( p = 1, Q = 1 \), and \( \varepsilon, \eta \) would have the elasticities of demand and (ex-ante) supply respectively. For small risk, the market equilibrium will be close to this point, and the elasticities of demand and supply will likewise be approximately \( \varepsilon, \eta \) at and near the equilibrium. The two polar cases of risk we shall consider are pure demand risk and perfectly correlated supply risk.

4.1 Pure Demand Risk

In this case output is certain so

\[
\bar{Q} = 1 - \eta + \eta \hat{p}
\]

but the demand is risky, so that the coefficient of variation of price is \( \sigma_p \), or, from (21), since \( \bar{p} \sim 1 \),
\[ \text{Var } p = E u^2 = \sigma_p^2. \]

With pure demand risk, it is true quite generally that the action certainty equivalent price is the futures price (Danthine, 1978). The argument goes as follows. Let \( U(y) \) be the utility of income \( y \), then Earners choose inputs \( x \) and futures sales \( z \) to

\[ \begin{align*}
\max_{x, z} & \quad EU \{ p^f(x) - wx - z(p - p^f) \}\.
\end{align*} \]

The first order conditions are

\[ f' \quad EU'p = w \quad EU' \]

\[ EU'p = p^f \quad EU' \]

hence

\[ p^f \quad f'(x) = w, \quad p = p^f. \]

4.2 Perfectly Correlated Supply Risk 

If farmers experience perfectly correlated supply risk, so that their output is

\[ q^1 = \theta \quad f^1(x), \quad E\theta = 1, \quad \text{Var } \theta = a^2. \]
then their action certainty equivalent price, \( \hat{p} \), is, from (16) and (8);

\[
\hat{p} = \mathbb{E} \theta - Af \left\{ \text{Var} \ p \theta - \frac{\text{Cov}(p, p \theta)}{\text{Var} \ p} \right\} - \left( \overline{p} - p^f \right) \cdot \frac{\text{Cov}(p, p \theta)}{\text{Var} \ p}
\]

For perfectly correlated pure supply risk, the correlation between price and output is \( r = -1 \), and \( \alpha_p = \alpha \). From Appendix A

\[
\text{Var} \ p \theta = \frac{\alpha_p^2}{\varepsilon^2} \left\{ (1 - \varepsilon)^2 + 2\sigma^2 \right\} - \frac{\alpha_p^2 \sigma^2}{\varepsilon^2} (1 - \varepsilon)^2
\]

\[
\text{Cov} \ (p, p \theta) = \frac{\alpha_p^2 \sigma^2}{\varepsilon^2} (1 - \varepsilon) = (1 - \varepsilon) \text{Var} \ p
\]

so, ignoring terms in \( \alpha^4 \)

\[
\hat{p} = \mathbb{E} \theta - (1 - \varepsilon) \left( \overline{p} - p^f \right)
\]

\[
\hat{p} = (1 - \varepsilon)p^f + \varepsilon \overline{p} \left( 1 - \sigma^2 / \varepsilon^2 \right)
\]

If producers experience additive risk, however, so that

\[
\tilde{q} = f(x) + \tilde{u}, \quad \mathbb{E} \tilde{u} = 0
\]

then \( \hat{p} \) is again \( p^f \), as can be seen from the first order conditions for expected utility maximization:
\[
\max_{\tilde{z}} \{ f(x) + \tilde{u} \} = -wx + z(p^f - \tilde{p}).
\]

(26)

4.3 Futures Market Manipulation by a Large Speculator

First, we examine the best trading strategy for a large risk neutral non-producing speculator in an otherwise competitive market of competitive producers described by equations (20) and (21), in the simpler case of pure demand risk. (The case of pure supply risk is left as an exercise.) If the large speculator sells \( S \) futures then equilibrium in the futures market is, from (8)

\[
S + \Sigma z^1 + \Sigma z^S = 0 = S + Q + \frac{p^f - \tilde{p}}{\alpha \text{Var} \ p}
\]

