Disinflation and the Supply Side

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Summary findings

Agénor and Pizzati study the dynamics of output, consumption, and real wages induced by a disinflation program based on permanent and temporary reductions in the nominal devaluation rate.

They use an intertemporal optimizing model of a small open economy in which domestic households face imperfect world capital markets, the labor supply is endogenous, and wages are flexible.

The model predicts that, with a constant capital stock and no investment, there is an initial reduction in real wages and output expands. Consumption falls on impact but increases afterward.

In addition, with a temporary shock, a current account deficit emerges and, later, a recession sets in, as documented in various studies.

With endogenous capital accumulation, numerical simulations show that the model can also predict a boom in investment.

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Disinflation and the Supply Side

Pierre-Richard Agénor* and Lodovico Pizzati**

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1 Introduction

Stabilization programs based on the use of the exchange rate as a nominal anchor have often been characterized by a boom-recession cycle, a real exchange rate appreciation, and persistent current account deficits. In the Southern Cone “tablita” experiments of the late 1970s in Argentina, Chile, and Uruguay, for instance, aggregate consumption increased in real terms by an average of ten percent in the first year following the implementation of the plan, before slowing down (see Calvo and Végh (1994)). An expansion in investment (often associated with an increase in imports of capital goods) and a rise in labor supply have also been observed in some of these programs (Roldós (1995)). The output and domestic absorption booms appear to have been observed in both successful and unsuccessful exchange rate-based stabilization attempts.¹

Various theories have been proposed to explain the boom-recession cycle in exchange-rate based stabilization programs. One branch of literature, developed in particular by Helpman and Razin (1987), emphasizes the wealth effects of stabilization programs. A second approach is the temporariness hypothesis, developed by Calvo and Végh (1993). A key feature of this approach is its emphasis on the interactions between the lack of credibility (modeled as a temporary policy change) and intertemporal substitution effects. A transitory reduction in the devaluation rate is equivalent to a temporary fall in present prices relative to the future, and induces an intertemporal substitution in consumption toward the present—leading to a rise in output, real exchange rate appreciation, and a current account deficit. However, evidence on the temporariness hypothesis is mixed. In particular, the econometric study by Reinhart and Végh (1995) suggested that although it can explain the behavior of consumption in some of the programs implemented in the 1980s, it is less useful for the tablita experiments of the late 1970s in Argentina, Chile, and Uruguay. Essentially, the low intertemporal substitution parameters estimated for these countries suggest that nominal interest rates would have had to fall by substantially more than they actually

¹Research by Easterly (1996) and Gould (1996) has shown, however, that output and absorption tend to rise at the inception of both money-based (MBS) and exchange-rate based (ERBS) stabilization programs. Gould (1996) for instance found that real output growth tends to increase in all the programs in his sample (except for the 1985 stabilization in Bolivia). Output growth, nevertheless, appears to be higher in the immediate aftermath of ERBS compared to MBS.
did to account for a sizable fraction of the consumption boom recorded in the data.

A third approach to the behavior of consumption and output in exchange-rate based programs emphasizes the supply-side effects of stabilization. Roldós (1995), in particular, analyzed these effects in a model of a dependent-economy with capital (which plays a dual role as a financial asset and a production input), endogenous labor supply, and a cash-in-advance constraint (following Stockman (1981)) on purchases of both consumption and capital goods. Roldós showed that, as a result of the cash-in-advance constraint, inflation creates a wedge between the real rate of return on foreign-currency denominated assets and that of domestic-currency denominated assets—which include money and capital. Thus, a stabilization program based on a permanent—and thus fully credible, in the Calvo-Végh sense—reduction in the devaluation rate (and ultimately the inflation rate) reduces the wedge and leads to an increase in the desired capital stock in the long run. In the short run, consumption and investment increase, causing a real appreciation, a current account deficit, and an increase in output of home goods. During the transition period, firms increase gradually their purchases of capital goods and their capital stock (which is constant on impact), drawing labor resources into the (capital-intensive) tradables sector, raising wages and leading to further appreciation of the real exchange rate. Over time, the increase in output of tradable goods lowers the initial current account deficit generated on impact by the increase in aggregate demand. However, this model does not predict a recession at a later stage as suggested by the evidence. In a subsequent paper, Roldós (1997) showed that a gradual and permanent reduction in the nominal devaluation rate leads to an initial boom when the intertemporal elasticity of substitution in labor supply is larger than that in consumption. The expansion in output occurs in both the tradable and nontradable production sectors, as a result of a reduction in real wages. The reduction in the devaluation rate lowers inflation and raises the marginal value of wealth, thereby raising the opportunity cost of leisure and inducing an increase in labor supply in the initial phase of the program. The continued reduction over time in the rate of devaluation and the rate of inflation leads to further increases in the supply of labor and downward pressure on wages.

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2 Uribe (1997) also analyzes the dynamics of exchange-rate based stabilization in a model in which inflation influences the rate of return on domestic capital as opposed to the rate of return of foreign assets.

3 By contrast, Roldós (1995) found that real wages increase during the transition (after
The purpose of this paper is to explore further the role of supply-side factors in the dynamics of output and absorption in exchange-rate based stabilization programs. As in some existing studies—such as those by Calvo and Végh (1993), Lahiri (1994), and Roldós (1995)—we explicitly model labor supply decisions, labor market equilibrium, and capital accumulation in an infinite-horizon, intertemporal optimizing framework. The analysis, however, departs from the current literature by assuming (following Agenor (1997)) that domestic households face imperfect world capital markets. As a result of these imperfections, capital mobility is imperfect, and domestic interest rates are determined by the equilibrium condition of the domestic money market. A key difference between our model and those that assume perfect capital mobility is that our framework does not require the rate of time preference to be equal at all times to the world interest rate. In optimizing models with infinite horizon and perfect capital mobility, it is necessary to impose equality between the (constant) rate of time preference and the world interest rate to get a finite and positive level of consumption in the steady state. A problem with that approach, however, is that the steady state—and the adjustment path to it—depends not only on the dynamic structure of the model but also on the economy's initial conditions. This "hysteresis" phenomenon, as pointed out by Turnovsky and Sen (1991) in a related context, is what results in temporary shocks having permanent effects.

The remainder of the paper is organized as follows. Section II presents the basic model, which assumes that the capital stock is constant, and derives its dynamic form. Section III examines the short- and long-term effects of a reduction in the nominal devaluation rate in this setting. Section IV extends the basic framework to account for capital accumulation in the presence of installation costs, using a Tobin-q approach to investment decisions. A key feature of the extended model is that, with endogenous labor supply, investment decisions are not independent of consumption decisions. Changes in falling on impact), whereas the long-run effect on labor supply is ambiguous.

4 An analysis of alternative models based on simulation techniques by Rebelo and Végh (1997) emphasized the importance of supply-side factors (notably the role of real wages) in explaining the boom-recession cycle.