(27)

where \( \alpha \) is defined in (11) and (certain) production is

\[
Q = 1 - \eta + \eta p^f.
\]

(28)

The expected price in the cash market, \( \tilde{p} \) is given by (28) and (21):

\[
\tilde{p} = 1 + \frac{\tilde{p}}{\tilde{z}^f} - \frac{\eta}{\kappa} \tilde{p}^f.
\]

(29)

Competitive equilibrium with a risk-neutral speculator would yield an unbiased futures market, \( p^f = \tilde{p} = 1 = Q = -S \). The expected income of the large speculator is, using (29):
Although the large speculator manipulates the market when comparing his behaviour to the competitive equilibrium, his presence improves arbitrage, or reduces the magnitude of the bias, \( \bar{p} - p^f \), for in his absence the competition equilibrium could be

\[
p^f = 1 - \frac{1}{\gamma}, \quad \bar{p} = 1 + \frac{\eta}{\gamma C}.
\]  

(33)

The presence of the large speculator clearly harms other speculators (and will thus tend to make the market thinner), but in the present model he induces a positive supply response which benefits consumers. It is a somewhat delicate question whether he benefits or harms other producers, since their average price falls, but they obtain better risk sharing facilities via the less biased futures market. In the Appendix the gain to producers of allowing the large speculator to trade is

\[
\frac{1}{2\gamma} - \frac{3\eta}{8\gamma^2} - \frac{2(1 + \eta/\varepsilon)^2}{8A\gamma^2}.
\]  

(34)

which is small, but for at least some parameters, positive. Thus, somewhat surprisingly, in this model allowing a large speculator to manipulate the futures market (as opposed to excluding him) can benefit everyone except the existing speculators, assuming all agents have rational expectations. If, however, the futures market is thin, and hence quite biased, and supply relatively more elastic than I assumed, then, as the
Appendix shows, producers would benefit from prohibiting the large speculator.

\[ \text{4.4 Futures Market Manipulation by a Large Producer} \]

The previous section demonstrated that large risk neutral agents have a direct incentive to manipulate futures markets. We now ask whether they have an additional incentive if they are large producers. This will be the case if their choice of futures sales (or purchases) depends on their level of production. The intuitive reason why this is beneficial is that by changing the futures price the dominant producer can reduce fringe supply and hence increase his profits as a producer.

To explore this, we need a slightly different model of supply. Suppose aggregate fringe supply is given by

\[ \tilde{Q} = \mu(1 - \eta + \eta \hat{P}) \]  

(cf. (19)). The parameters are chosen so that at an equilibrium quantity and price of unity, the elasticity of fringe supply is \( \eta \), and market share \( \mu \). See Appendix B for details) Demand is again given by (21), whilst the dominant producer’s production function is

\[ F(x) = m (1 - \lambda) + \sqrt{2m \lambda x + k} \]  

(The parameters \( m, \lambda \) are to be chosen so that in the absence of risk, the market equilibrium price is unity. The parameter \( \lambda \) would then be the
elasticity of supply were the producer to behave competitively. \( K \) is an arbitrary constant \( \text{which} \) does not affect the equilibrium output.) Since it is by no \text{means} obvious that the \text{dominant} producer \text{will} benefit \text{from} futures \text{market} manipulation as a producer (and not just as a risk-neutral agent) it is important to consider different types of risk specification.

4.5 \textbf{Pure Demand Risk}

Fringe supply is \textit{riskless} and equal to

\[
Q = \mu (1 - \eta + \eta p^f). \tag{37}
\]

If the \textit{dominant} producer produces \( q \) and sells \( S \) futures, equilibrium \textit{in} the futures market requires (cf 27):

\[
S + Q + \frac{p_f - \bar{p}}{\alpha \nu} = 0, \tag{38}
\]

which, substuting for \( Q \), gives

\[
p_f = \bar{p} - \alpha \nu S - \mu \alpha \nu (1 - \eta) \frac{1}{1 + \mu \alpha \nu \eta}, \tag{39}
\]

and hence

\[
Q = \frac{\mu (1 - \eta + \eta (\bar{p} - \alpha \nu S))}{1 + \mu \alpha \nu \eta}. \tag{40}
\]
Equilibrium in the cash market requires the average price to satisfy

$$\bar{p} = 1 + \frac{1}{\varepsilon} - \frac{1}{\varepsilon} (q + Q)$$

(41)

which, with (40), gives

$$Q = \frac{\mu(1 + \frac{\mu}{\varepsilon} - \frac{\eta}{\alpha} q - \eta \alpha v S)}{1 + \mu (\alpha \eta + \eta / \varepsilon)}$$

(42)

The dominant producer's expected profits are

$$y = \bar{p}q - \alpha x + S (p_f - \bar{p})$$

which, from (38), can be written

$$y = \bar{p}q - \alpha x - \alpha v S (S + Q)$$

(43)

Since from (42) Q is linear in q and S, so is $\bar{p}$.

Consequently expression (43) contains terms in Sq, and hence the optimizing choice of S will depend on q, and vice versa. Therefore the producer's choice of future trades, S, will depend on his choice of output, q, and vice versa. To quantify this interdependence, maximize y in (43) with respect to $\bar{p}$

$$\frac{\partial y}{\partial S} = 0 = \frac{\partial \bar{p}}{\partial Q} \frac{\partial Q}{\partial S} - \alpha v (2S + Q + S \frac{\partial Q}{\partial S})$$

(44)
where from (41) \( \partial p / \partial Q = -1/\varepsilon \). This can be solved to yield

\[
S = -\frac{1}{2} \left( -\frac{\kappa + \varepsilon}{\kappa + \varepsilon / \mu} + \frac{\mu \eta q}{\varepsilon + \mu \eta} \right).
\]

(45)

Note that when \( \mu = 1, q = 0 \), so that there is no dominant producer, the result collapses into the previous case of a large speculator, for whom \( S = -\frac{1}{2} \).

The expected profit maximizing choice of input, \( x \), satisfies

\[
\frac{\partial \pi}{\partial x} = 0 = \{ \tilde{p} + q \frac{\partial \tilde{p}}{\partial q} - \alpha v s \frac{\partial q}{\partial q} \} F'(x) = 0
\]

where the term in braces is the dominant producer's action certainty equivalent marginal revenue, \( MR \). Substituting for \( \tilde{p} \) and \( \partial \tilde{p} / \partial q \) this can be written

\[
MR = 1 + \frac{1}{\varepsilon} - \frac{1}{\varepsilon} (q + Q) - \frac{q}{\varepsilon} (1 + \frac{\partial q}{\partial q}) - \alpha v s \frac{\partial q}{\partial q}
\]

whilst from (36)

\[
q = \eta (1 - \lambda + \lambda MR)
\]

These two equations can be solved to yield

\[
2 \mu \eta \alpha v s = \left[ (\frac{\kappa}{\lambda \eta} + 2) \phi - \frac{2 \mu \eta}{\varepsilon} \right] q + \mu (1 + \frac{\eta}{\varepsilon}) (1 + \frac{\kappa}{\lambda}) \phi
\]

(46)
where

$$\phi = 1 + \mu \frac{x + \eta}{\eta + \varepsilon}.$$ 

Equations (45) and (46) can be solved for $S$ and $q$, but one conclusion is immediate; $S$ increases with $q$. In the previous section we saw that a large speculator would still be a net futures buyer, so $S$ would be negative. If he is also a producer, then he buys fewer futures. The effects of this, from (42), is to reduce $Q$ and hence increase his market power in the cash market.

4.5.1 The Consequences of Excluding the Dominant Producer from the Futures Market

If the dominant producer is somehow excluded from the futures market, then the market equilibrium can be found from (46) and (42) by setting $S = 0$. The dominant producer acts rather like another competitive speculator (so far as arbitraging prices) provided $S < 0$.