5 Of course, alternative approaches, such as those based on Uzawa preferences or finite lifetimes—as in the Blanchard-Yaari framework used by Helpman and Razin (1987)—could be adopted to alleviate this problem.

6 Another problem, specific to the Calvo-Végh model, is that the boom-recession cycle in consumption takes the form of step changes—rather than the inverted-U shaped path suggested by the evidence (Végh, 1992).
consumption change the marginal utility of leisure, and this alters the supply of labor at any given wage. As a result, the marginal product of capital changes and this in turn affects investment—to an extent that depends on capital installment costs. Because of the complexities of the resulting model, the transitional dynamics associated with a reduction in the nominal devaluation rate are examined in Section V using numerical simulations. Finally, Section VI summarizes the main results of the analysis and offers some suggestions for further research.

2 The Basic Framework

Consider a small open economy in which perfect foresight prevails and four types of agents operate: households, producers, the government, and the central bank. The nominal exchange rate $E$ (defined as the home-currency price of foreign currency) is depreciated by the central bank at a constant rate, $\varepsilon$. The economy produces a traded good, whose foreign-currency price is constant and normalized to unity. The capital stock is fixed initially, and labor is homogeneous. Domestic production takes place under decreasing returns to labor.

Households hold two categories of financial assets in their portfolios: domestic money (which bears no interest) and domestic government bonds. They also borrow on world capital markets, subject to a rising risk premium. The government consumes the domestic good, pays interest on its domestic debt, and balances its budget by levying lump-sum taxes on households.

2.1 Households

Assuming that government expenditure does not yield direct utility, the representative household's discounted lifetime utility can be written as

$$\int_0^\infty \left[ \frac{c^{1-\eta}}{1-\eta} + \ln \lambda^\alpha m^{1-\alpha} \right]^{-\rho} dt,$$

where $\rho > 0$ denotes the rate of time preference (assumed constant), $c$ consumption expenditure, $\lambda$ leisure, $m$ real money balances, $\eta > 0$ and $\eta \neq 1$, ...
and \(0 < \alpha < 1\). The instantaneous utility function is assumed to be additively separable in consumption, leisure, and real money balances.

Real wealth of the representative household \(a\) is defined as

\[
a = m + b - l^*,
\]

where \(b\) denotes real holdings of government bonds (which consist of claims of infinite maturity), and \(l^*\) foreign borrowing measured in foreign-currency terms. The flow budget constraint is given by

\[
\dot{a} = wn^s + \Pi + ib - c - \tau - (i^* + \theta)l^* - (m + b)\varepsilon,
\]

where \(n^s\) denotes labor supply, \(\Pi\) firms' profits, \(\tau\) lump-sum taxes, \(i\) the domestic nominal interest rate, and \(\varepsilon \equiv \dot{E}/E\) the predetermined rate of devaluation of the exchange rate. The term \(-(m + b)\varepsilon\) accounts for capital losses on real money balances and the stock of domestic bonds resulting from inflation. The effective cost of borrowing faced by the representative household on world capital markets is equal to \(i^* + \theta\), where \(i^*\) is the risk-free rate and \(\theta\) a risk premium, which is positively related to the household’s level of foreign debt:

\[
\theta = \theta(l^*, \cdot), \quad \theta_1 > 0.
\]

Thus, domestic households are able to borrow more on world capital markets only at a higher cost. The size of the premium is related positively to the risk of default on loan obligations.\(^9\)

\(^7\)Except when otherwise indicated, partial derivatives are denoted by corresponding subscripts, while the total derivative of a function of a single argument is denoted by a prime. A sign over a variable refers to the sign of the corresponding partial derivative. We also define \(\dot{z} \equiv dz/dt\).

\(^8\)The money-in-the-utility-function approach adopted here is actually less restrictive than it may appear; it is, in particular, functionally equivalent to a shopping-time model (Croushore (1993)). Feenstra (1986) showed that the cash-in-advance approach used for instance by Calvo and Végh (1993) can be replicated in a money-in-the-utility-function formulation if households maximize a Leontief instantaneous utility function of the form \(u[\min(c, m)]\). As argued by Feenstra, this utility function can be approximated by a concave utility function of the form \(u(c, m)\) with \(u_{em} > 0\).

\(^9\)The premium can also be assumed to be a convex function of \(l^* (\theta_1 > 0)\) over the efficient portion of the supply curve of funds. The other factors that are not specified in (4) refer for instance to the proportion of gross assets (or collateral) that lenders can seize in case of default. Agénor (1997) provides a more thorough discussion of this approach to individual risk on world capital markets, and contrasts it with the conventional country risk approach.
Households treat \( w, \Pi, \varepsilon, i, i^* \) and \( \tau \) as given, internalize the effect of their portfolio decisions on \( \theta \), and maximize (1) subject to (3) and (4) by choosing a sequence \( \{c, m, \lambda, b, l^*\}_{t=0}^\infty \). Let \( r = i - \varepsilon \) denote the domestic real rate of interest, and \( \sigma = 1/\eta \) the intertemporal elasticity of substitution. Suppose also that the household has a fixed time endowment which is normalized to one, so that \( n^s = 1 - \lambda \). The first-order optimality conditions are given by:

\[
(1 - \alpha)c^n/m = i, \tag{5}
\]

\[
\alpha c^n/(1 - n^s) = w, \tag{6}
\]

\[
i = (i^* + \theta + \varepsilon) + l^*\theta_{l^*}, \tag{7}
\]

\[
\dot{c}/c = \sigma(r - \rho), \tag{8}
\]

together with the transversality condition \( \lim_{t \to \infty} (e^{-\rho t}a) = 0 \).

Equation (5) equates the marginal rate of substitution between consumption and money to the opportunity cost of holding money, the domestic nominal interest rate. It implies that money demand is positively related to consumption and negatively related to the domestic interest rate:

\[
m^d = m^d(c, i). \tag{9}
\]

Equation (6) indicates that the marginal rate of substitution of leisure for consumption is equal to the going wage. It yields

\[
n^s = n^s(w, c), \tag{10}
\]

which indicates that labor supply is positively related to the real wage and negatively to consumption.

Equation (7) is the interest rate parity condition that holds under the assumption of imperfect world capital markets. It equates the marginal cost of borrowing abroad and the marginal rate of return on domestic assets. In turn, the marginal cost of foreign borrowing is given by the effective cost of borrowing, \( i^* + \theta \), plus the devaluation rate and the increase in the cost of servicing the existing stock of foreign loans induced by the marginal increase in the risk premium (itself resulting from the marginal increase in borrowing), \( l^*\theta_{l^*} \).
Since the premium is endogenous, the optimal condition for foreign borrowing (equation (7)) determines implicitly the private demand for foreign loans. Taking a linear approximation to \( i^* \) yields

\[
I^* = \frac{(i - i^* - \varepsilon)}{\gamma},
\]

(11)

where \( \gamma = 2\theta_1 > 0 \). Equation (11) shows that the optimal level of foreign borrowing is proportional to the conventionally-measured covered interest rate differential, given by the difference between the domestic interest rate and the sum of the safe interest rate and the devaluation rate. When the premium is independent of the household's level of borrowing (that is, when \( d_1 = 0 \)), equation (11) yields the uncovered interest parity condition \( i = i^* + \varepsilon \).