If, on the other hand, $S > 0$ the dominant producer would be amplifying the bias in the futures market, to the disadvantage of consumers and possibly producers. In such a case, banning the dominant producer from trading may improve matters. However, it can be shown that it is never desirable for the producer to be a net futures seller, and so, perhaps surprisingly, allowing the dominant producer to trade on the futures market always reduces the degree of bias of the market in this model. There is another or additional way of reducing the bias, and that

1/ As shown above, reducing the bias reduces producer risk but may, via the supply response, reduce their income and lower utility. See Appendix B.

2/ See Appendix C.
is to reduce \( \alpha \), the measure of market risk aversion, by increasing the accessibility of the market and reducing transactions or entry costs.

**Numerical Example**

It \( \mu = 1/2, \varepsilon = \eta = 1, \lambda = 1, \omega = 3/4 \), then in the absence of risk, \( p = 1, q = 1/2, Q = 1/2 \) and the market would be equally shared between the fringe and the dominant producer. If now \( \omega \alpha = 1/20 \), then the manipulated equilibrium is:

\[
q = 0.5, \quad S = -0.1667, \quad Q = 0.4945 \\
\overline{p} = 1.0055, \quad p^f = 0.9891
\]

If the dominant producer is present from \( \omega \) to \( \omega \alpha \), then the equilibrium is

\[
q = 0.5020, \quad Q = 0.4911 \\
\overline{p} = 1.0069, \quad p^f = 0.9823
\]

4.6 **Perfectly Correlated Supply Risk**

As the other extreme polar case, suppose all producers experience perfectly correlated supply risk, \( \Theta \), and no demand risk, so that \( u \) is (21) is identically zero. Average fringe supply is, from (2) and (25)

\[
\overline{Q} = 1 = \eta + \eta (1 - \varepsilon) p^f + \varepsilon \eta (1 - \sigma^2/\varepsilon) \overline{p} \overline{p}
\]

(47)

Equilibrium in the futures market in which the dominant producer sells \( S \) requires, from (10) and (24)
The average cash market clearing price satisfies

\[ \bar{\rho} = 1 + \frac{1}{\varepsilon} - \frac{1}{\varepsilon} (\bar{Q} + \bar{q}) \]

so

\[ \bar{p} \left\{ 1 + \alpha \eta (1 - \varepsilon) \right\} = \left\{ 1 + \alpha \eta (1 - \varepsilon) \right\} \left\{ 1 + \frac{1}{\varepsilon} - \frac{\bar{q}}{\varepsilon} \right\} - \frac{1}{\varepsilon} \left\{ 1 - \eta \alpha \eta (1 - \varepsilon) S \right\} \]

The dominant producer again wishes to maximize expected profit

\[ y = q \, \bar{p} \, \text{Ep} - \omega x + S (p^f - \bar{p}) . \]

This can be written, using (48), as

\[ y = q \, \bar{p} \left( 1 - \frac{\sigma^2}{\varepsilon} \right) - \omega x - \alpha \eta \bar{q} \left\{ S + \bar{Q} (1 - \varepsilon) \right\} . \]
Again, since $\bar{p}$ and $Q$ are linear in $\bar{q}$ and $S$ from (49) and (50), $y$ contains terms in $qS$, and hence the profit maximizing choice of futures sales, $S$, depends on planned production, $\bar{q}$, and vice versa. Thus the dominant producer does not act just as a large speculator, but chooses his speculative position to benefit his role as a producer. It is clear that this argument is robust, for it merely require $\partial^2 y / \partial q \partial S$ to be non-zero, which, given that income is bilinear in prices and quantities, and that prices depend on quantities, is virtually guaranteed as the only point to check is that all the terms do not cancel. Since they do not cancel in the simplest linear examples, in which any cancelling is most likely to occur, we can conclude that the futures and cash markets positions of the dominant producer are interdependent.

The next question to resolve is whether the dominant producer's interventions reduce or increases the bias in the futures market. The simplest test of his position is to ask in which direction he wishes to move starting with zero futures sales, i.e. what is the sign of

$$\left. \frac{\partial y}{\partial S} \right|_{S=0} = \bar{q}(1 - \sigma^2/\varepsilon) \frac{\partial \bar{p}}{\partial S} - \alpha \nu (1 - \varepsilon) \bar{q} \quad (53)$$

from (52). From (50)

$$\frac{\partial \bar{p}}{\partial S} = \frac{\alpha \nu (1 - \varepsilon)}{\varepsilon \psi} \quad , \quad \psi = 1 + \alpha \nu (1 - \varepsilon)^2 + \eta (1 - \sigma^2/\varepsilon)$$

After some manipulation this can be written
\[
\frac{\partial v}{\partial S} \bigg|_0 = -\frac{\alpha v(1-\epsilon)}{\phi} \left[1 - \eta(1 - \frac{\sigma^2}{\epsilon}(1 + \frac{1}{\epsilon})) - (1-\sigma^2/\epsilon)(1+\eta/\epsilon)q\right]
\]  

which has the sign of \(-(1-\epsilon)\). \(^1\) It follows from (48) that the dominant producer is on the opposite side of the futures market to the hedging element of the fringe farmers — i.e., if the farmers are hedging by selling futures, which they will if the market is inelastic \((\epsilon < 1)\) then the dominant producer will be buying futures, and vice versa (if \(\epsilon > 1\)). Thus the effect of the dominant producer is again to further arbitrage prices, and, assuming this to be desirable, his presence is welcome (provided, of course, that everyone shares the same information and knows his objectives). Moreover, as equations (49), (50), (52) and (54) show, as the futures market becomes more efficient (as its risk aversion, \(a\), tends to zero), to the manipulative power of the dominant producer in the futures market goes to zero (see especially (54), where \(S = 0\) if \(a = 0\)).

4.7 Conclusions

Even when all agents share the same information and hence can predict his strategy, the dominant producer still has an incentive to manipulate the futures market, beyond the point he would choose if he were merely a large speculator. Nevertheless, his intervention always has the

\(^1\) For bounded price, as \(q \to 1\), \(\epsilon > 1\).
effect of reducing the bias on the futures market (though not as much as if he rather acted competitively, or as large speculator), and hence excluding him from the futures market, would on the assumption here of full information, reduce the efficiency of the futures market. Finally, his market power in the futures market diminishes with the risk aversion of that market (i.e. as the number of speculators increases, and/or their risk aversion decreases).

5. Incentives for Destabilizing Markets

If dominant producers can predict planned fringe supply (or can do so as well as the fringe farmers can), but fringe farmers cannot observe the dominant producer's current production plans, but only his average planned production, then the dominant producer might have an incentive to randomize his supply. This incentive is easiest to study in the absence of futures markets and storage, but the argument carries over to these more complex cases. For example, where the dominant producer stores, he may have an incentive to randomize his storage decision, and where there is a futures market, he may benefit from further destabilizing the cash market. In all cases, the motive is the same — instability is costly to fringe producers who will respond by reducing their supply to the benefit of the dominant producer. However, it is also costly to the dominant producer and will not necessarily be worthwhile (in contrast to futures market manipulation discussed above, which is always worthwhile).