Finally, equation (8) shows that total consumption rises or falls depending on whether the domestic real interest rate exceeds or falls below the rate of time preference. The size of the intertemporal elasticity of substitution \( \sigma \) determines the extent to which households adjust their consumption profiles in response to changes in the differential between the domestic real interest rate and the rate of time preference. Because the desired path for labor supply and real money balances depends in part on the desired degree of consumption smoothing, the intertemporal elasticity of substitution plays an important role in determining the overall dynamics of the economy, as discussed below.

2.2 Firms and the labor market

The production technology is characterized by decreasing returns to labor:

\[
y = y(n), \quad y' > 0, \quad y'' < 0,
\]

(12)

assuming for the moment that the capital stock is given.

From (12), labor demand is given by \( n^d = n^d(w) \), with \( n^d = 1/y'' < 0 \). The equilibrium condition of the labor market is given by, using (10):

\[
n^d(w) = n^s(w, c).
\]

(13)

\(^{10}\)Note that without separability in consumption and leisure, the rate of consumption growth would also depend on the rate of real wage growth. This would be the case, for instance, if the instantaneous utility function were to take the form

\[
\left\{c^\alpha \lambda^{1-\alpha}\right\}^{1-\eta}/(1-\eta) + \ln m, \quad 0 < \alpha < 1.
\]
This condition can be solved, under perfect wage flexibility, for the equilibrium wage:

\[ w = w(c), \quad w' > 0. \]  \hspace{1cm} (14)

Substituting this result in equation (10) shows that an increase in consumption has both a direct (negative) effect on labor supply as well as an indirect, positive effect. On the one hand, the increase in consumption increases the demand for leisure and directly reduces labor supply. This reduction, on the other, creates excess demand for labor and requires a rise in the market-clearing wage to maintain equilibrium—thereby increasing the supply of labor. It can be verified that the direct effect dominates the indirect effect, so that an increase in consumption unambiguously lowers labor supply. This relationship can be written as

\[ n^s = n^s[w(c), c] = N(c). \quad N' < 0. \]  \hspace{1cm} (15)

Substituting the market-clearing wage given by equation (14) in the labor demand equation gives \( n^d = n^d(c) \), with \( n^d < 0 \). Substituting this result in equation (12) yields:

\[ y^s = y^s(c), \]  \hspace{1cm} (16)

with \( y'' < 0 \).

### 2.3 The Central Bank and the Government

The central bank, in addition to devaluing the exchange rate at a constant rate, engages in nonsterilized intervention; that is, it ensures the automatic and costless conversion at any given moment in time of domestic currency holdings into foreign currency (and vice versa) at the prevailing exchange rate. Because there is no credit, the real money supply is equal to

\[ m^s = R^*, \]  \hspace{1cm} (17)

where \( R^* \) is the central bank's stock of net foreign assets, measured in foreign currency terms. The central bank receives interest on its holdings of foreign assets, \( i^* R^* \). Capital gains on official foreign reserves, \( ε R^* \), normally contribute to an increase in the central bank's net worth, measured in domestic-currency terms. In what follows we assume that the central bank's
net worth remains constant, and that net income—inclusive of capital gains, that is, \((i^* + \varepsilon)R^*\)—is transferred to the government.

The government consumes a quantity \(g\) of the domestic good, and maintains a balanced budget by levying lump-sum taxes on households. Setting the constant stock of domestic bonds to zero for simplicity, the budget constraint of the government can be written as

\[
\tau = g - (i^* + \varepsilon)R^*. \tag{18}
\]

Finally, to close the model requires specifying the equilibrium condition of the money market. From equation (5), the market-clearing interest rate is given by:

\[
i = i(t, \bar{m}), \tag{19}
\]

which shows that the nominal interest rate depends positively on consumption expenditure and negatively on real money balances.

### 2.4 Dynamic Structure

Substituting (17) in (2) yields, given the normalization rule for \(b\):

\[
D^* = -a = l^* - R^*, \tag{20}
\]

which shows that, since the net worth of the central bank and the government does not change over time (given the transfer rule and a continuously balanced budget), the private sector’s net financial liabilities consist of the economy’s net stock of foreign debt (measured in foreign-currency terms), \(D^*\), which is defined as the difference between private foreign liabilities and official foreign assets.

Substituting equations (18) and (20) in (3) yields the consolidated budget constraint of the economy:

\[
\dot{D}^* = i^*D + \theta l^* + c + g - y^*, \tag{21}
\]

which indicates that the current account deficit (whose counterpart is the change in foreign debt) is the sum of the trade deficit and interest payments on the outstanding stock of debt held by households and the central bank.\(^{11}\)

\(^{11}\)Integrating equation (21) yields the economy’s intertemporal budget constraint

\[
D_0 = \int_0^\infty (y^* - c - g - \theta l^*)e^{-\int_0^t i^* dt} dt + \lim_{t \to \infty} D^* e^{-\int_0^t i^* dt}.
\]

10
Equations (8), (11), (16), (17), (19) and (21) describe the evolution of the economy along any perfect foresight equilibrium path. The system can be re-written as:

\[ I^* = [i(c, R^*) - i^* - \varepsilon] / \gamma, \]  
\[ \dot{c} = \sigma[i(c, R^*) - \varepsilon], \]  
\[ \dot{D}^* = i^* D + \theta l^* + c + g - y^*(c), \]  

with equation (18) determining residually lump-sum taxes.