We shall examine a model with demand risk, and no futures market, in which the dominant producer can choose his average output, q.
and the magnitude of the variation about this, \( \hat{\theta} \), measured by its variance, \( \hat{s}^2 \). The total price variance is, from (21), \( \nu = \text{Var} \ u + \hat{s}^2/\hat{\varepsilon}^2 \), on the (reasonable) assumption that demand and supply risk are uncorrelated. Demand and (certain) fringe supply can be written

\[
p = 1 + \frac{1}{\varepsilon} + \frac{\hat{u}}{\varepsilon}(q + \theta + Q), \quad \text{E} u^2 = \sigma^2, \quad \text{E} \theta = s^2, \quad \text{E} \hat{u} = 0
\]

\[
Q = \mu(1 - \eta + \eta \hat{p})
\]

where the action certainty equivalent price is

\[
\hat{p} = \text{E} p - A Q \text{Var} \ p, \quad \text{Var} \ p = \sigma^2 + \frac{s^2}{\varepsilon^2} = \nu.
\]

Thus

\[
Q = \frac{\mu(1 - \eta + \eta \hat{p})}{1 + \mu \eta \nu A}, \quad (55)
\]

\[
\hat{p} = \frac{(1 + \frac{1}{\varepsilon} (1 - q))(1 + \mu \eta A) - \frac{\mu}{\varepsilon} (1 - \eta)}{1 + \mu \eta A + \mu \eta / \varepsilon}, \quad (56)
\]

The dominant producer chooses \( q \) (or more correctly, \( \text{E} x \), average input) and \( s^2 \) (determined by the variance of input) to maximize expected profit:

\[
y = \text{E} p (q + \theta) - \text{E} wx
\]

\[
= \hat{p} q - \frac{s^2}{\varepsilon^2} - \text{E} wx
\]
where from (36), if, as assumed, \( w = 1 \)

\[
\frac{\partial y}{\partial s^2} \bigg| _0 > 0 \quad \text{or} \quad q \frac{\partial \bar{p}}{\partial v} \cdot \frac{\partial v}{\partial s^2} - \frac{1}{\epsilon^2} - \frac{1}{2\lambda m^2} > 0. \tag{57}
\]

Now, from (56)

\[
\frac{\partial \bar{p}}{\partial v} = \frac{\mu \eta A \{1 + \frac{1}{\epsilon}(1 - q)\} - \bar{p} \mu A}{1 + \mu \eta A + \mu \eta / \epsilon}.
\]

which, if risk is small, has \( \bar{p} \sim 1 \), and \( 1 - q \sim \mu \), so

\[
\frac{\partial \bar{p}}{\partial v} \sim \frac{\mu^2 \eta A / \epsilon}{1 + \mu \eta A + \mu \eta / \epsilon}.
\]

Hence, a necessary condition for profitable destabilization is

\[
\frac{\mu^2 \eta A}{\epsilon^3 (1 + \mu \eta A + \mu \eta / \epsilon)} > \frac{1}{\epsilon^2} + \frac{1}{2\lambda m^2}
\]

or

\[
A > \frac{(1 + \mu \eta A + \mu \eta / \epsilon)}{(1 - \mu) \mu^2 \eta} \{ \epsilon + \frac{\epsilon^3}{2\lambda m^2} \}. \tag{58}
\]
Thus if fringe producers are sufficiently risk averse, then it will be desirable for the dominant producer to exploit this risk aversion by destabilizing the market. Since $A$ is not dimensionless, it is best replaced by $R = A\Pi$, where $\Pi$ is fringe income, approximately $\mu/2$ near $Q = \mu$, $\rho = 1$, and $R$ is the coefficient of relative risk aversion. Then the condition becomes

$$R > \frac{1 + \mu\eta/\varepsilon + 2\pi\varepsilon^2}{2\mu(1 - \mu)\eta} \left\{ \varepsilon + \frac{\varepsilon^3}{2\lambda m^2} \right\}$$

Taking the rather favorable (for destabilization) parameters used before, $\mu = 1/2$, $\varepsilon = \eta = 1$, $A = 1$, $m = 3/4$, $v = 1/10$, then the initial value of $R$ is 9.107, which is very high (empirical estimates cluster around values of 1–2. See Newbery and Stiglitz, 1981, Ch. 7).

5.1 Conclusions About Destabilizing Markets

For destabilization to be profitable, fringe producers must be more risk averse than some critical level, which, in the absence of storage, is (implausibly) high. In the presence of a futures market, destabilization is likely to be even less attractive as fringe producers are partially insured, and the costs to the dominant producer are of the same magnitude as before. Similarly, destabilizing storage is, at the margin, presumably as costly as destabilizing future supply (since they are, at the margin, substitutes). Hence this form of market manipulation appears a theoretical rather than a practical possibility.


Appendix A
Properties of Joint Normal Distributions

Let \( x \) and \( y \) are jointly normally distributed about the origin with SDs \( \sigma_1, \sigma_2 \), and correlation coefficient \( p \), then write this

\[
N(0, 0, \sigma_1^2, \sigma_2^2, \rho).
\]

The moment generating function (m.g.f.) is

\[
M(t_1, t_2) = \exp\left\{ t_1 \sigma_1^2 + 2 \rho \sigma_1 \sigma_2 t_1 t_2 + \sigma_2^2 t_2^2 \right\}
\]

and then

\[
\mu_{rs} = \text{Ex}^{r}y^{s} = \text{coefficient of } \frac{t_1^r t_2^s}{r! s!}
\]

in the expansion of the m.g.f. Therefore

\[
\mu_{11} = \rho \sigma_1 \sigma_2; \quad \mu_{22} = (1 + 2 \rho^2) \sigma_1^2 \sigma_2^2; \quad \mu_{12} = \mu_{21} = 0.
\]

To evaluate \( \text{Ep} \theta, \text{Var}(p \theta), \text{Cov}(p, p \theta) \), write

\[
p = \tilde{p}(1 + x), \quad \theta = 1 + y
\]

\[
\text{Ep} \theta = \tilde{p} \tilde{E}(1 + x)(1 + y) = \tilde{p}(1 + \mu_{11}) = \tilde{p}(1 + \rho \sigma_1 \sigma_2)
\]
\[ \text{var}(p \theta) = \bar{p}^2 E(1 + x + y + xy - 1 - \rho \sigma_1 \sigma_2)^2. \]

\[ \text{var}(p \theta) = \bar{p}^2 \{ \sigma_1^2 + 2 \rho \sigma_1 \sigma_2 + \sigma_2^2 + (1 + \rho^2) \sigma_1^2 \sigma_2^2 \} \]

\[ \text{cov}(p, p \theta) = \bar{p}^2 \mathbb{E}(x + y + xy - \rho \sigma_1 \sigma_2). \]

\[ \text{cov}(p, p \theta) = \bar{p}^2 (\sigma_1^2 + \rho \sigma_1 \sigma_2). \]
Appendix B

Finding Market Equilibrium Without Risk

Equations (37) and (41) together with \( p = \hat{p} = p^* \) imply

\[
p = 1 + \frac{1}{\varepsilon} - \frac{\lambda}{\varepsilon} - \frac{\mu}{\varepsilon} (1 - \eta + \eta p)
\]

or

\[
p(1 + \mu \eta / \varepsilon) = 1 + \frac{1}{\varepsilon} (1 - \mu(1 - \eta)) - q / \varepsilon
\]

This gives the net demand facing the dominant producer, for whom marginal revenue is

\[
\lambda R = 1 + \frac{1 - \mu - 2q}{\varepsilon + \mu \eta}
\]

But

\[
q = m(1 - \lambda + \lambda MR) = 1 - \mu \text{ in equilibrium}
\]

so

\[
\frac{1}{m} = \frac{1}{1 - \mu} - \frac{\lambda}{\varepsilon + \eta \mu}
\]  \hspace{1cm} (B.1)

This determines \( m \) given \( \mu, \lambda, \varepsilon, \eta \).
Appendix C

The Welfare Effect of Speculative Manipulation

With pure demand risk and optimal hedging the farmer's welfare is

\[ W = \bar{p}q - \bar{w}x + z(p^f - \bar{p}) - \frac{1}{2} \alpha \nu (q - z)^2, \]
\[ q - z = \frac{\bar{p} - p^f}{\alpha \nu}, \quad \nu = \text{Var } p = \text{Var } u. \]