The dynamic form of the model can be further reduced to a system involving two variables: consumption \(c\), which may jump in response to new information, and net external debt \(D^*\), which is a predetermined variable that can change only over time through current account deficits and surpluses. To begin with, note that

\[ R^* = -(l^* - R^*) + l^* = -D^* + l^*, \]

or, using equation (22):

\[ R^* = -D^* + [i(c, R^*) - i^* - \varepsilon] / \gamma, \]

so that

\[ R^* = \varphi(\hat{c}, \bar{D}^*; \bar{\varepsilon}), \]  
where, setting \(\beta \equiv 1/(\gamma - i_m) > 0:\)

\[ \varphi_c = \beta i_c, \quad \varphi_D = -\beta \gamma, \quad \varphi_e = -\beta. \]

Substituting this result in equation (23) yields

\[ \dot{c} / c = \sigma[i(c, \varphi(c, D^*, \varepsilon)) - \varepsilon]. \]

The economy cannot maintain indefinitely a positive or negative net debtor position with the rest of the world, so the second term on the right-hand side in the above expression must be zero. Thus, the current level of foreign debt must be equal to the discounted stream of the excess of future production over domestic absorption plus premium-related interest payments on private foreign borrowing.
so that

\[ \dot{c} = G(\dot{c}, \dot{D}^*; \varepsilon), \]

where, with \( \Delta = \sigma \bar{c} \gamma \beta \):

\[ G_c = i_c \Delta, \quad G_{D^*} = -i_m \Delta, \quad G_e = -\Delta. \]

Substituting equation (25) into (22) yields

\[ I^* = \Lambda(\dot{c}, \dot{D}^*; \varepsilon), \]

where

\[ \Lambda_c = i_c \beta, \quad \Lambda_{D^*} = -i_m \beta, \quad \Lambda_e = -\beta. \]

Using (27), equation (24) can be written as

\[ \dot{D}^* = i^* D^* + \theta(\Lambda(c, D^*, \varepsilon))\Lambda(c, D^*, \varepsilon) + c + g - y^s(c), \]

which can be rewritten as

\[ \dot{D}^* = \varPsi(\dot{c}, \dot{D}^*; \varepsilon), \]

where, with a 's' denoting initial steady-state values:

\[ \varPsi_c = 1 - y^s + (\bar{\theta} + \bar{\iota}^\prime \theta^\prime_s)\lambda_c, \quad \varPsi_{D^*} = i^* + (\bar{\theta} + \bar{\iota}^\prime \theta^\prime_s)\lambda_c, \quad \varPsi_e = (\bar{\theta} + \bar{\iota}^\prime \theta^\prime_s)\alpha_e. \]

Equations (26) and (28) form a dynamic system in consumption and net external debt, which can be linearized around the steady state to give

\[ \begin{bmatrix} \dot{c} \\ \dot{D}^* \end{bmatrix} = \begin{bmatrix} G_c & G_{D^*} \\ \varPsi_c & \varPsi_{D^*} \end{bmatrix} \begin{bmatrix} c - \bar{c} \\ D^* - \bar{D}^* \end{bmatrix}. \]

Since \( c \) is a jump variable whereas \( D^* \) is predetermined—evolving continuously from its initial level \( D^*_0 \)—saddlepath stability requires one unstable (positive) root.\(^{12}\) To ensure that this condition holds, the determinant

\(^{12}\)Note that although the overall net stock of external debt \( D^* \) cannot change on impact, both official reserves and private foreign borrowing may shift discretely in response to changes in interest rates because intervention is unsterilized.
of the matrix of coefficients in (29)—which is equal to the product of the roots—must be negative: $\Psi_{D^*}G_c - \Psi_cG_{D^*} < 0$. This condition is interpreted graphically in Figure 1.

The steady-state solution is obtained by setting $\dot{c} = \dot{D}^* = 0$. From equation (23), the real interest rate is equal to the rate of time preference:

$$\tilde{r} = \tilde{\tau} - \varepsilon = \rho. \quad (30)$$

Substituting this result in (22) yields

$$\tilde{l}^* = (\rho - \tilde{\tau}^*)/\gamma, \quad (31)$$

which indicates that the steady-state level of private foreign debt is positive as long as the rate of time preference of domestic consumers is sufficiently high—that is, if domestic households value the future sufficiently.

In the long-run the current account must be in equilibrium, so that, from (24):

$$\tilde{c} = \tilde{y}^* - g - \tilde{\tau}^* \tilde{D}^* - \tilde{\theta} \tilde{l}^*. \quad (32)$$

Finally, from (9) and (30), long-run real money balances are given by

$$\tilde{m} = m(\tilde{c}, \rho + \varepsilon). \quad (33)$$

The steady-state equilibrium of the model is depicted in Figure 1. In the North-East quadrant, the locus $[\tilde{D}^* = 0]$ gives the combinations of $c$ and $D^*$ for which net private financial liabilities (or, equivalently here, the economy’s net stock of foreign debt) remain constant, whereas the locus $[\dot{c} = 0]$ depicts the combinations of $c$ and $D^*$ for which consumption does not change over time. Since $\Psi_{D^*} > 0$, $\dot{D}^*$ is positive (negative) when $D^*$ is to the right (left) of the $[\tilde{D}^* = 0]$ locus. The positive sign of $G_D$ also implies that $\dot{c}$ is positive to the right of the $[\dot{c} = 0]$ locus and negative to the left of it. This explains the directions of the arrows.

Saddlepath stability requires that the $[\dot{c} = 0]$ locus be steeper than the $[\tilde{D}^* = 0]$ locus. The stable path has a negative slope and is denoted $SS$. In the North-West quadrant, the upward-sloping curve $LL$ traces the combinations of $w$ and $c$ for which the labor market is in equilibrium, as indicated in equation (14). Finally, the South-West quadrant traces the negative relation between wages and output supply, obtained by substituting the demand for labor function $n^d = n^d(w)$ in equation (12). The initial steady-state level of output is obtained at point $Q$. 

13
3 Reduction in the Devaluation Rate

Consider first a permanent and unanticipated reduction in the nominal devaluation rate \( \varepsilon \), with no discrete change in the level of the exchange rate. As can be inferred from (30) and (31), the reduction in \( \varepsilon \) has no long-run effect on the real interest rate (which is tied to the rate of time preference) or private foreign borrowing. But because the nominal interest rate falls in the same proportion as the devaluation rate, thereby reducing the opportunity cost of holding money, the demand for domestic cash balances—at the initial level of consumption—rises (equation (33)). The official stock of reserves must therefore increase; and because private foreign borrowing does not change in the long run, the economy’s overall stock of debt must fall in the new steady state, implying a lower deficit in the services account. To maintain external balance, as implied by equation (32), the initial trade surplus must fall—or equivalently private consumption must rise. The increase in private expenditure also raises the demand for domestic cash balances, lowers the supply of labor, and raises real wages. The fall in employment is associated with a reduction in domestic output.

On impact, as shown in Appendix I. For a given level of the domestic nominal interest rate, the reduction in \( \varepsilon \) leads to a discrete increase in private foreign borrowing. This leads in turn to an offsetting increase in official reserves (and thus a rise in the real money stock) to maintain the economy’s stock of foreign debt constant on impact. For a given level of consumption, the nominal interest rate falls. For foreign borrowing to increase, the domestic real interest rate must increase; in turn, this implies that the nominal interest rate must fall by less than the reduction in the devaluation rate. The increase in the real interest rate creates an incentive for households to shift consumption toward the future. Consumption therefore falls on impact. As a result of the reduction in private spending, labor supply increases, reducing wages and stimulating output. The trade balance therefore improves. At the same time, the negative income effect associated with the increase in the premium-related component of interest payments (itself resulting from the increase in private foreign borrowing) raises the initial deficit of the services account. Nevertheless, the increase in the trade surplus outweighs the adverse movement in the services account, and the current account improves, leading to a fall in external debt (\( \dot{D}_s < 0 \)). Because the shock is permanent, the current account remains in surplus throughout the adjustment process. After its initial drop, consumption begins increasing, and the real interest
rate falls gradually toward the rate of time preference.