From the production function with \( n \) identical farmers

\[ W = \frac{n(p^f)^2}{2n} - \frac{nk}{2n}, \]

and

\[ nq = 1 - \eta + np^f. \]

Hence, to an arbitrary constant, aggregate producer welfare is

\[ nW = p^f(l - \eta + \frac{n}{2} p^f) + \frac{(\bar{p} - p^f)^2}{2\alpha \nu} \]

where \( \bar{a} = \frac{A}{n} \approx 2b \) is a measure of risk aversion (where \( \bar{a} \) is the dimensionless coefficient of relative risk aversion, \( -y \frac{u''(y)}{u'(y)} = XY \), and each farmer has net income of roughly \( n/2 \) in equilibrium for small risk).
In the absence of the large speculator

\[ p^f = 1 - \frac{1}{Y}, \quad \bar{p} = 1 + \frac{n/e}{Y}, \quad \bar{p} - p^f = \frac{1}{Y} (1 + n/e), \]

\[ \gamma = \frac{1 + n/e}{av} + \eta, \]

whilst with the large speculator

\[ p^f = 1 - \frac{1}{2Y}, \quad \bar{p} = 1 + \frac{n/e}{2Y}, \quad \bar{p} - p^f = \frac{1}{2Y} (1 + n/e) \]

Let \( n, c \) superscripts denote the manipulated and competitive equilibria, then

\[ n(\dot{w}^m - \dot{w}^c) = (1-\eta)(\frac{1}{2Y}) + \frac{n}{2} \left\{ (1-\frac{1}{2Y})^2 - (1-\frac{1}{Y})^2 \right\} + \frac{(1+n/e)^2}{2av} \left( \frac{1}{4Y^2} - \frac{1}{Y^2} \right) \]

\[ = \frac{1}{2Y} - \frac{3n}{8Y^2} - \frac{3(1+n/e)^2}{8Y^2av} \]

For an agricultural commodity, plausible values for the parameters might be:

\[ \eta = 0.5, \quad e = 0.5, \quad R = 1, \quad a = 2, \quad v = 0.1, \quad \alpha = 0.5, \quad \gamma = 20.5 \]

(where we have assumed that small speculators contribute as much risk sharing as the farmers, i.e. \( m = n \).) In this case the expression is positive, but it would be negative if
\[ 4 \gamma cn < 3 (1 + \eta / \varepsilon)^2 \]

or

\[ \frac{n + m}{n} (1 + \eta / \varepsilon) + 2 R \nu n < \frac{3}{4} (1 + \eta / \varepsilon)^2 \]

This is quite possible with elastic supply, \( \eta \), inelastic demand, \( \varepsilon \), and few speculators, \( m \), relative to farmers, \( n \), and small risk, \( \nu \), or risk aversion \( R \).
Appendix D

Proof the the Dominant Producer is a Net Purchaser of Futures

Proof by contradiction. Consider the boundary case $S = 0$. From

(45)

$q = \frac{1}{2} \left( 1 + \frac{\epsilon}{\eta} \right),$ so $\epsilon < \eta.$

Substitute for $q$ and $m$ in (46) and rearrange to solve for $A$

$$
\lambda \left\{ \frac{2(1 - \eta/\epsilon)}{\epsilon} + \frac{1 + \mu(\alpha \eta + \eta/\epsilon)}{\epsilon + \eta \mu} \right\} = \frac{1 + \mu(\alpha \eta + \eta/\epsilon)}{1 - \mu}
$$

Since $\eta > \epsilon$

$$
\lambda > \frac{\epsilon + \eta \mu}{1 - \mu}
$$

which implies from (B.1) that

$$
m < 0,
$$

which is impossible, therefore $S < 0$. (The same argument: hold with greater force if it is assumed that $S > 0$).