The dynamics of consumption and net external debt are illustrated in Figure 2. Both the $[\dot{c} = 0]$ and $[\dot{D}^* = 0]$ curves shift to the left. Consumption jumps downward from point E to point A on impact, and begins rising afterward. The economy's stock of foreign debt falls continuously during the transition to the new steady state, which is reached at point $E'$. Real wages jump downward on impact from point H to point F on the LL curve, and starts rising toward its new long-run equilibrium position, point $H'$. Output rises on impact from point Q to point M, and begins falling afterward (toward point $Q'$), as a result of the gradual increase in wages.

Consider now a temporary reduction for the period $(0, T)$ in the devaluation rate. As is well-known in this type of models, the adjustment path depends on the length of the period during which the devaluation rate is reduced. Figure 2 illustrates one particular outcome, in which, consumption, after falling from point $E$ to point $A$, begins to increase.\(^\text{13}\) The current account moves at first into surplus ($\dot{D}_0 < 0$) and output increases. The economy reaches the original saddlepath $SS$ at point $B$, exactly at period $T$. During the first part of the adjustment process, between $A'$ and $B$, consumption increases and output falls. After point $B$, consumption starts falling and output once again. The current account moves into deficit, and the stock of foreign debt increases.

Thus, the results associated with a temporary shock indicate that the model is capable of reproducing some of the "stylized facts" of exchange-rate based stabilization programs mentioned in the introduction: a boom-recession cycle in output (associated with a reduction and a subsequent increase in real wages) and a current account deficit in the second stage of the adjustment process. Although consumption drops on impact (whose magnitude depends on the size of the intertemporal elasticity of substitution), it follows afterward the inverted-U shape pattern observed in some actual programs.\(^\text{14}\) In addition, the assumption of a constant capital stock in the above framework prevents it from explaining another important feature of exchange rate-based programs, namely, an expansion in investment. To do so requires endogenizing capital accumulation and investment decisions.

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\(^{13}\)See Agénor (1997, 1998) for a more detailed discussion of the dynamics associated with a temporary reduction in the devaluation rate in this type of models.

\(^{14}\)Given that it considers only one good, the present model cannot reproduce the real exchange rate appreciation that has also characterized exchange rate-based programs.
4 Capital Accumulation

To account for capital accumulation in the framework developed above, we write the production function in the form

\[ y = y(n, k), \] (34)

where \( k \) is the capital stock. The function \( y(\cdot) \) is assumed to possess all the standard neoclassical properties. In particular, it exhibits constant returns to scale and production inputs are gross complements \( (y_{nk} > 0) \). For simplicity, we also assume that the capital stock does not depreciate over time.

To model investment decisions, we follow the literature on installation costs, and assume that in order to invest \( I \) units of output, the representative firm must spend \( I_G = I[1 + \Phi(\cdot)] \) units of the good, with \( \Phi(\cdot) \) denoting the installment cost function. More specifically, \( \Phi(\cdot) \) is assumed to depend linearly on the ratio of investment to the existing capital stock, \( I/k \):

\[ I_G = I(1 + \frac{\phi I}{2k}), \] (35)

where \( \phi \) is a positive constant, which determines the degree of intertemporal substitution in production, that is, the response of investment to a given increase in the shadow value of capital.\(^{15}\)

Using (35), the firm's cash flow is thus given by

\[ y(n, k) - wn - I(1 + \frac{\phi I}{2k}), \] (36)

which is linearly homogeneous in capital, labor and investment.

Suppose now that the representative firm maximizes its net present worth (that is, the net present value of future earnings), given by its production technology and capital stock, which evolves over time according to

\[ \dot{k} = I. \] (37)

\(^{15}\)See Abel and Blanchard (1983) and, more recently, Servén (1996). Convex adjustment costs give firms an incentive to smooth the adjustment path of the stock of capital over a period of time, with the optimal amount of adjustment depending on the differential between the equity price of capital over the unitary replacement cost. By contrast, non-convexities in the adjustment cost function would tend to cause discontinuous adjustment in the capital stock.
The firm's optimal behavior can thus be described as

\[ V = \max_{I,n} \int_t^\infty \left\{ y(n, k) - wn - I(1 + \frac{\phi I}{2k}) \right\} e^{-\rho t} dt, \]
given \( k \).\(^{16}\) The optimality conditions are given by

\[ y_n(n, k) = w, \Rightarrow n^d = n^d(\bar{w}, \bar{k}), \quad (38) \]

\[ I = \frac{k}{\phi}(q - 1), \quad (39) \]

\[ q = \int_t^\infty \left\{ y_k(n, k) - \frac{\phi I}{2k} \right\} e^{-\rho(t-s)} ds, \quad (40) \]

together with the transversality condition \( \lim_{t \to \infty} (e^{-\rho t}k) = 0 \).

Equation (38) is the demand for labor, derived by equating the marginal productivity of labor \( y_n(n, k) \) to the going wage, \( w \). Equation (39) is the demand function of investment, where \( q \) is the ratio of the value of installed capital to its replacement cost—that is, Tobin's \( q \), which, as shown in (40), is such that the shadow value of capital equals the present value of its marginal product.\(^{17}\) As shown by Hayashi (1982), under the assumptions of a constant-returns-to-scale technology and homogeneity of degree one of the installation cost function (as is the case here), the marginal value of installed capital will equal to the average value of installed capital. Put differently, \( q \) can also be interpreted as the market value of an equity claim on a unit of installed capital.

Equation (40) yields

\[ \dot{q} = \rho q - y_k(n, k) + \frac{\phi I}{2k} \tag{41} \]

\(^{16}\)For simplicity, the firm's discount factor is assumed to be same as the one used by the representative household.

\(^{17}\)Equation (39) can be rewritten as \( q = 1 + \phi I/k \), where the right-hand side term is the marginal increase in gross investment resulting from a marginal increase in net investment. Thus, the optimal investment rate is determined by setting the marginal benefit equal to the marginal investment cost.
where the last two terms represent the difference between the marginal cost of adding new capacity and the marginal product of capital.\textsuperscript{18}

Households behave as specified before. In particular, labor supply is again given by equation (10). However, the representative household’s real wealth (equation (2)) is now given by

\[ a = m + b + qk - l^*. \]  

(42)

In addition, since labor demand depends now on the capital stock, the equilibrium wage rate obtained by solving (13) is now also a function of the capital stock, in addition to consumption:

\[ w = w(\bar{c}, \bar{k}). \]  

(43)

Substituting equation (43) in (38) and using (34) yields\textsuperscript{19}

\[ y^* = y^*(\bar{c}, \bar{k}), \quad y_k(n, k) = h(\bar{c}, \bar{k}). \]  

(44)

Using (39) and (44), equation (41) can be written as

\[ \dot{q} = \rho q - h(c, k) + \frac{1}{2\phi}(q - 1)^2. \]  

(45)

Finally, substituting (39) in (35) and (37) yields gross investment as

\[ I_G = \frac{k}{\phi}(q - 1)(1 + \frac{q - 1}{2}) \equiv kv(q), \]  

(46)

with \( \nu' > 0 \) and \( \nu(1) = 0 \). The dynamics of the capital stock are governed by, from (37) and (39):

\[ \dot{k} = \frac{k}{\phi}(q - 1). \]  

(47)

With the same assumptions as before regarding the government and the central bank, and using (42), (44), and (46), the consolidated flow budget constraint of the economy is now given by, instead of (21):

\[ \dot{D}^* = \theta^* D^* + \theta^* l^* + c + g + kv(q) - y^*(c, k). \]  

(48)

\textsuperscript{18}Note that, from (39), if \( \phi = 0 \), no investment will take place, \( q \) will always be equal to unity and the capital stock will be constant. At any point in time, the value of the capital stock will equal its replacement cost. The reason, of course, is that it makes sense to have less than full adjustment of the capital stock only when adjustment entails some cost.

\textsuperscript{19}In signing \( h_k \), it is assumed that the direct effect of an increase in \( k \) on the marginal product of capital outweighs its indirect effect on the marginal productivity of labor.
The dynamic model now consists of (22), (23), (45), (47) and (48). By eliminating \( l^* \) as before, this set of equations can be further condensed into a differential equation system involving four variables: private consumption \( c \), the ratio of the value of installed capital to its replacement cost \( q \), the economy's net stock of external debt, \( D^* \), and the capital stock, \( k \). The system possesses two jump variables, \( c \) and \( q \), whereas \( k \) and \( D^* \) are predetermined variables. Saddlepath stability therefore requires two positive and two negative roots.

The steady-state values of the system are given as before by equations (30), (31), (33), together with

\[
\begin{align*}
\hat{q} &= 1, \quad (49) \\
h(c, \bar{k}) &= \rho \Rightarrow \bar{k} = s(c), \quad (50)
\end{align*}
\]

with \( s' < 0 \). Finally, (32) is replaced by

\[
\bar{c} = y^*(\bar{c}, \bar{k}) - g - i^* \bar{D}^* - \bar{I}^*, \quad (51)
\]

since \( \bar{I}_G = 0 \).

Conditions for saddlepath stability are discussed in Appendix II. In general, only sufficient conditions can be established from qualitative analysis. As shown also in Appendix II, the steady-state effects of a reduction in the devaluation rate are qualitatively similar to those obtained in the model without capital accumulation (see Appendix I).

Given the complexity of the model, it is not possible to analyze explicitly its transitional dynamics in response to any given shock. As a result, numerical simulations were used to study the adjustment process associated with an exchange-rate based stabilization.

## 5 Numerical Simulations

To analyze numerically the effects of a reduction in the devaluation rate in the above setting, we used Portable TROLL.\(^2\)\(^1\) The program uses a Newton stacked-time algorithm to derive its solutions. The procedure involves stacking the time-dependent equations of this non-linear forward looking model,

\(^{20}\)This is intuitively clear from (49) and (50) which show that \( q \) is invariant to any shock, and the capital stock varies inversely with consumption; as a result, solving for the steady-state effect involves solving simultaneously only for \( c \) and \( D^* \).

\(^{21}\)Distributed by Intex Solution, Needham, MA 02494.
such that each endogenous variable is represented by an independent equation. The stacked structure is then simultaneously solved using a Newton procedure. The effects of both permanent and temporary reductions in the devaluation rate are evaluated using two alternative values of the intertemporal elasticity of substitution, \( \sigma \).

The numerical values assigned to the variables and parameters of the system are as follows. The initial devaluation (or inflation) rate is 3 percent, and both a permanent and a temporary drop by one percentage point in this rate are considered. On the supply side of the model, output \( y \) is normalized to 1000; the production function is assumed to have a constant elasticity of substitution between labor and capital equal to 0.56. The real interest rate, \( r \), is initially set to 4 percent, the same value as the rate of time preference, \( \rho \). Consumption, \( c \), is calibrated to have an initial value equal to 80 percent of total output (\( c = 800 \)), whereas the intertemporal elasticity of substitution, \( \sigma \), takes the values of 0.3 and 0.8 for the two scenarios under consideration.

Net external debt, \( D \), is set at an initial value of 700, with private external liabilities, \( l^* \), taking the value of 1000; these values imply therefore that \( R^* = m = 1000 - 700 = 300 \). In equation (21), the term capturing the risk premium, \( \theta(\cdot)l^* \), is assumed to take the quadratic form \( \gamma l^{*2}/2 \). Finally, the parameter \( \phi \), which represents the installation cost of investment, takes a value of 0.1.

Calibration of the model around these initial values produces the baserun solution. For the permanent disinflation shock, the initial value of the devaluation rate drops permanently by 1 percentage point for all 100 periods considered in this simulation. For the temporary shock, the devaluation rate drops by 1 percentage point only for the first ten periods, and then returns to its initial value of 3 percent. The effect of these shocks on output, consumption, Tobin's \( q \), investment, domestic absorption, the real interest rate, external debt, and the capital stock are reported in Figures 3 and 4. In both cases, the key variable that drives the short-run dynamics is the real interest rate (which increases on impact) because of its effect on intertemporal decisions.

\[22\] See Armstrong et al. (1998) for a detailed discussion of this procedure.

\[23\] For the numerical simulation, the instantaneous utility function appearing in equation (1) has been modified to \( c^{1-\eta}/(1 - \eta) + \ln \lambda m^\eta \), in order to distinguish between labor supply and money demand elasticities. Consequently, the money demand function is derived from the condition \( i = \alpha c^\eta/m \) and the labor supply function from \( n^* = T - n^* \), where \( \lambda = T - n^* \) and \( T \) denotes the time endowment.

20
Consider first a permanent shock to the devaluation rate. As shown in Figure 3, consumption, output, external debt and the real interest rate behave in the same pattern as in the initial model without capital accumulation. In fact, the mechanism behind these dynamics has not changed. The drop in the devaluation rate leads to an initial increase in private foreign borrowing which drives an increase in official reserves (and consequently a rise in the real money stock) since foreign debt remains constant on impact. This leads to a fall in the nominal interest rate, but to a smaller amount than the fall in the devaluation rate (this is due to money demand elasticity and to the risk premium), so that the real interest rate unambiguously rises on impact. This drives the rest of the economy. Households have an incentive to shift consumption toward the future. The fall in initial consumption increases labor supply, which lowers the real wage rate and boosts production. The long-run equilibrium is also in line with the new steady-state results of the initial model.

Introducing capital accumulation, however, may alter the effect on the current accounts and the external debt. The behavior of investment subsequent to a devaluation shock is in line with the evidence with the boom-recession cycle. As labor supply and output rise, so does investment due to its complementarity to the production process. Since in the extended model investment enters the external debt equation (see Equation (48)), the overall effect on the current account is ambiguous and will depend on the values of parameters. With a low level of intertemporal elasticity of substitution in consumption (in line with empirical evidence; see Agenor and Montiel (1999)), the initial drop in consumption is relatively small (compare in Figure 3 the results for the different values of $\sigma$). Instead, given a relatively low installation cost, investment have a more pronounced boom that outweighs consumption in shaping the behavior of domestic absorption (which in Figure 3 follows a boom-recession pattern). Depending on the parameters used (in Figure 3 only $\sigma$ varies), domestic absorption may outweigh the effects of output causing trade deficit and consequently a current account deficit.

Figure 4 shows the effects of a temporary disinflation shock, that is, a reduction in the devaluation rate for ten periods only. As can be observed, the results obtained for the first phase of the adjustment process are qualitatively similar to those obtained with a permanent shock. A key difference in the present case is that the real interest rate experiences a discrete drop once the shock is removed, that is, at period 10. On impact, and during the first few periods of the adjustment process, all variables behave in a way similar
to what is obtained with the permanent shock. Over time, all variables now return to their initial baserun values, including the level of external debt and the stock of capital. In particular, the initial sequence of current account surpluses is followed by a sequence of current account deficits.

In interpreting movements in the current account balance in this model, it is important to keep in mind that we have assumed the elasticity of substitution between labor and capital to be relatively high (0.56). With lower values of that elasticity, it would be possible for the model to generate trade and current account deficits on impact with a temporary shock, and possibly with a permanent shock as well; the reason is that with now two negative roots, adjustment even with a permanent shock does not need to be monotonic.\textsuperscript{24} With a temporary shock, and even with the current value of the capital-labor elasticity of substitution, it is also possible to generate external deficits on impact depending on the length of the period during which the shock is maintained—very much like in the experiment described in Figure 2.

Thus, numerical simulations of a temporary reduction in the devaluation rate appear to replicate fairly well the boom-recession cycle in output and domestic absorption associated with the type of exchange rate-based stabilization programs often implemented in developing countries. Although consumption, once again, displays a small downward jump on impact, it does follow afterward an inverted-U pattern, as suggested by the evidence. The key assumptions needed for obtaining these results are a low value of the intertemporal elasticity of substitution in consumption (as observed in empirical studies) and low investment installation costs.

6 Summary and Conclusions

This paper studied the role of supply-side factors in the dynamics of output and domestic absorption in exchange-rate based stabilization programs. First, a basic framework was presented to illustrate the main differences between this study and some of the existing analytical literature on these programs. In particular, the economy was modeled using an infinite-horizon, intertemporal optimizing framework in which domestic households face imperfect world capital markets. With imperfect capital mobility, domestic

\textsuperscript{24}In the model without capital accumulation, a permanent shock could induce only monotonic movements in the stock of external debt because of the existence of only one negative root.
interest rates are determined by the equilibrium condition of the domestic money market. Consequently, this framework does not require the rate of time preference to be equal at all times to the world interest rate to ensure a stationary solution. Therefore, it has the advantage of not having steady-state solutions that depend on initial values. It was shown that a temporary reduction in the devaluation rate leads to a boom-recession cycle in output and consumption (in the latter case following an initial, short-lived downward movement) and a sequence of current account deficits, all in line with evidence.

The basic framework was then further extended to account for capital accumulation in the presence of investment installation costs. We showed that the extended model was able to replicate a boom-recession behavior for investment as well, as suggested by the evidence.

The major economic variable that our analysis of exchange-rate based stabilization fails to describe is the real exchange rate. This limitation is due to the fact that we model a one-good economy only. However, this framework can be further developed to include traded and non-traded sectors, as in some existing contributions. With sticky prices in the nontradables sector, for instance, a reduction of the devaluation rate would directly affect prices in the traded sector only, implying that overall inflation would not fall by as much as the devaluation rate. This would lead create an appreciation of the real exchange rate, as documented in the evidence. The distinction between tradables and nontradables would provide further insight on the behavior of consumption and the current account, notably by allowing for intratemporal substitution, in addition to consumption smoothing considerations.
Appendix I
Basic Framework:
Impact and Steady-State Effects

The saddlepath solution to the system is given by

\[ c - \tilde{c} = \kappa(D^* - \tilde{D}^*), \quad (A1) \]

where \( \kappa \equiv (\nu - \Psi_{D^*})/\Psi_c = G_{D^*}/(\nu - G_c) < 0 \) and \( \nu \) denotes the negative root of the system.

It can be established that

\[ \frac{d\tilde{c}}{d\varepsilon} = (\Psi_c G_{D^*} - \Psi_{D^*} G_c)/\Omega < 0, \quad (A2) \]

\[ \frac{d\tilde{D}^*}{d\varepsilon} = (\Psi_c G_{\varepsilon} - \Psi_{\varepsilon} G_c)/\Omega, \quad (A3) \]

where, as shown in the text, \( G_{\varepsilon}, \Psi_{\varepsilon} < 0 \) and (for stability) \( \Omega = \Psi_{D^*} G_c - \Psi_{\varepsilon} G_{D^*} < 0 \). To show that \( \frac{d\tilde{c}}{d\varepsilon} < 0 \) requires showing that \( \frac{\Psi_c G_{D^*} - \Psi_{D^*} G_c}{\Omega} > 0 \) or that

\[ \frac{\Psi_c}{\Psi_{D^*}} > \frac{G_{\varepsilon}}{G_{D^*}} = \frac{(i_m \varphi_{\varepsilon} - 1)/i_m \varphi_{D^*}}{1/i_m}, \]

or equivalently

\[ (\bar{\theta} + \bar{l}^* \theta_{l^*}) \Lambda_{\varepsilon} > i_m^{-1} \left[ i^* + (\bar{\theta} + \bar{l}^* \theta_{l^*}) \Lambda_{D^*} \right]. \]

Again, with \( \Lambda_{D^*} = -i_m \beta \) and \( \Lambda_{\varepsilon} = -\beta \):

\[ -\beta (\bar{\theta} + \bar{l}^* \theta_{l^*}) > i_m^{-1} \left[ i^* - i_m \beta (\bar{\theta} + \bar{l}^* \theta_{l^*}) \right], \]

or \( i^*/i_m < 0 \), which always holds because \( i_m < 0 \). From the steady-state condition (30), \( \tilde{d}/d\varepsilon = 1 \). From (33):

\[ \tilde{m}/d\varepsilon = m_c \tilde{c}/d\varepsilon + m_i < 0, \]

which implies that \( \tilde{R}^*/d\varepsilon = \tilde{m}/d\varepsilon < 0 \). This, in turn, implies that, because \( \tilde{d}/d\varepsilon = 0 \):

\[ \tilde{D}^*/d\varepsilon = -\tilde{R}^*/d\varepsilon > 0. \]
On impact, using (A1) and (A2) and noting that $dD_0^*/d\varepsilon = 0$ and $G_D + \kappa G_c = \kappa \nu$:

$$dc_0/d\varepsilon = d\bar{c}/d\varepsilon - \kappa (d\bar{D}^*/d\varepsilon) = -\nu (G_e - \kappa \Psi_e)/\Omega > 0.$$  

From the equilibrium condition of the money market

$$di_0/d\varepsilon = (i_c + i_m \varphi_c) (dc_0/d\varepsilon) + i_m \varphi_e > 0,$$

(A4)

Since $i_c + i_m \varphi_c$ and $i_m \varphi_e$ are both positive. It can be established that $di_0/d\varepsilon \rightarrow 1$ when $\gamma \rightarrow 0$, and that $di_0/d\varepsilon < 1$ for $\gamma > 0$.

Finally, from equation (22), and given that $di_0/d\varepsilon < 1$:

$$dl^*_0/d\varepsilon = \gamma^{-1} \{(di_0/d\varepsilon) - 1\} < 0.$$  

Because $dD_0^*/d\varepsilon = 0$,

$$dR_0^*/d\varepsilon = dm_0/d\varepsilon = dl^*_0/d\varepsilon < 0.$$
Appendix II
Extended Framework:
Stability Conditions and Steady-State Effects

Using the solution for $l^*$ derived in Section III, the current account equation (48) can be written as

$$\dot{D}^* = i^*D^* + \theta[\Lambda(c, D^*, \varepsilon)] \Lambda(c, D^*, \varepsilon) + c + g + kv(q) - y^s(c, k),$$

that is

$$\dot{D}^* = \Psi(\dot{c}, \dot{D}^*, \dot{q}, \dot{k}, \varepsilon),$$  \hspace{1cm} (B1)

with $\Psi_{D^*}$ and $\Psi_{\varepsilon}$ as defined in the text and now

$$\Psi_c = 1 - (\partial y^s/\partial c) + (\tilde{\theta} + i^*\theta^*) \Lambda_c, \quad \Psi_q = \tilde{k}v', \quad \Psi_k = -\partial y^s/\partial k,$$

The dynamic system now consists of (26), (45), (47), and (B1). Taking a linear approximation around the initial steady state yields

$$\begin{bmatrix} \dot{c} \\ \dot{q} \\ \dot{D}^* \\ \dot{k} \end{bmatrix} = \begin{bmatrix} G_c & 0 & G_{D^*} & 0 \\ -h_c & \rho & 0 & -h_k \\ \Psi_c & \Psi_q & \Psi_{D^*} & \Psi_k \\ 0 & \tilde{k}/\phi & 0 & 0 \end{bmatrix} \begin{bmatrix} c - \tilde{c} \\ q - 1 \\ D^* - \tilde{D}^* \\ k - \tilde{k} \end{bmatrix},$$  \hspace{1cm} (B2)

where, in the neighborhood of the initial steady state, $\Psi_q = \tilde{k}/\phi$ and $\Psi_k = -\rho$.

Let $\mathbf{A}$ denote the matrix of coefficients on the right-hand side of (B2). We have

$$\text{tr} \mathbf{A} = G_c + \Psi_{D^*} + \rho > 0, \quad (B3)$$

$$\det \mathbf{A} = \frac{\tilde{k}}{\phi} [h_cG_{D^*}\Psi_k + h_k(G_c\Psi_{D^*} - \Psi_{c}G_{D^*})] > 0, \quad (B4)$$

where in (B4) it is assumed that the condition $G_c\Psi_{D^*} - \Psi_{c}G_{D^*} < 0$, which was shown to be necessary for saddlepath stability in the basic model, holds. From (B3), it follows that there is at least one positive characteristic root. Given (B3), (B4) indicates that there are either four positive roots, or two positive and two negative roots.\(^{25}\) The former case implies that the system

\(^{25}\)The case of four negative roots, which also gives $\det \mathbf{A} > 0$, can be excluded given (B3).
is unstable and can be ruled out. But the conditions guaranteeing two pairs of roots of opposite sign cannot be pinned down analytically. However, the values used in the numerical simulations always generated stable adjustment paths.\textsuperscript{26}

The steady-state effects of the extended model are parallel to those obtained with the basic framework. Because the value of the Tobin's q remains unchanged in the steady state ($q = 1$), the steady-state value of capital is a function of consumption only ($k = s(c)$ where $s' < 0$; see (50)). Because of this inverse relationship between consumption and capital, the steady-state condition that determines the stock of external debt remains qualitatively unchanged and can be also reduced to be a function of consumption only ($D^* = d(c)$, where $d' < 0$). Finally, the steady-state value of consumption can be shown to be negatively related to a reduction in the devaluation rate, just as in Appendix I.

\textsuperscript{26}The "saddlepath" is, in this case, a two-dimensional subspace of the four-dimensional space spanned by the extended model.
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——, “On Gradual Disinflation, the Real Exchange Rate, and the Current Account,” *Journal of International Money and Finance*, 16 (February 1997), 37-54.
Figure 1
Steady-State Equilibrium
Figure 2
Reduction in the Devaluation Rate

Graph showing economic analysis with axes for wages (w), output (D*), consumption (c), and savings (y_s). Points H, F, E, and B represent different economic states and transitions under the conditions of devaluation rate changes (\( \dot{c} = 0 \) and \( \dot{D} = 0 \)). The diagram illustrates the impact of devaluation on the economy's equilibrium and the adjustment process.
Figure 3. Permanent Shock
Figures are in percentage change from base run value (except Real Interest Rate)

- **REAL OUTPUT**
- **NET INVESTMENT**
- **CONSUMPTION**
- **DOMESTIC ABSORPTION**
- **TOBIN'S Q**
- **REAL INTEREST RATE**
- **EXTERNAL DEBT**
- **CAPITAL STOCK**

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*Net Investment is presented in proportion to Output.*
Figure 4. Temporary Shock
Figures are in percentage change from base run value (except Real Interest Rate)

* Net Investment is presented in proportion to Output.
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